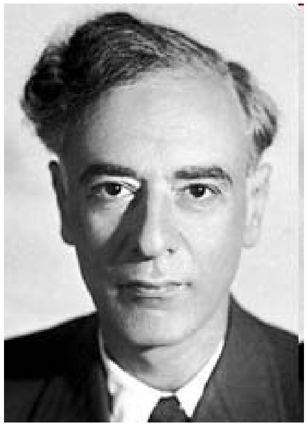
25. Landau Theory of Phase Transitions

Jan. 17, 2018

Landau theory of phase transitions

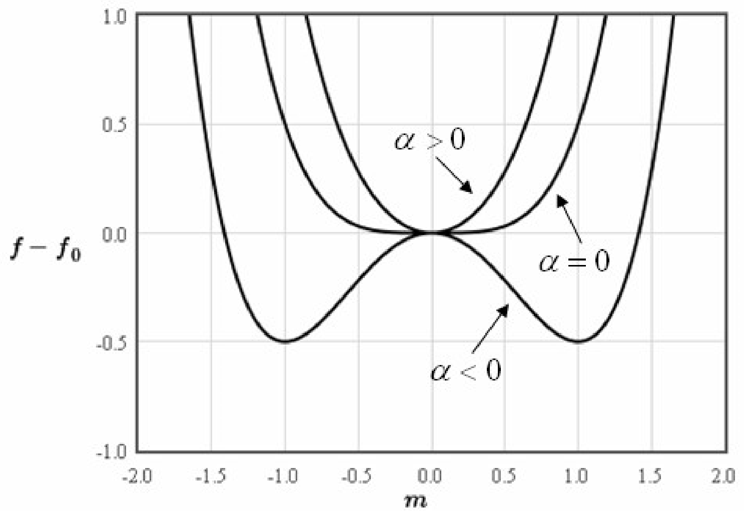
A phase transition is associated with a broken symmetry.

magnetism	direction of magnetization
cubic - tetragonal	different point group
water - ice	translational symmetry
ferroelectric	direction of polarization
superconductivity	gauge symmetry



Lev Landau

Temperature dependence of the order parameter



At $T = T_c$ $\alpha = 0$

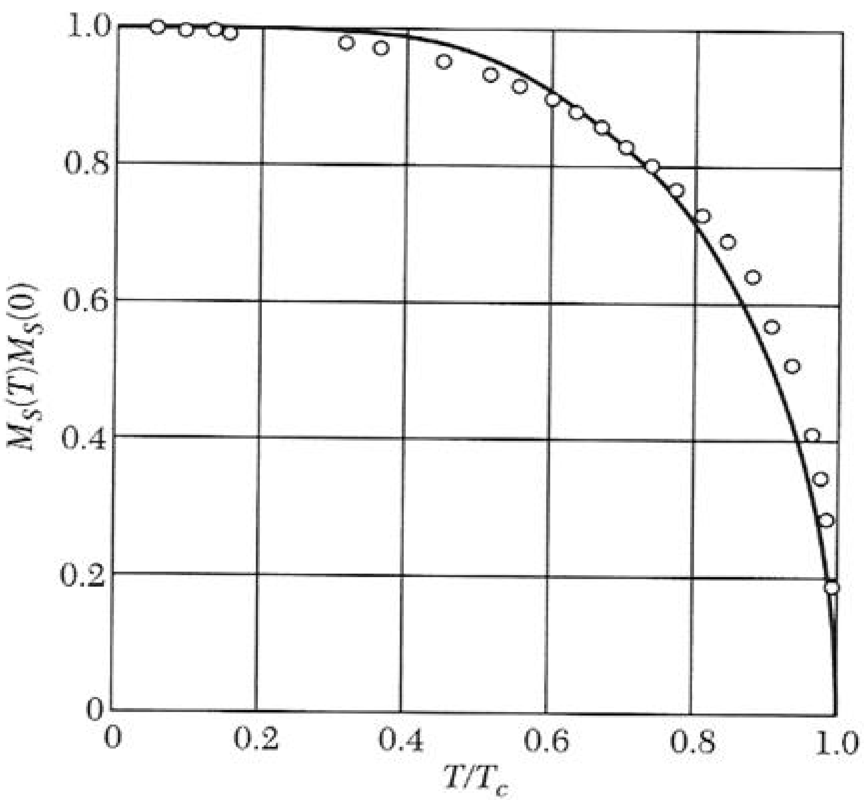
Expand α interms of T - T_c. Keep only the linear term. m and T - T_c are both small near Tc.

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2} \beta m^4 + \cdots$$

The temperature dependence of the magnetization is

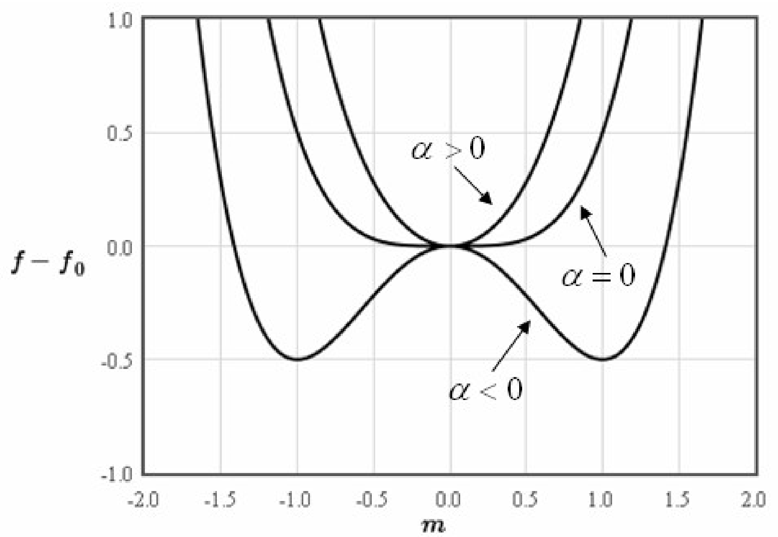
$$m = \pm \sqrt{\frac{\alpha_0 (T_c - T)}{\beta}} \qquad T < T_c$$

