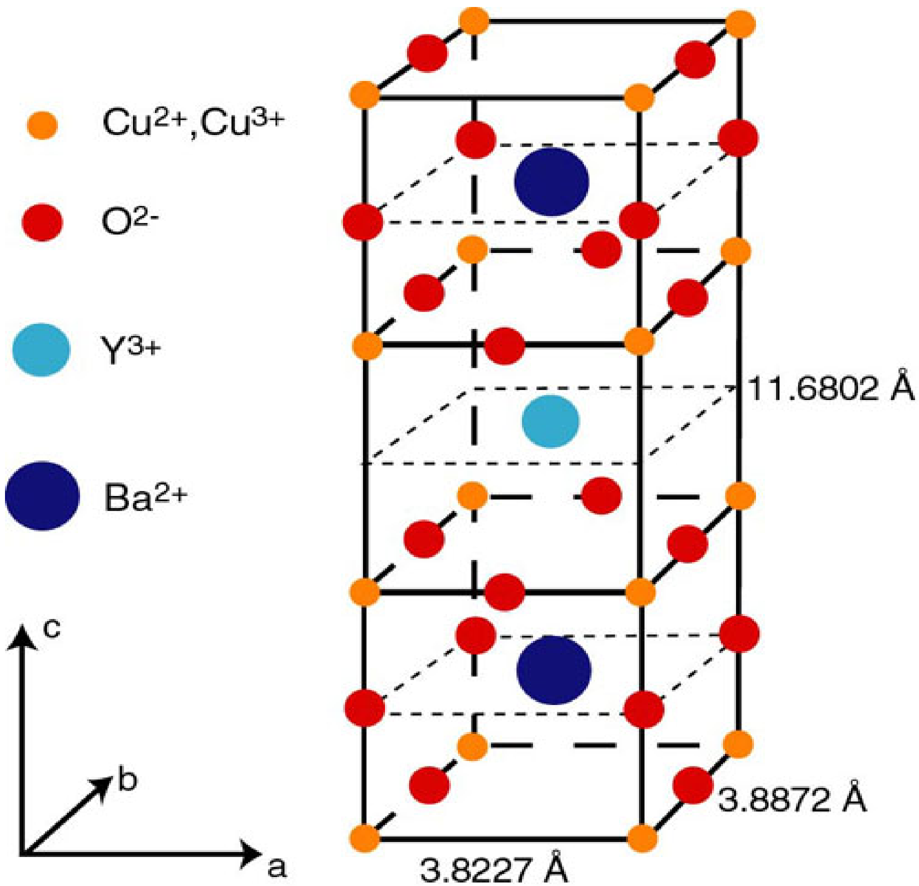
Technische Universität Graz

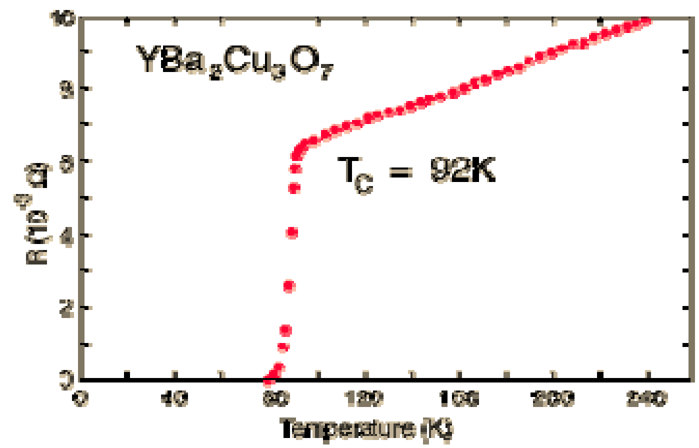
18. Superconductivity

Dec. 3, 2018

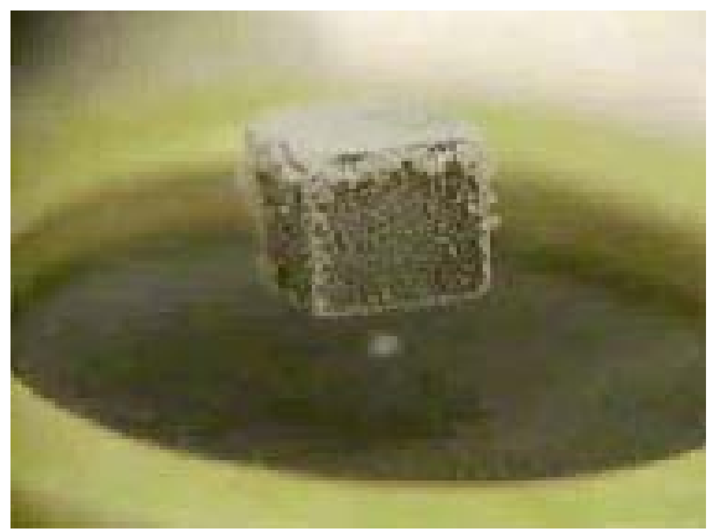
$YBa_2Cu_3O_x$



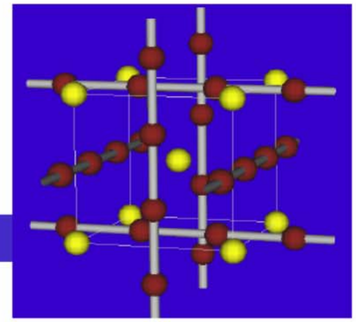
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

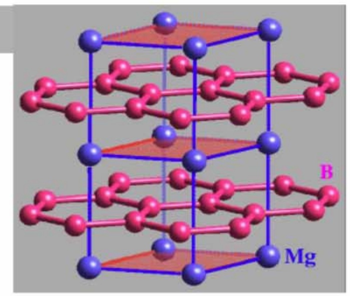


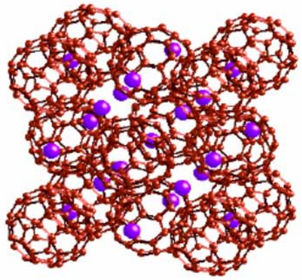
YBa2Cu3O7
Tc = 92K
Temperature (K)



	Material	T_c
• Legierung:	NbTi	9,6 K
• Verbindungen:	NbN	16,0 K
Borocarbide:	$(Lu/Y)Ni_2B_2C$	16,0 K
"A15"-Strukturen:	Nb_3Sn	18,0 K
(= β-Wolfram-Struktur)	Nb_3Al	18,7 K
	Nb_3Ge	22,5 K
neu:	MgB_2	39 K
Fullerene:	Cs_2RbC_{60}	33 K
+ Druck 15 kbar:	Cs_3C_{60}	40 K