Landau theory of phase transitions



$$m = \sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$

Free energy

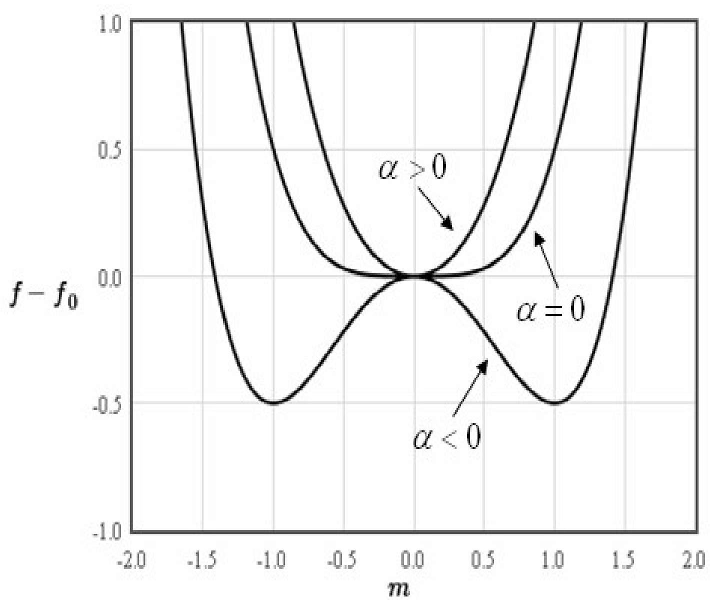


$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2} \beta m^4 + \cdots$$

$$m = \pm \sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$

$$f = f_0 - \frac{\alpha_0^2 \left(T - T_c\right)^2}{\beta} + \cdots$$

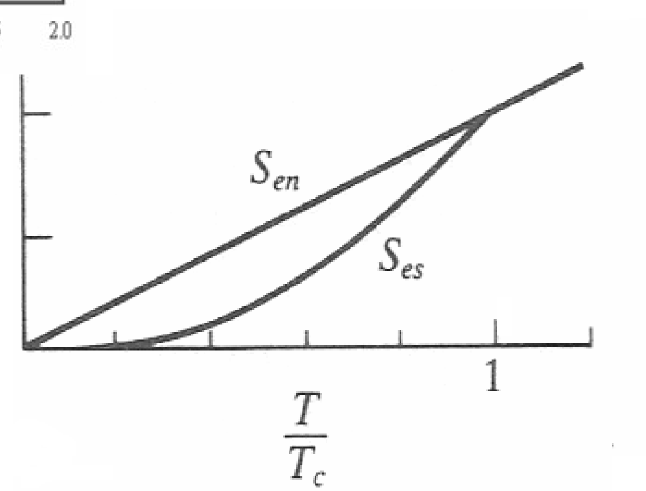
Entropy



$$f = f_0(T) - \frac{\alpha_0^2 (T - T_c)^2}{\beta} + \cdots$$

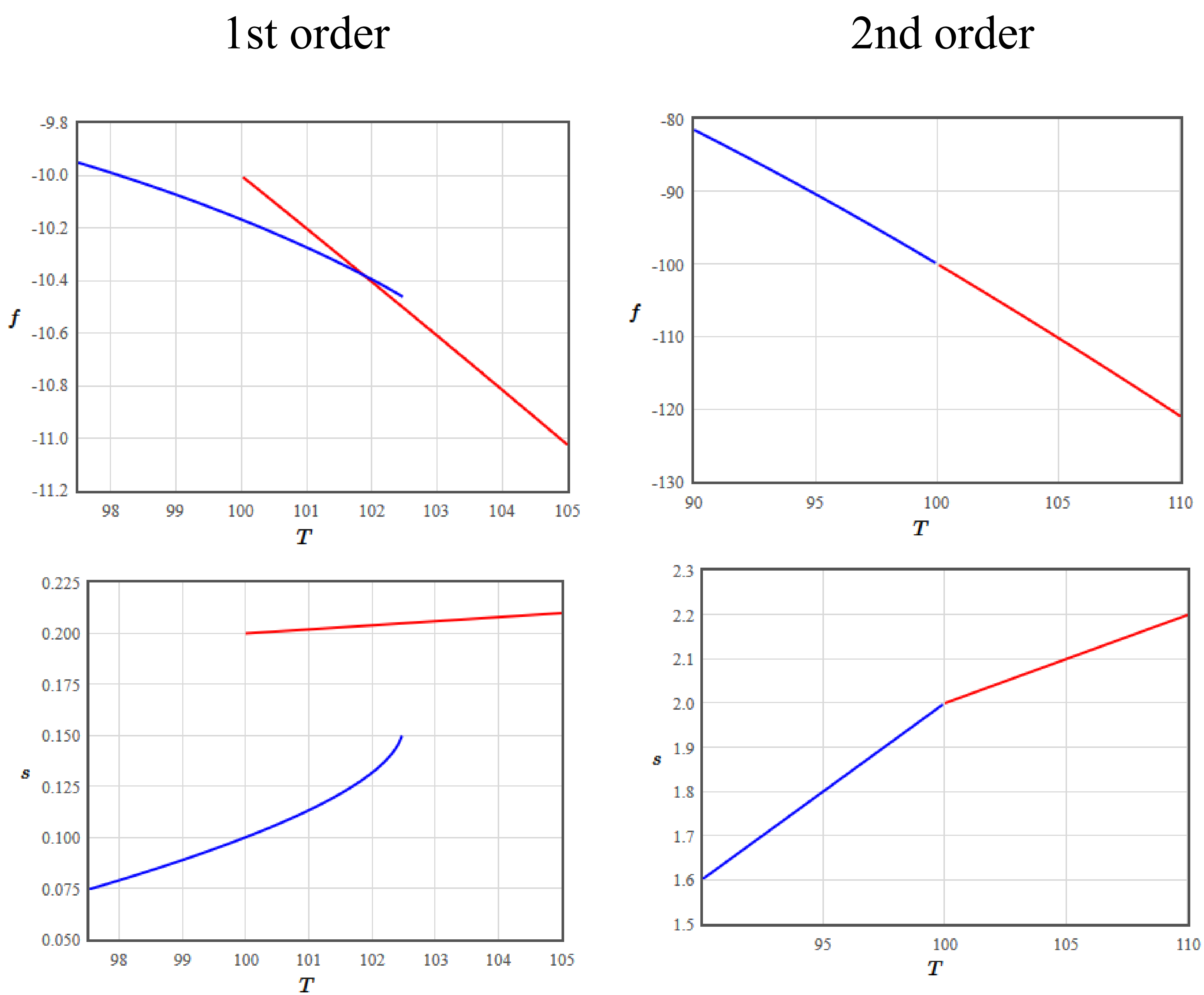
$$s = -\frac{\partial f}{\partial T} = s_0(T) + \frac{2\alpha_0^2 (T - T_c)}{\beta} + \cdots$$

Kink in the entropy



$$L = T_c (S_A - S_B) = 0$$

This is a second order phase transition


1st order
2nd order
-9.8
-10.0
-10.2
-10.4
-10.6
-10.8
-11.0
-11.2
f
98
99
100
101
102
103
104
105
T
-80
-90
-100
-110
-120
-130
f
90
95
100
105
110
T
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
s
98
99
100
101
102
103
104
105
T
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
s
95
100
105
110
T