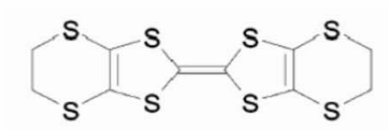


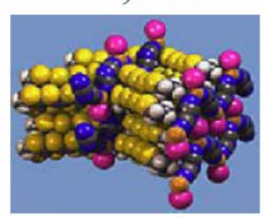




Organische Supraleiter:

$(BEDT\text{-}TTF)_2Cu[N(CN)_2]Br$ 11,2 K





Polymere

hochdotierte Halbleiter

http://www.wmi.badw.de/teaching/Lecturenotes/index.html

Compound	T_c, in K	Compound	T_c, in K
Nb_3Sn	18.05	V_3Ga	16.5
Nb_3Ge	23.2	V_3Si	17.1
Nb_3Al	17.5	$YBa_2Cu_3O_{6.9}$	90.0
NbN	16.0	Rb_2CsC_{60}	31.3
K_3C_{60}	19.2	MgB_2	39.0

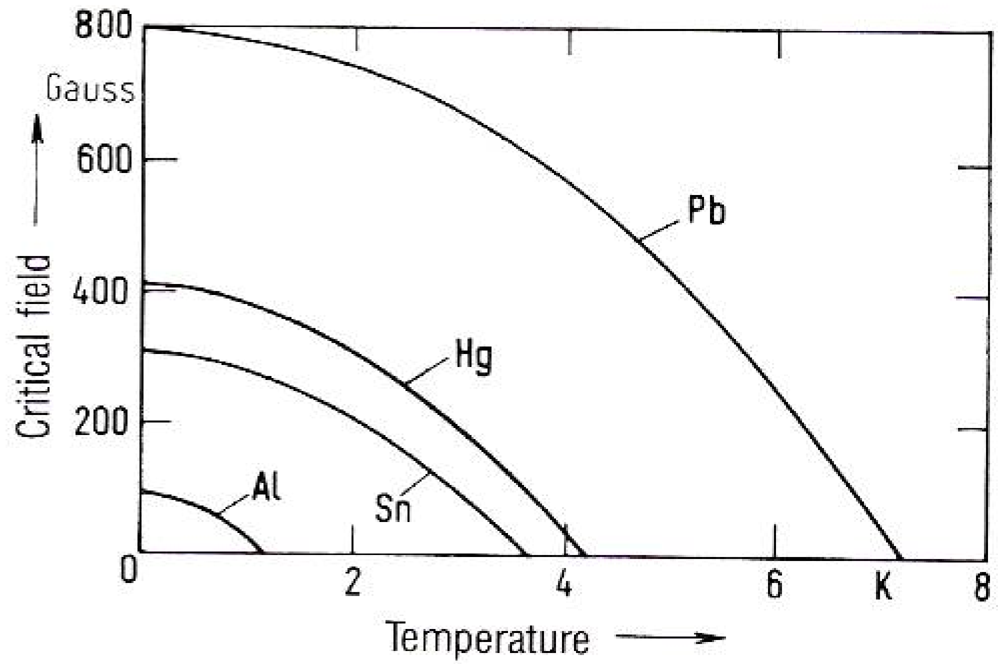
$BaPb_{0.75}Bi_{0.25}O_3$	$T_c = 12$ K	[BPBO]
$La_{1.85}Ba_{0.15}CuO_4$	$T_c = 36$ K	[LBCO]
$YBa_2Cu_3O_7$	$T_c = 90$ K	[YBCO]
$Tl_2Ba_2Ca_2Cu_3O_{10}$	$T_c = 120$ K	[TBCO]
$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	$T_c = 138$ K	
$(Sn_5In)Ba_4Ca_2Cu_{10}O_y$	$T_c = 212$ K	

Superconductivity

Critical temperature T_c

Critical current density J_c

Critical field H_c



$$n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \tfrac{1}{2} nmv^2 = \frac{m}{2ne^2} J_c^2$$

Superconductivity

Perfect diamagnetism

Jump in the specific heat like a 2nd order phase transition, not a structural transition

Superconductors are good electrical conductors but poor thermal conductors, electrons no longer conduct heat

There is a dramatic decrease of acoustic attenuation at the phase transition, no electron-phonon scattering

Dissipationless currents - quantum effect

Electrons condense into a single quantum state - low entropy.

Electron decrease their energy by Δ but loose their entropy.

Probability current

Schrödinger equation for a charged particle in an electric and magnetic field is

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - qA)^2\psi + V\psi$$

write out the $(-i\hbar\nabla - qA)^2\psi$ term

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-\hbar^2\nabla^2 + i\hbar qA\nabla + ihq\nabla A + q^2A^2\right)\psi + V\psi$$

write the wave function in polar form

$$\psi = |\psi|e^{i\theta}$$

$$\nabla\psi = \nabla|\psi|e^{i\theta} + i\nabla\theta|\psi|e^{i\theta}$$

$$\nabla^2\psi = \nabla^2|\psi|e^{i\theta} + 2i\nabla\theta\nabla|\psi|e^{i\theta} + i\nabla^2\theta|\psi|e^{i\theta} - \left(\nabla\theta\right)^2|\psi|e^{i\theta}$$

Probability current

Schrödinger equation becomes:

$$i\hbar\frac{\partial|\psi|}{\partial t}-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(\nabla^2|\psi|+2i\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)$$

$$+i\hbar qA\left(\nabla|\psi|+i\nabla\theta|\psi|\right)+ihq\nabla A|\psi|+q^2A^2|\psi|\Big]+V|\psi|$$

Real part:

$$-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{-\hbar^2}{2m}\left(\nabla^2-\left(\nabla\theta-\frac{q}{\hbar}\vec{A}\right)^2\right)|\psi|+V|\psi|$$

Imaginary part:

$$\hbar\frac{\partial|\psi|}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(2\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)+2\hbar qA\nabla|\psi|+\hbar q|\psi|\nabla A\Big]$$

Probability current

Imaginary part:

$$\hbar \frac{\partial |\psi|}{\partial t} = \frac{1}{2m}\left[-\hbar^2 \left(2\nabla\theta\nabla|\psi| + i\nabla^2\theta|\psi| - (\nabla\theta)^2|\psi| \right) + 2\hbar q A\nabla|\psi| + \hbar q|\psi|\nabla A \right]$$

Multiply by |ψ| and rearrange

$$\frac{\partial}{\partial t}|\psi|^2 + \nabla\cdot\left[\frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)\right] = 0$$

This is a continuity equation for probability

$$\frac{\partial P}{\partial t} + \nabla\cdot\vec{S} = 0$$

The probability current: $$\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$$

Probability current / supercurrent

The probability current: $\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$

This result holds for all charged particles in a magnetic field.

In superconductivity the particles are Cooper pairs $q = -2e$, $m = 2m_e$, $|\psi|^2 = n_{cp}$.

All superconducting electrons are in the same state so

$$\vec{j} = -2en_{cp}\vec{S}$$

$$\boxed{\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)}$$

London gauge $\nabla\theta = 0$

$$\vec{j} = \frac{-2n_{cp}e^2}{m_e}\vec{A} = \frac{-n_s e^2}{m_e}\vec{A} \qquad n_s = 2n_{cp}$$

1st London equation

Heinz & Fritz

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\frac{d\vec{j}}{dt} = \frac{-n_s e^2}{m_e} \frac{d\vec{A}}{dt} = \frac{n_s e^2}{m_e} \vec{E} \qquad \frac{d\vec{A}}{dt} = -\vec{E}$$

First London equation:

$$\boxed{\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e} \vec{E}}$$

Classical derivation:

$$-e\vec{E} = m \frac{d\vec{v}}{dt} = -\frac{m}{n_s e} \frac{d\vec{j}}{dt}$$

$$\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e} \vec{E}$$

2nd London equation

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \nabla \times \vec{A}$$

Second London equation:

$$\boxed{\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B}}$$

Meissner effect

Combine second London equation with Ampere's law

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B} \qquad \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla \times \nabla \times \vec{B} = \frac{-n_s e^2 \mu_0}{m_e} \vec{B}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \left(\nabla \cdot \vec{B} \right) - \nabla^2 \vec{B}$$

Helmholtz equation: $\lambda^2 \nabla^2 \vec{B} = \vec{B}$

London penetration depth: $\lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$

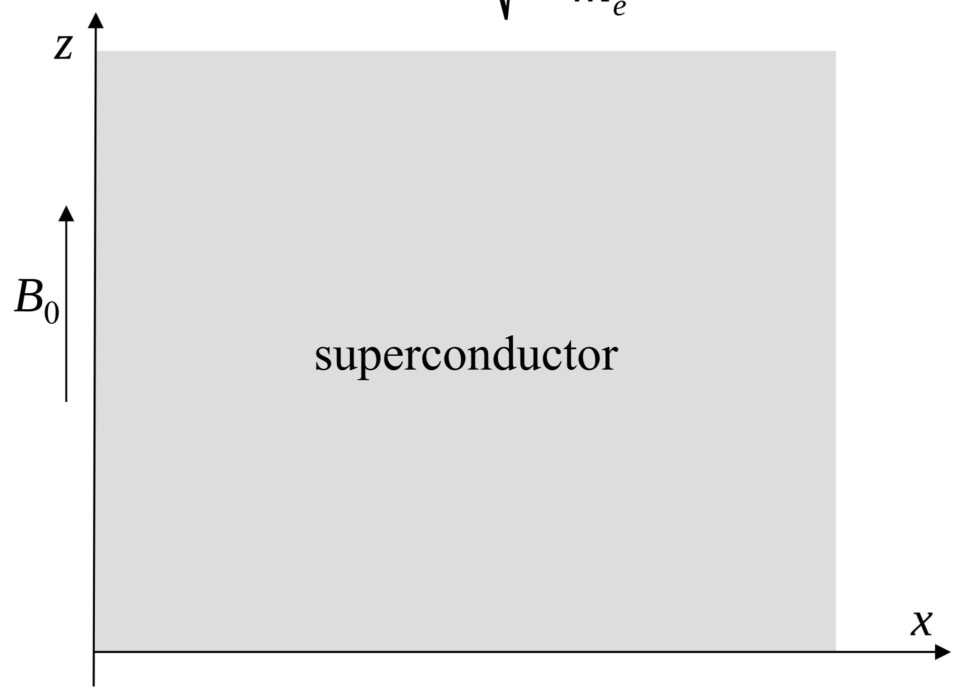
Meissner effect

$$\lambda^2 \nabla^2 \vec{B} = \vec{B} \qquad \lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$$

solution to Helmholtz equation:

$$\vec{B} = \vec{B}_0 \exp\left(\frac{-x}{\lambda}\right)\hat{z}$$

Al	λ = 50 nm
In	λ = 65 nm
Sn	λ = 50 nm
Pb	λ = 40 nm
Nb	λ = 85 nm



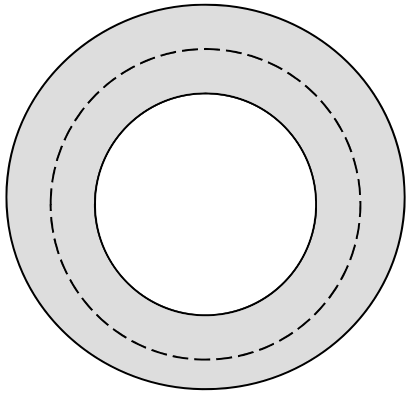
$$\nabla \times \vec{B} = \mu_0 \vec{j} \qquad \vec{j} = \frac{\vec{B}_0}{\mu_0 \lambda} \exp\left(\frac{-x}{\lambda}\right)\hat{y}$$

Flux quantization

$$\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$

For a ring much thicker than the penetration depth, $j = 0$ along the dotted path.

$$0 = \left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$



Integrate once along the dotted path.

$$\oint \nabla\theta \cdot d\vec{l} = -\frac{2e}{\hbar}\oint \vec{A}\cdot d\vec{l} = -\frac{2e}{\hbar}\int_S \nabla\times\vec{A}\cdot d\vec{s} = -\frac{2e}{\hbar}\int_S \vec{B}\cdot d\vec{s} = -\frac{2e}{\hbar}\Phi$$