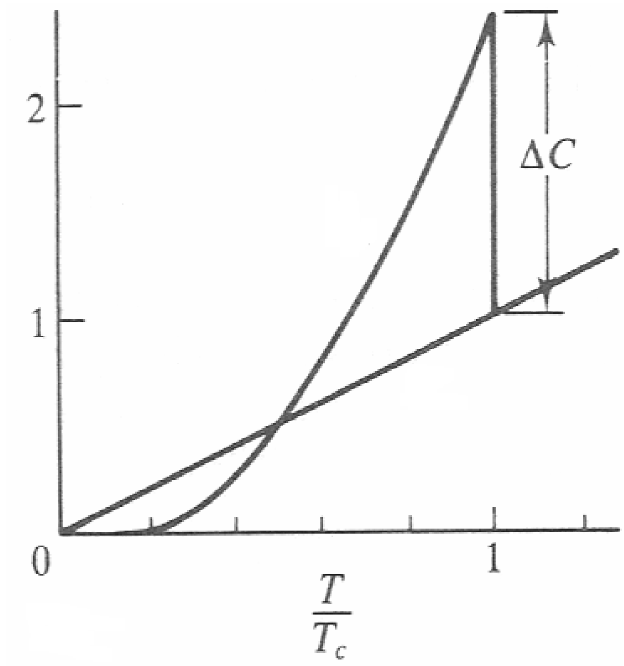
Specific heat

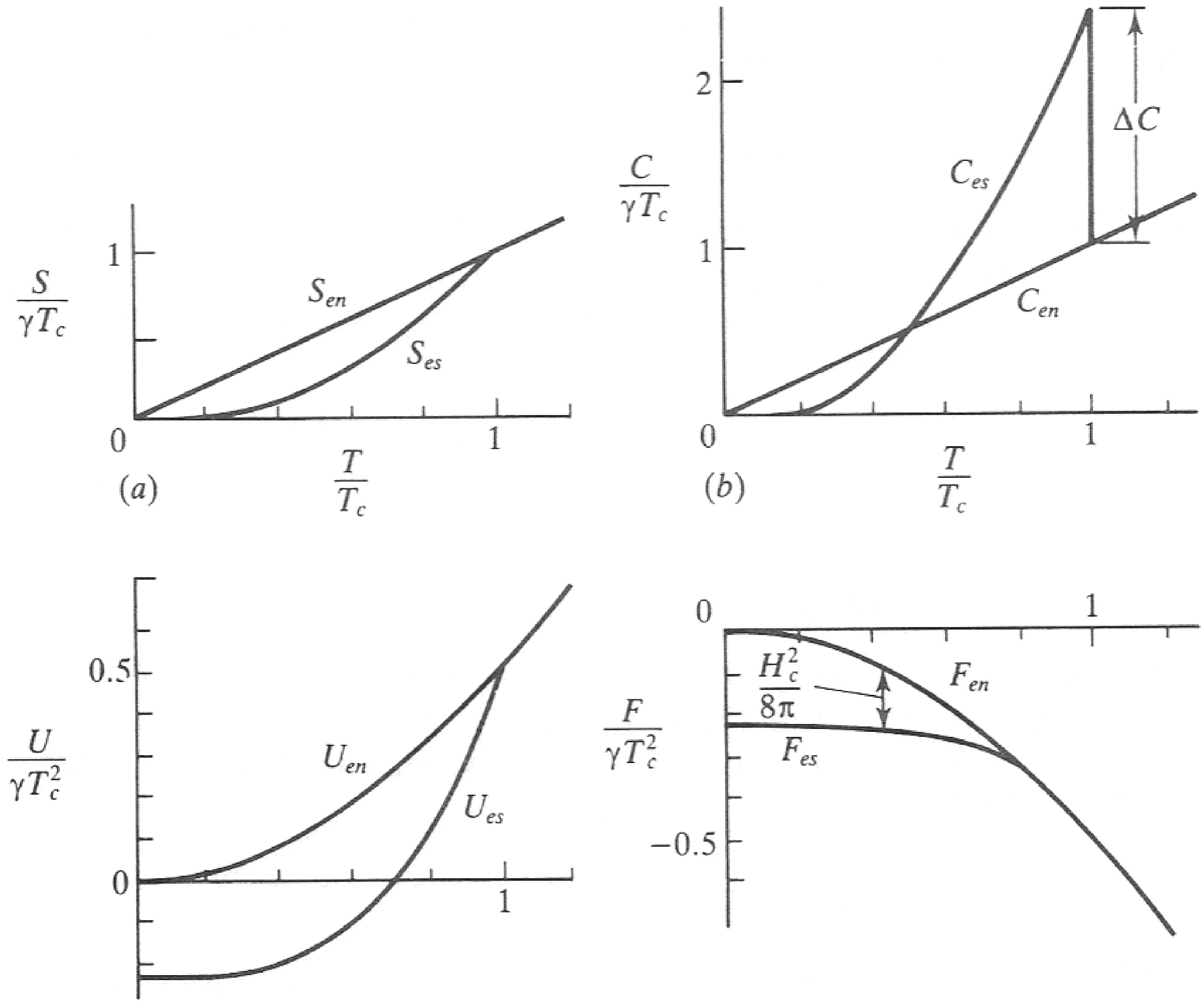
Entropy $$s = -\frac{\partial f}{\partial T} = s_0\left(T\right) + \frac{2\alpha_0^2\left(T - T_c\right)}{\beta} + \cdots$$

Specific heat $$c_v = T\frac{\partial s}{\partial T} = c_0\left(T\right) + \frac{2\alpha_0^2 T}{\beta} + \cdots \qquad T < T_c$$

There is a jump in the specific heat at the phase transition and then a linear dependence after the jump.