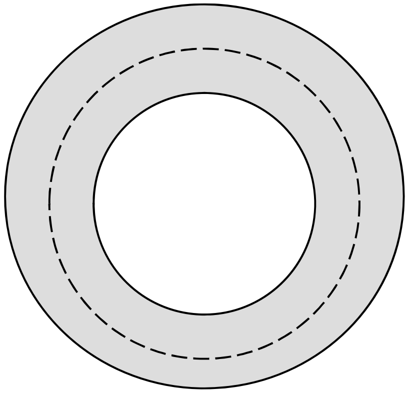
Stokes' theorem

magnetic flux

Flux quantization

$$\oint \nabla\theta \cdot d\vec{l} = 2\pi n = -\frac{2e}{\hbar}\Phi$$

$n = \ldots -2, -1, 0, 1, 2 \ldots$



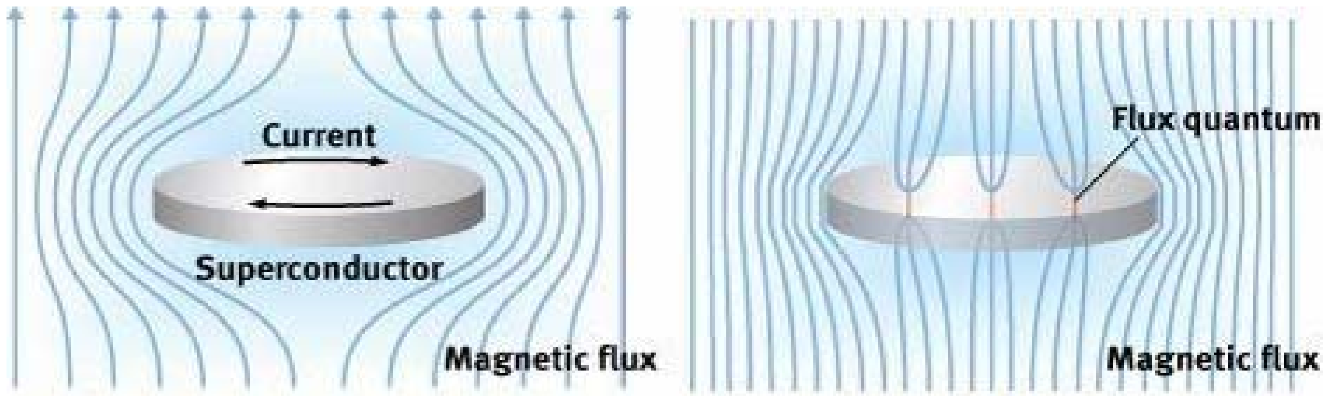
$$2\pi n = \frac{2e}{\hbar}\Phi = \frac{\Phi}{\Phi_0}$$

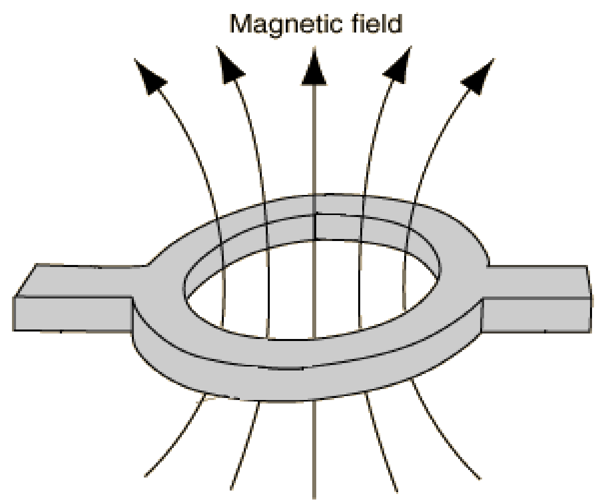
Flux quantization: $\boxed{\Phi = n\Phi_0}$

$$\Phi_0 = \frac{h}{2e} = 2.0679 \times 10^{-15}\ [\mathrm{W} = \mathrm{T\ m}^2]$$

Superconducting flux quantum

Flux quantization

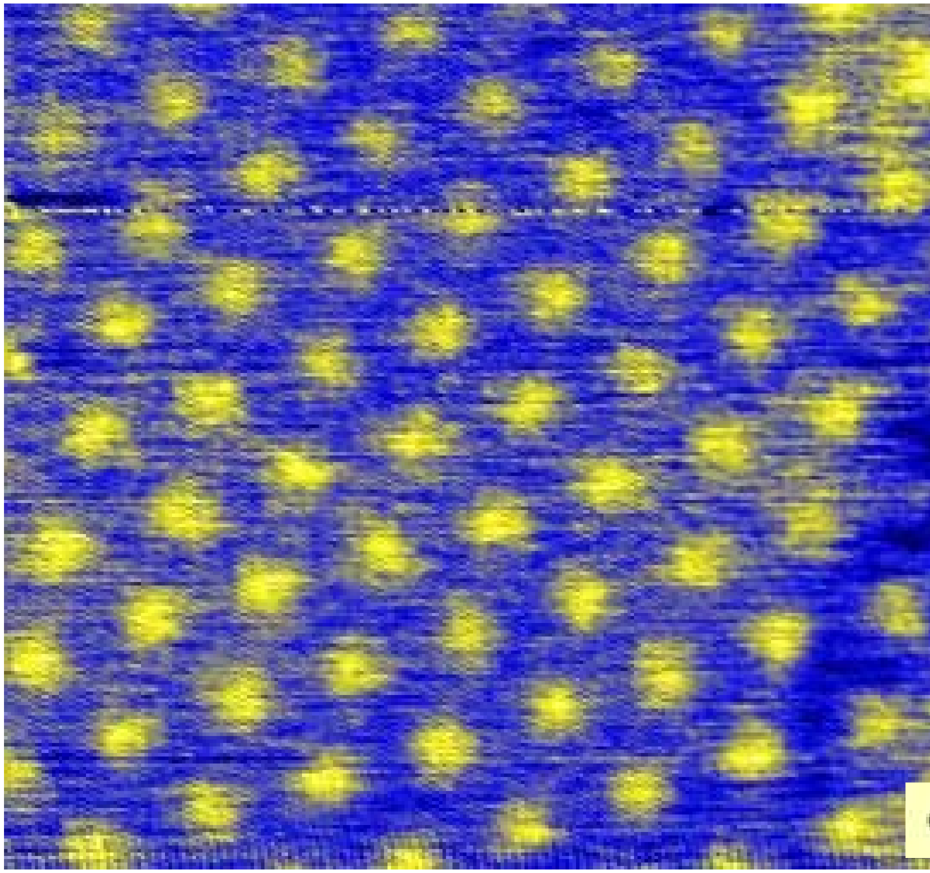


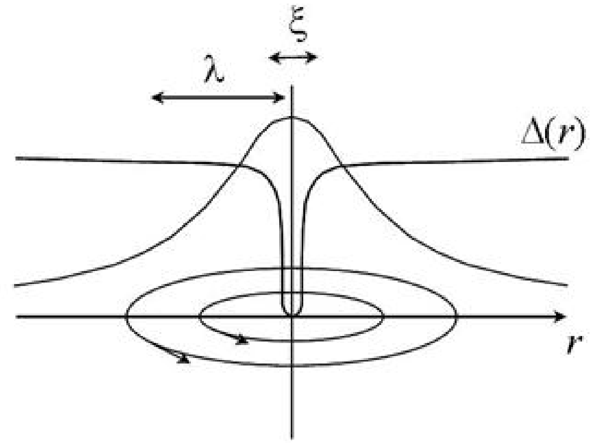


$$\Phi = n\Phi_0 = \frac{nh}{2e}$$

Flux is quantized through a superconducting ring.

Vortices in Superconductors



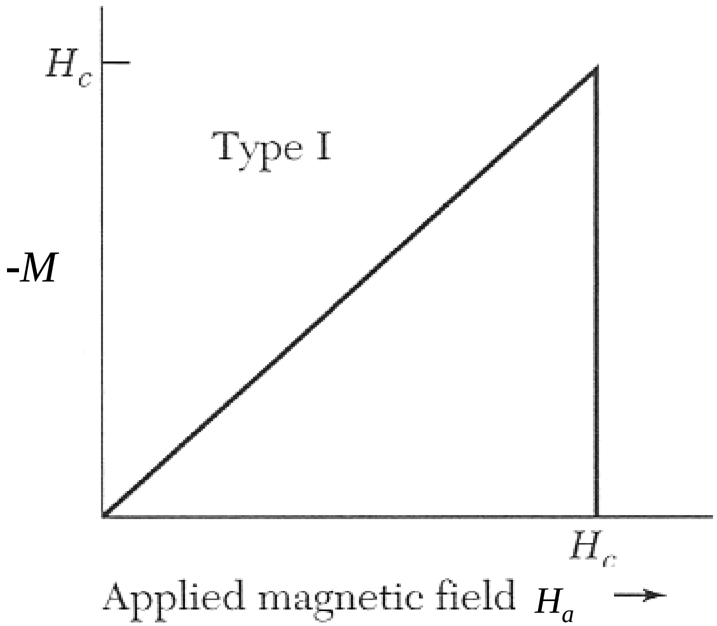


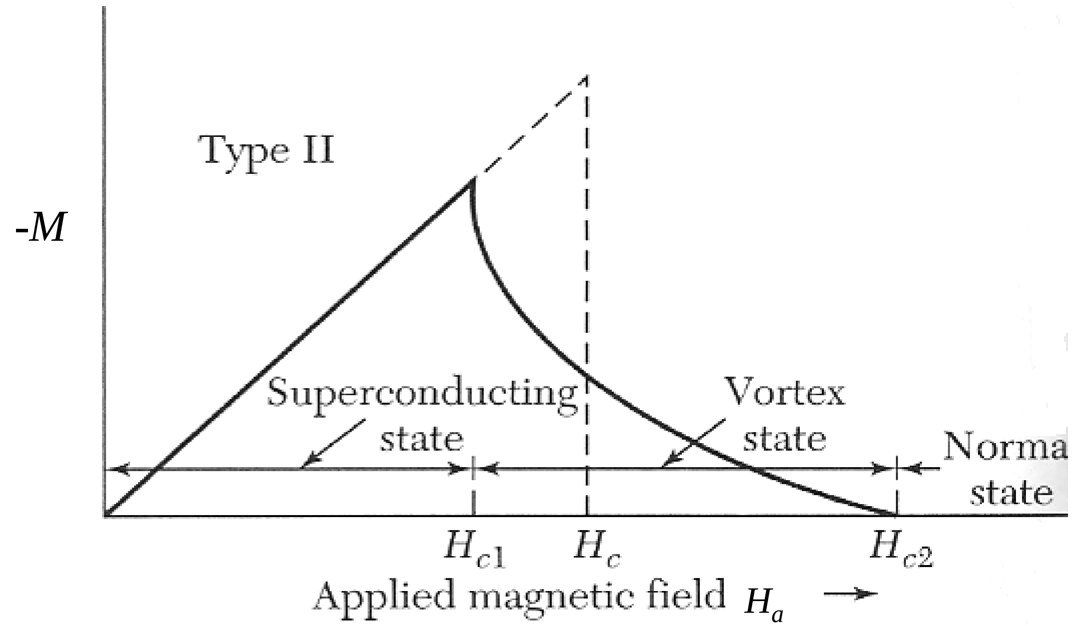
$$\Phi_0 = h/2e \simeq 2 \times 10^{-15} \text{ Tesla m}^2$$

STS image of the vortex lattice in $NbSe_2$.
(630 nm x 500 nm, B = .4 Tesla, T = 4 K)

http://www.insp.upmc.fr/axe1/Dispositifs%20quantiques/AxeI2_more/VORTICES/vortexHD.htm