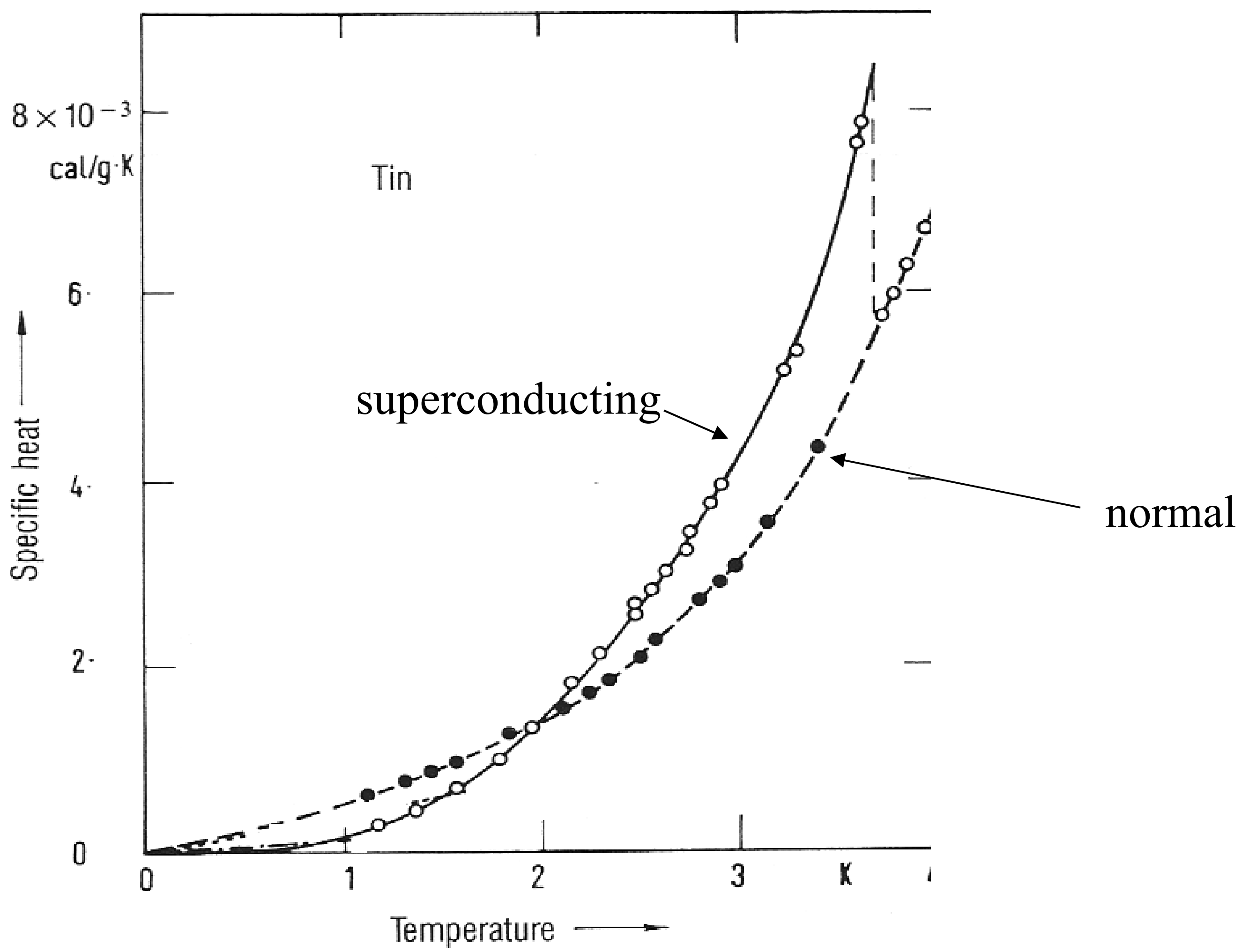
$$\Delta c_v = \frac{2\alpha_0^2 T_c}{\beta}$$