Type I and Type II

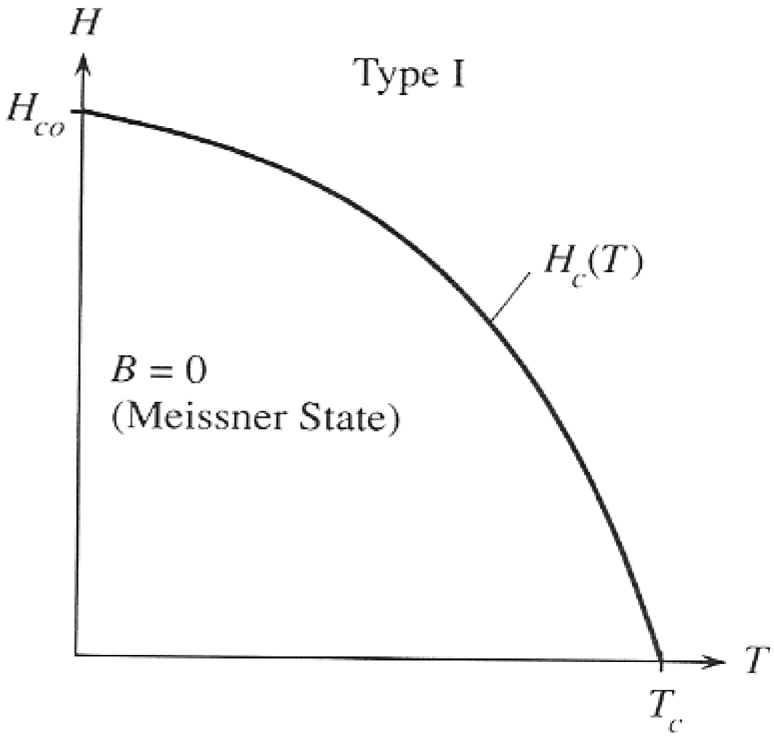


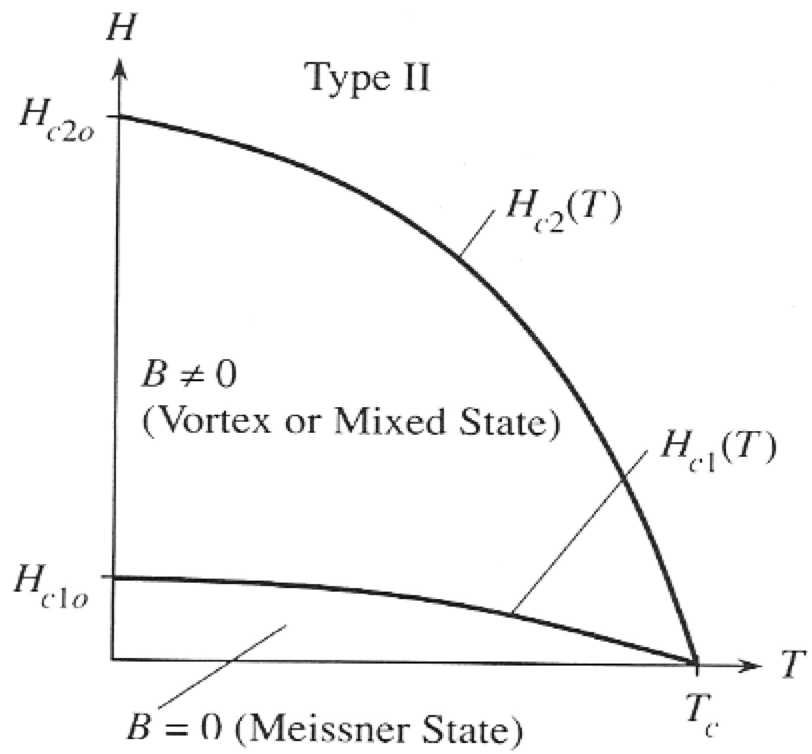


$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$$

Superconductors are perfect diamagnets at low fields.
B=0 inside a bulk superconductor.

Type I and Type II



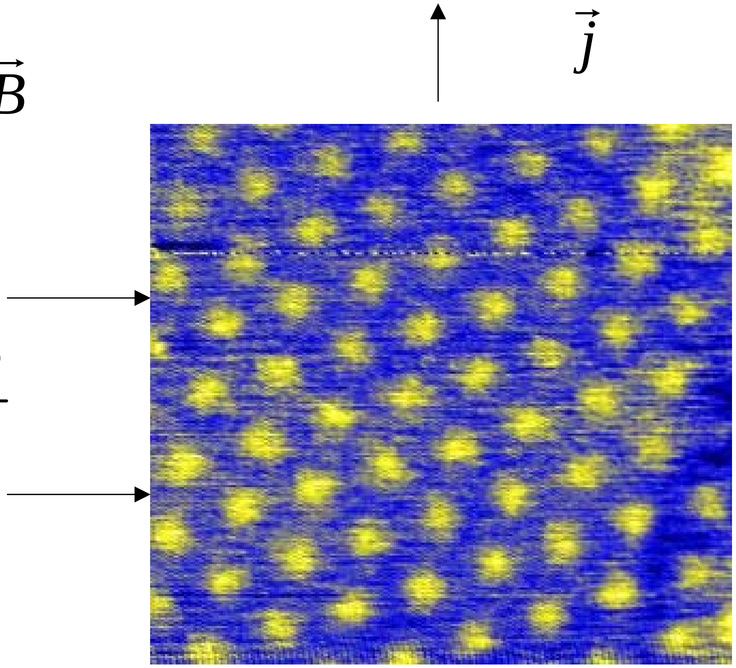


Vortices in Superconductors

Lorentz force $$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{j} = nq\vec{v}$$

$$\vec{F} = \frac{1}{n}\vec{j} \times \vec{B}$$



Faraday's law $$V = -\frac{d\Phi}{dt}$$

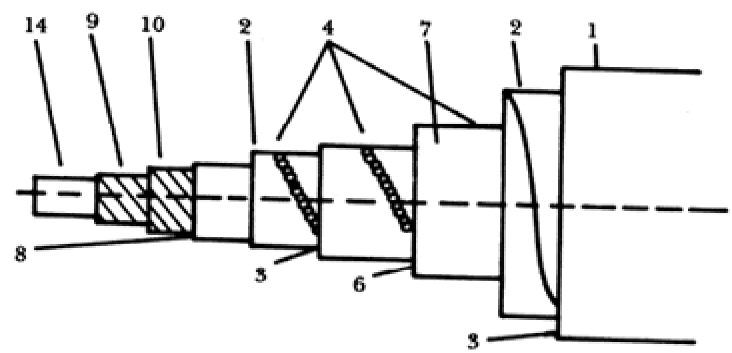
Defects are used to pin the vortices

Superconducting Magnets

Whole body MRI

Magnets and cables

Maglev trains



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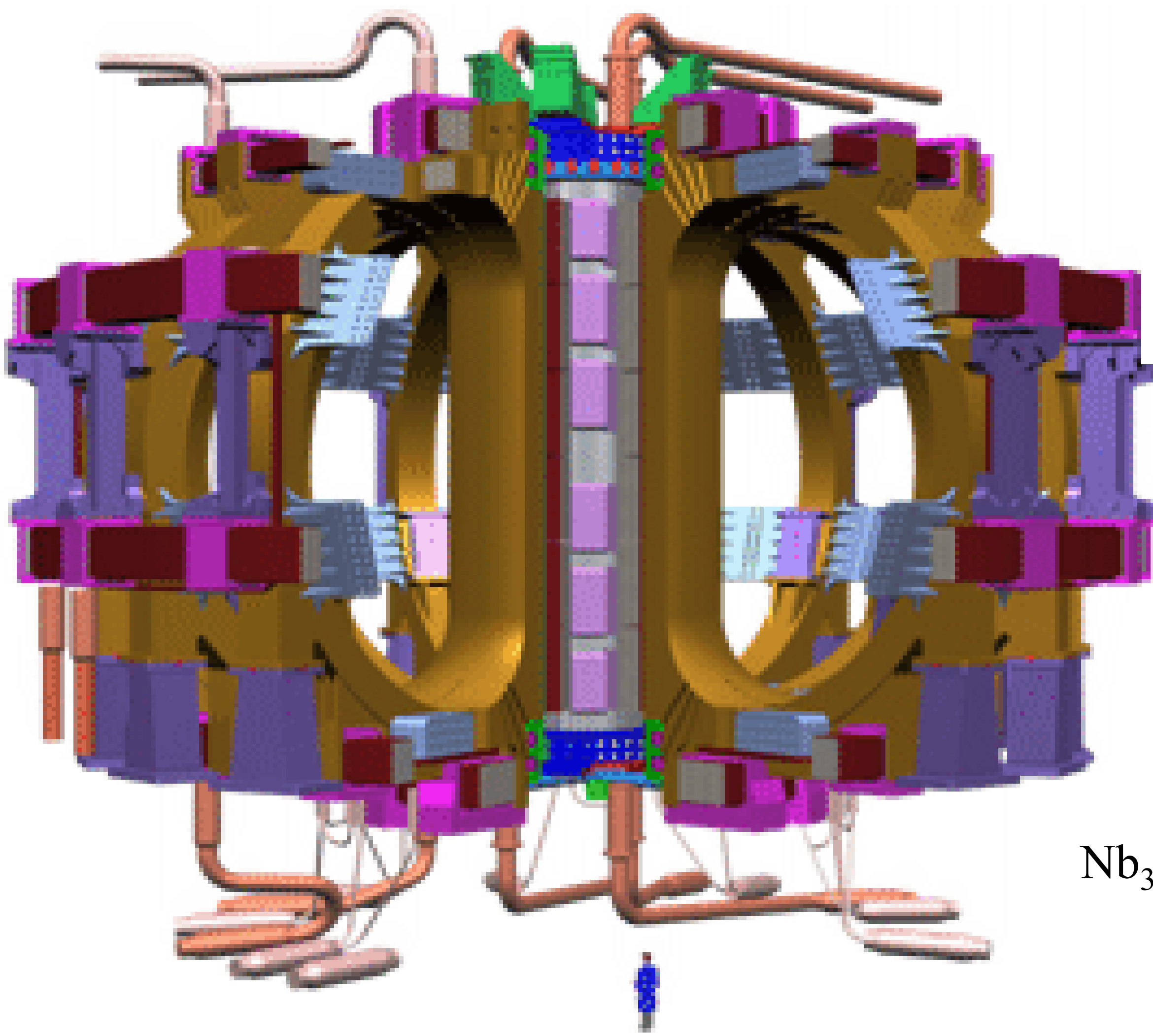
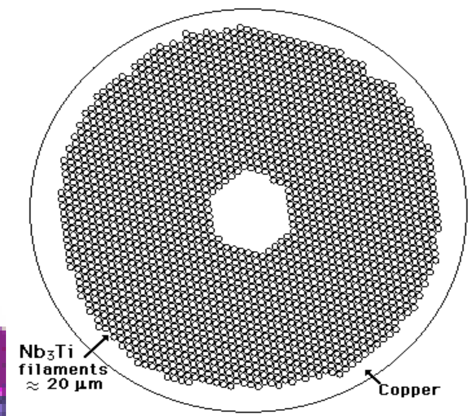
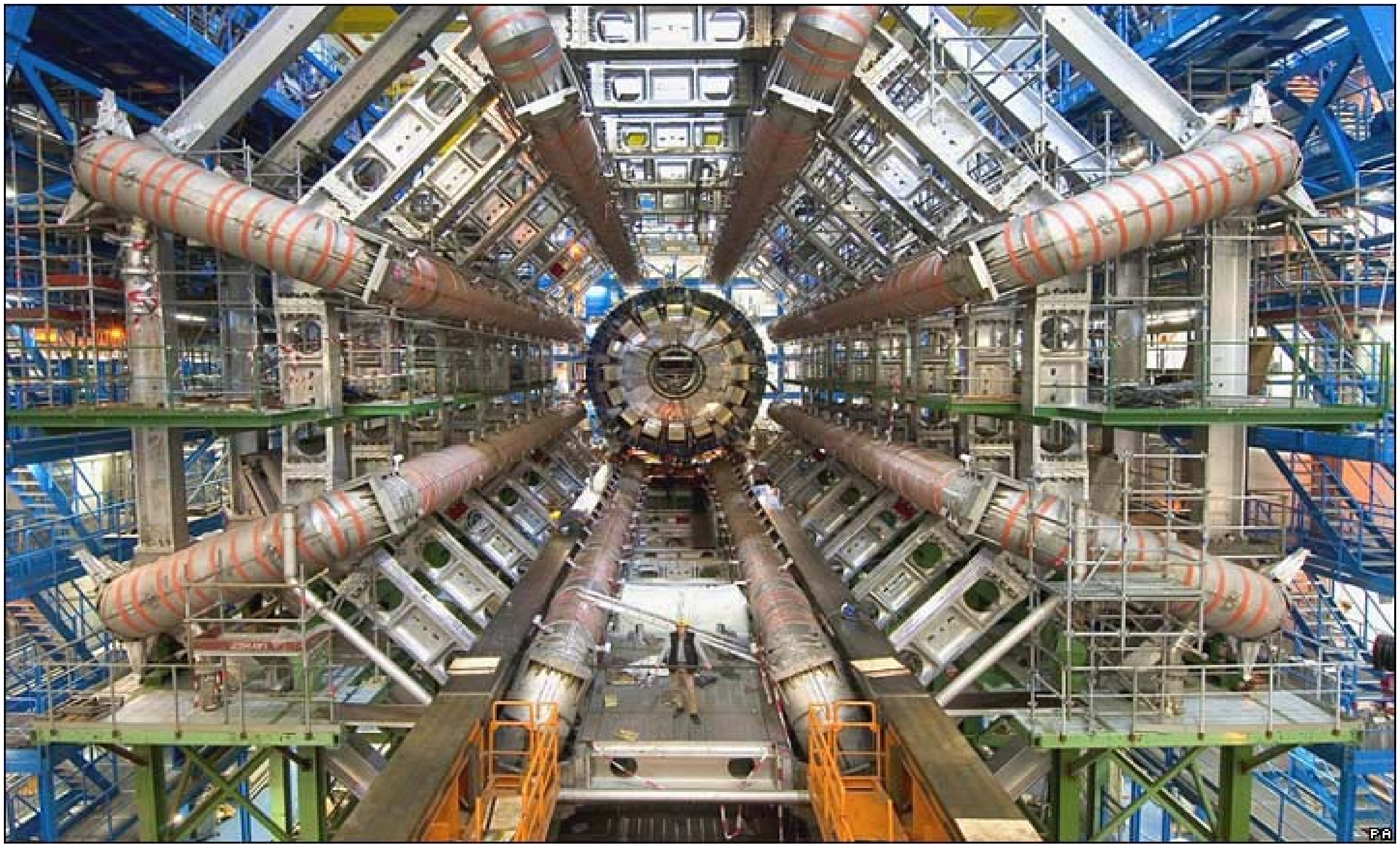