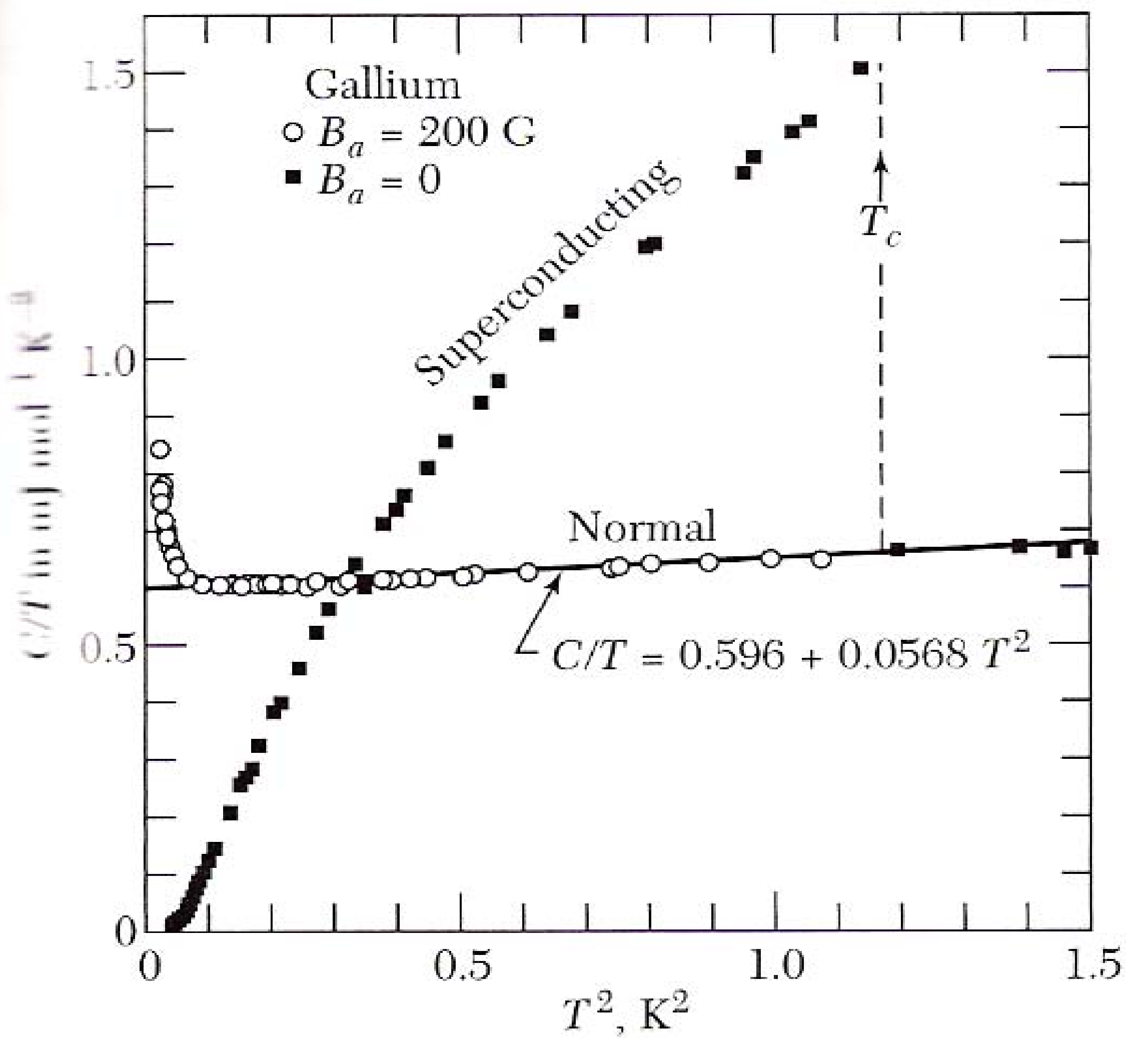


Introduction to Superconductivity, Tinkham

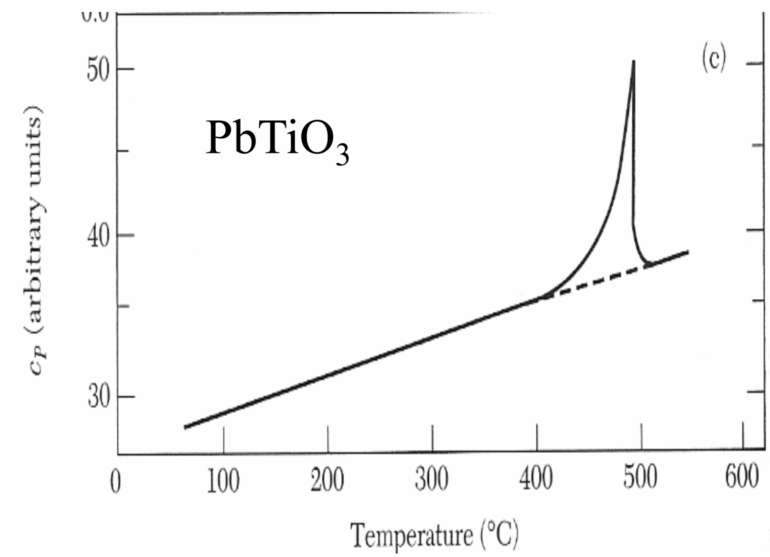
Specific heat

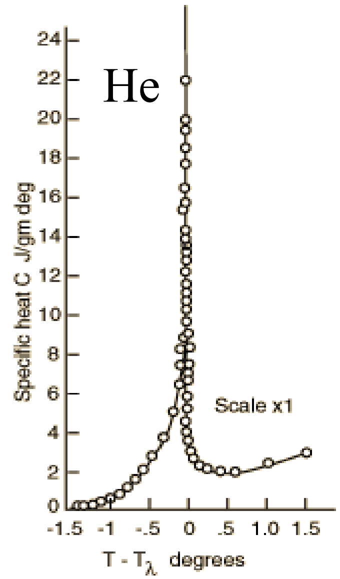


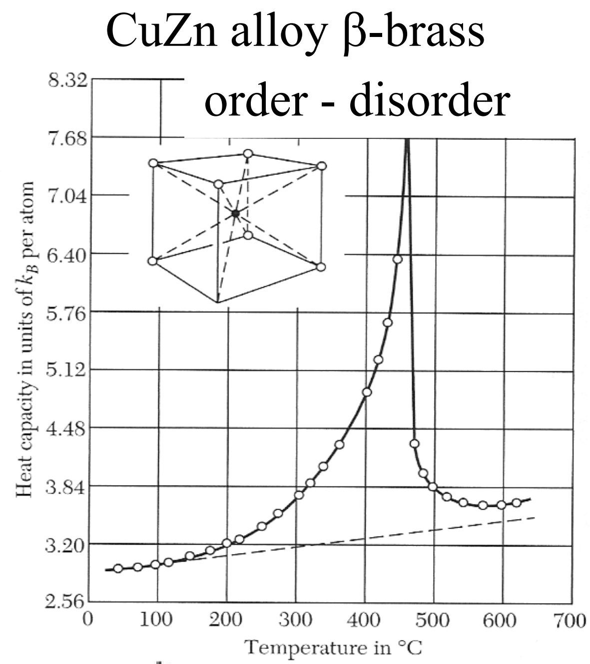
Specific heat



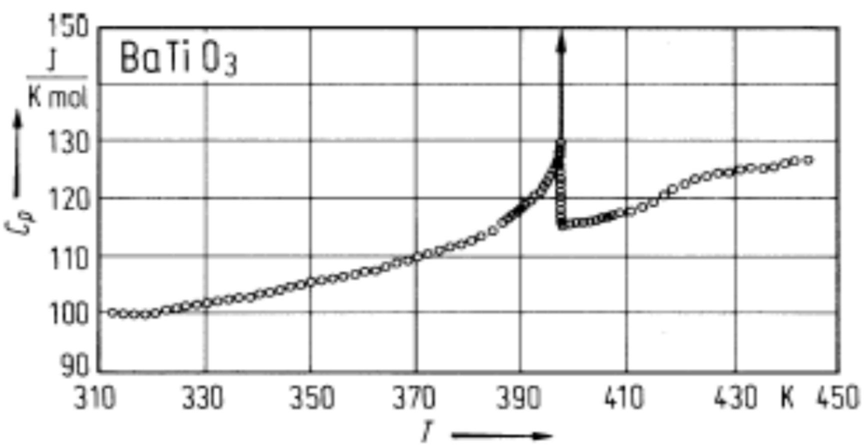
Specific heat

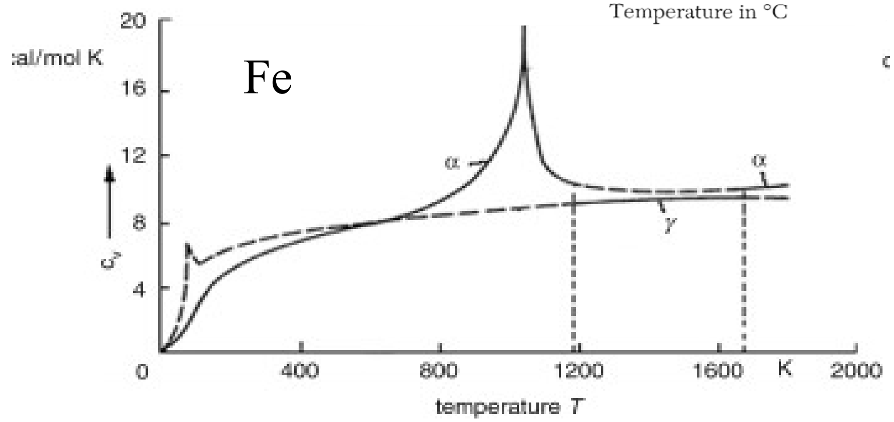




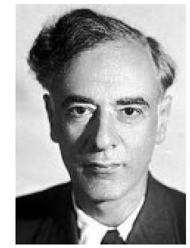


BaTiO$_3$. Heat capacity vs. temperature [76H].





Landau theory of second order phase transitions



Lev Landau

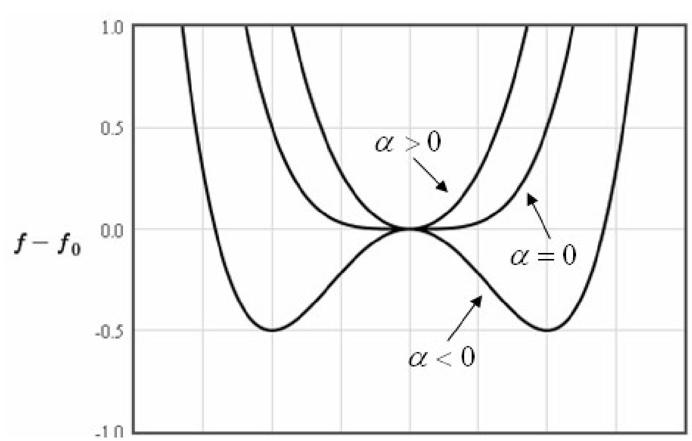
Normally, to calculate thermodynamic properties like the free energy, the entropy, or the specific heat, it is necessary to determine the microscopic states of system by solving the Schrödinger equation. For crystals, the microscopic states are labeled by k and the solutions of the Schrödinger equation are typically expressed as a dispersion relation where the energy is given for each k. The dispersion relation can be used to calculate the density of states and the density of states can be used to calculate the thermodynamic properties. This is typically a long and numerically intensive calculation.

Landau realized that near a phase transition an approximate form for the free energy can be constructed without first calculating the microscopic states. He recognized it is always possible to identify an order parameter the is zero on the high temperature side of the phase transition and nonzero on the low temperature side of the phase transition. For instance, the magnetization can be considered the order parameter at a ferromagnetic - paramagnetic phase transition. For a structural phase transition from a cubic phase to a tetragonal phase, the order parameter can be taken to be c/a - 1 where c is the length of the long side of the tetragonal unit cell and a is the length of the short side of the tetragoal unit cell.

At a second order phase transition, the order parameter increases continuously from zero starting at the critical temperature of the phase transition. An example of this is the continuous increase of the magnetization at a ferromagnetic - paramagnetic phase transition. Since the order parameter is small near the phase transition, to a good approximation the free energy of the system can be approximated by the first few terms of a Taylor expansion of the free energy in the order parameter.

$$f(T) = f_0(T) + \alpha m^2 + \tfrac{1}{2}\beta m^4 \qquad \alpha_0 > 0, \quad \beta > 0.$$

Here m is the order parameter, α and β are parameters, and $f_0(T)$ describes the temperature dependence of the high temperature phase near the phase transition. It is assumed that $\beta > 0$ so that the free energy has a minimum for finite values of the order parameter. When $\alpha > 0$, there is only one minimum at $m = 0$. When $\alpha < 0$ there are two minima with $m \neq 0$.



Landau theory, susceptibility

Add a magnetic field

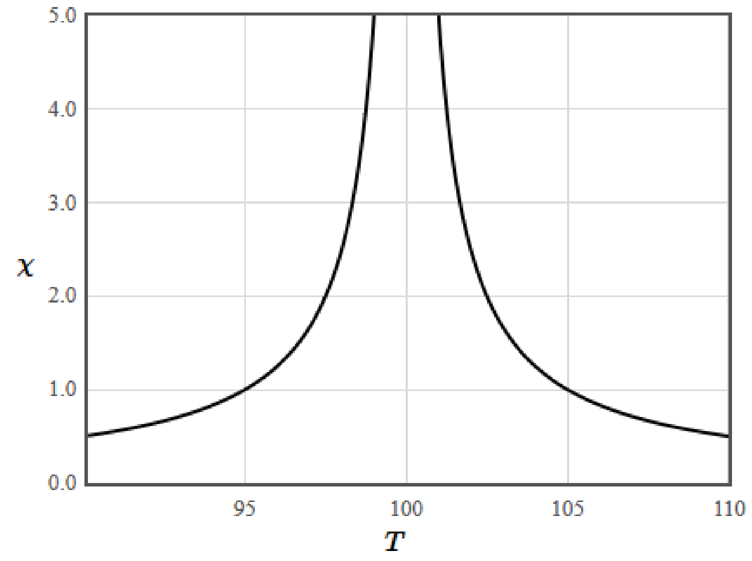
$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2} \beta m^4 - mB$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 - B = 0$$

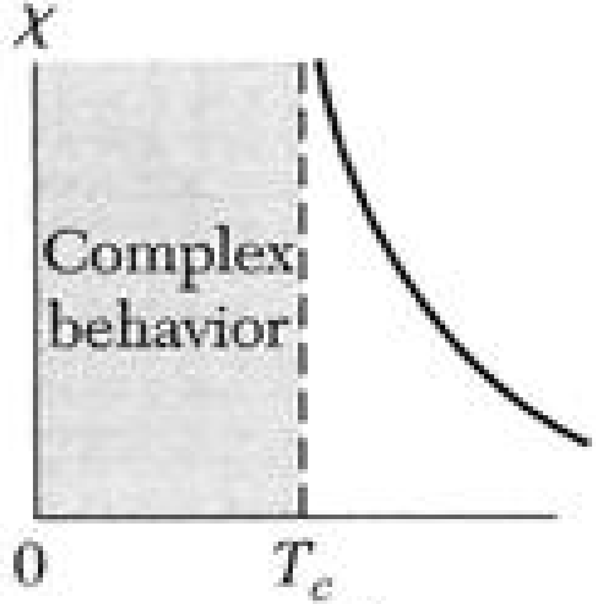
Above T_c, m is finite for finite B. For small m,