Magnet wire

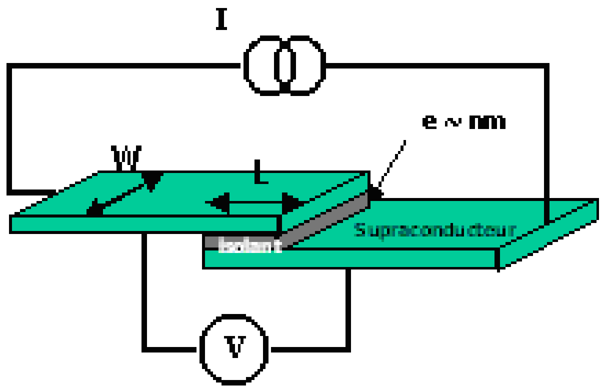
Nb_3Sn Magnet

Superconducting magnets

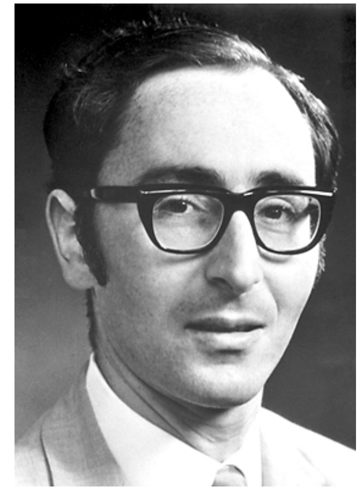


Largest superconducting magnet, CERN
21000 Amps

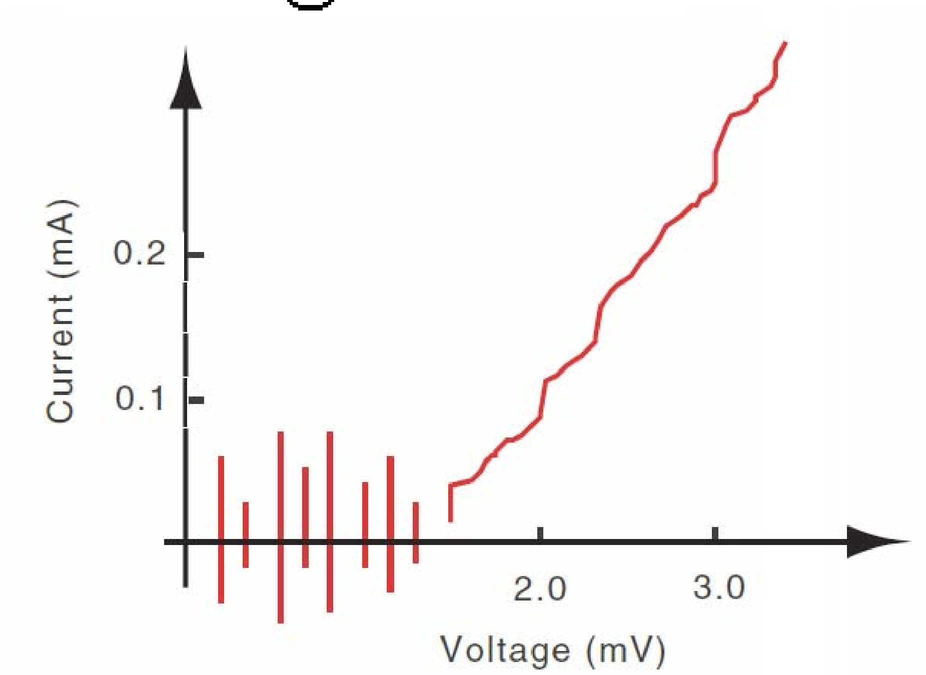
ac - Josephson effect



$$V = -\frac{d\Phi}{dt} = n\Phi_0 f = \frac{nhf}{2e}$$



Brian Josephson



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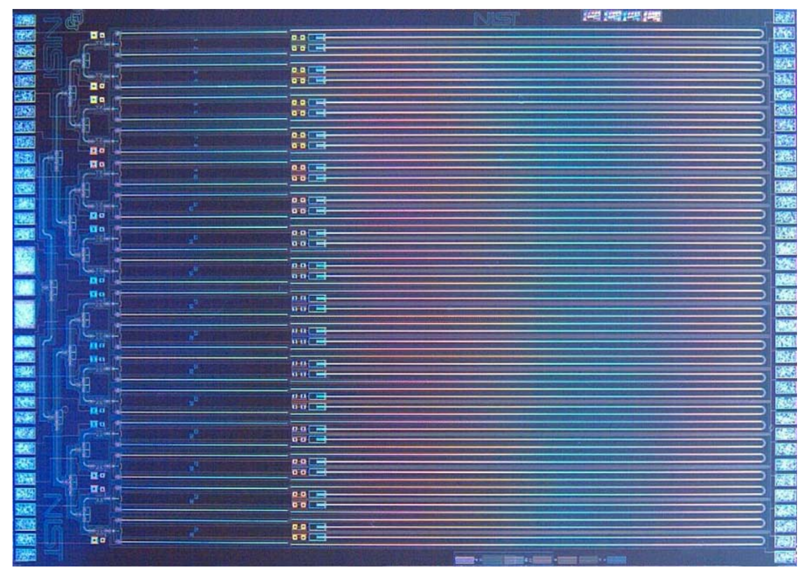

10 V standard

http://www.nist.gov/pml/history-volt/superconductivity_2000s.cfm

http://www.lne.eu/en/r_and_d/electrical_metrology/josephson-effect-ej.asp