$$m = \frac{B}{2\alpha_0 (T - T_c)} \qquad T > T_c$$

$$\chi = \frac{1}{2\alpha_0 (T - T_c)} \qquad \text{Curie-Weiss}$$



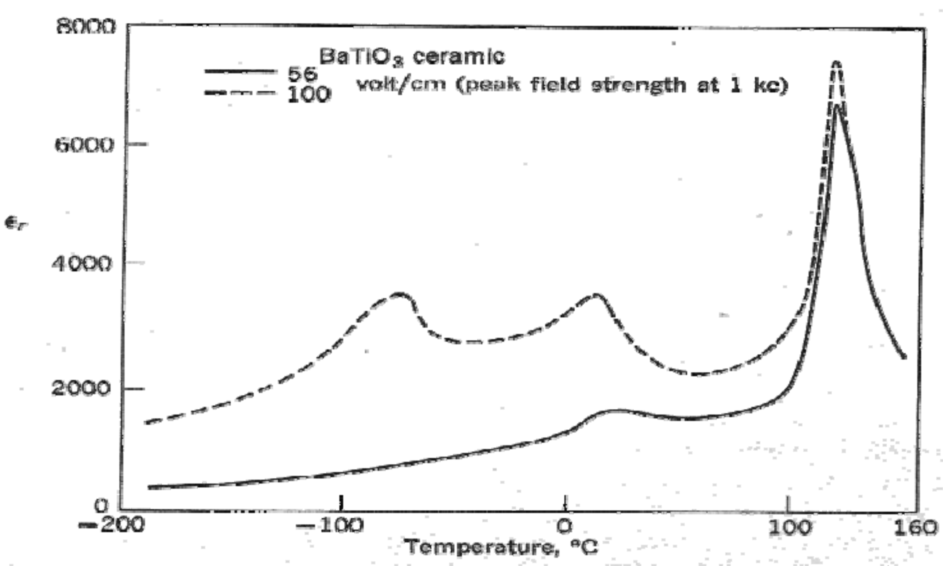
Ferromagnetism

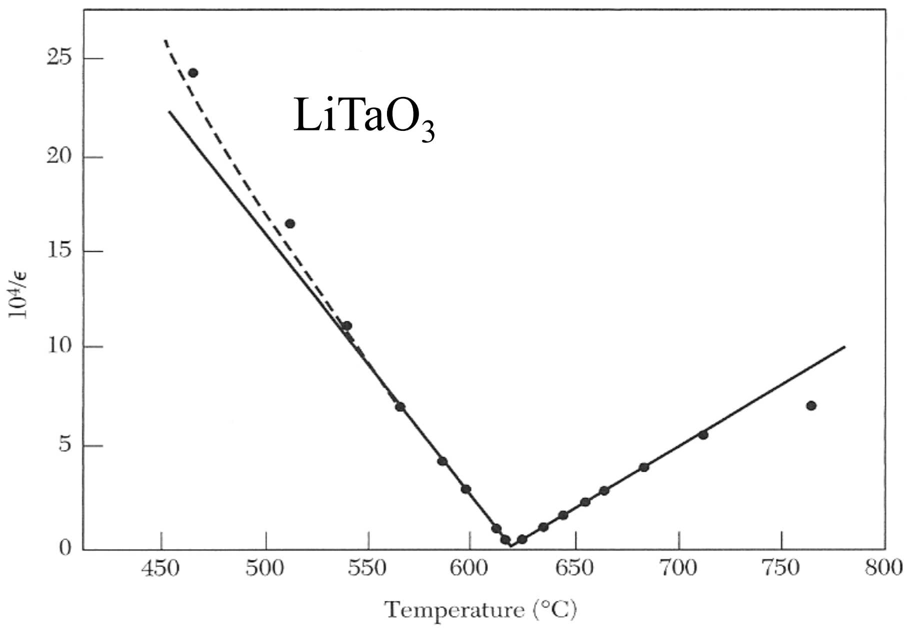


$$\chi = \frac{C}{T - T_c}$$

Curie-Weiss law
$(T > T_c)$

Landau theory of phase transitions



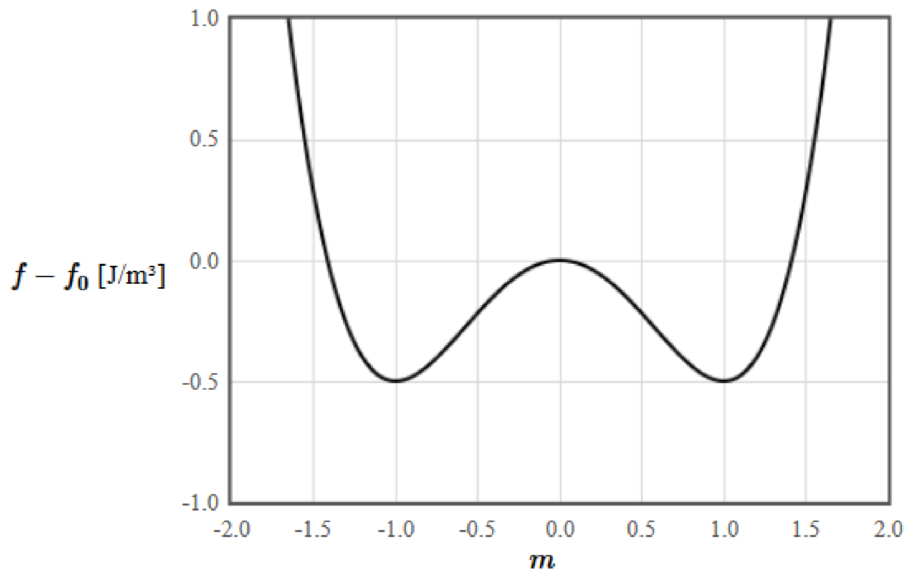


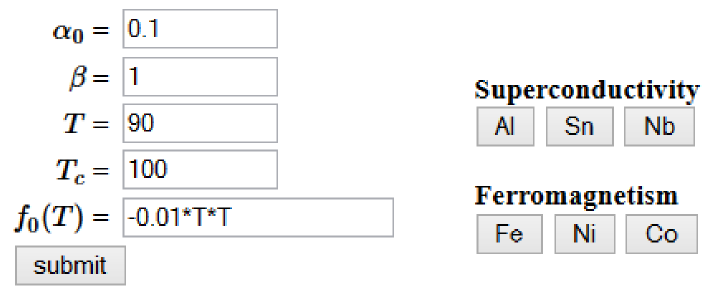
$$\varepsilon_r = 1 + \chi$$

$$\chi = \frac{1}{2\alpha_0 (T - T_c)}$$

Curie-Weiss law

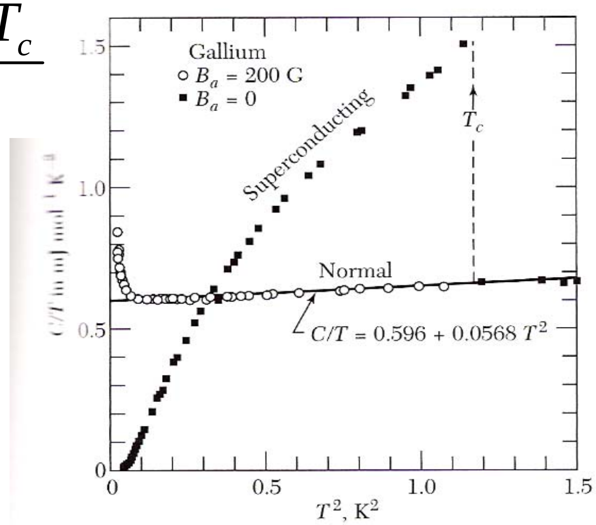
Fitting the α_0 and β parameters

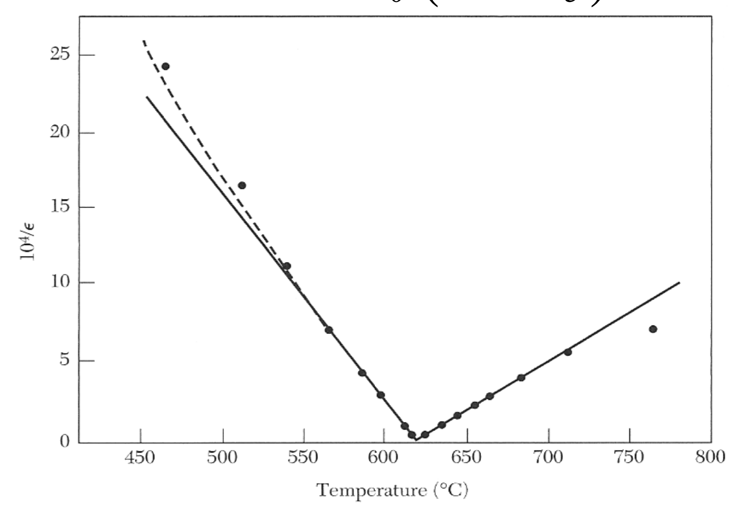




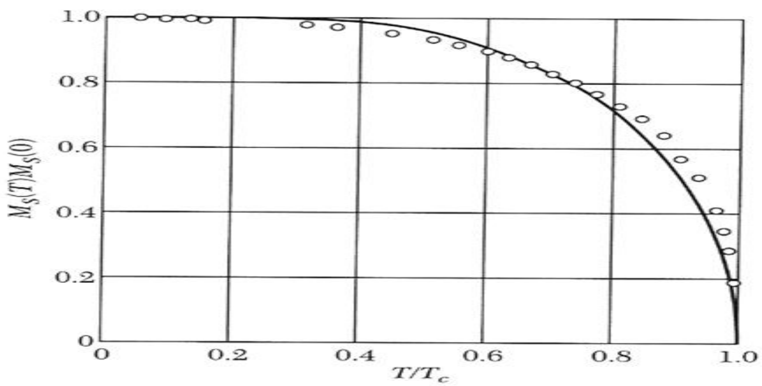
$$\chi = \frac{1}{2\alpha_0 \left(T - T_c\right)}$$

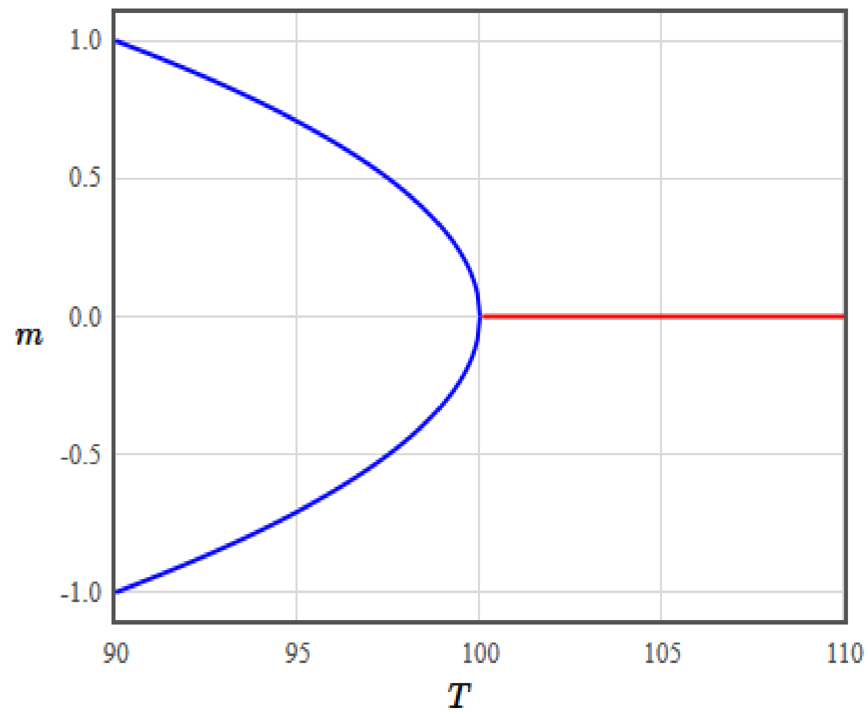
$$\Delta c_v = \frac{2\alpha_0^2 T_c}{\beta}$$





Landau theory of phase transitions



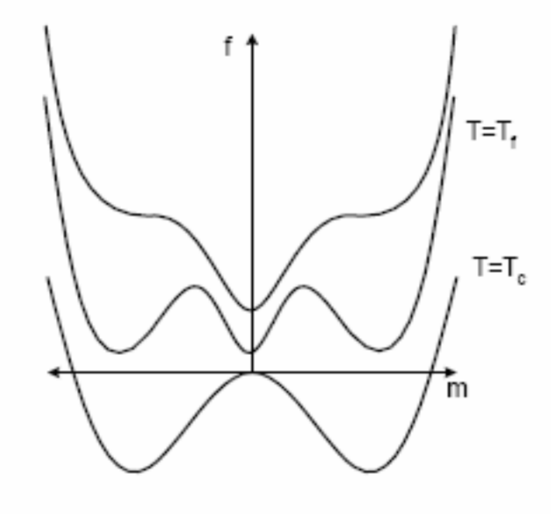


$$m = \sqrt{\frac{\alpha_0 \left(T_c - T\right)}{\beta}} \qquad T < T_c$$

$\dfrac{\alpha_0}{\beta}$ can be determined from the temperature dependence of the order parameter

First order transitions

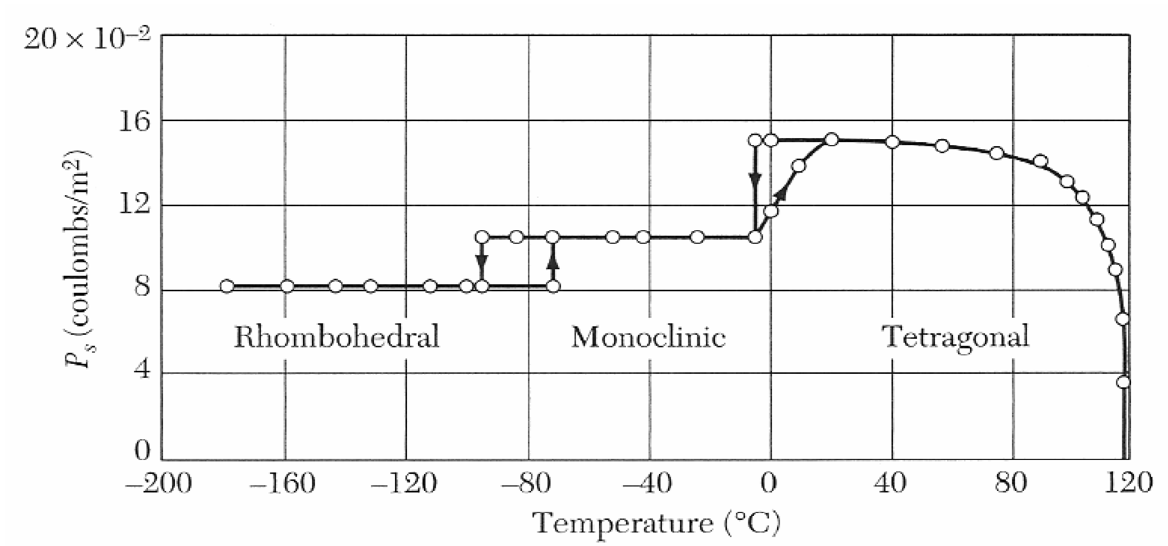
$$f = f_0 + \alpha_0 \left(T - T_c\right) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0$$

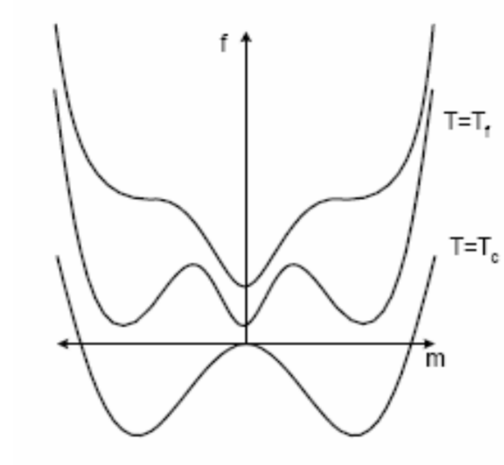


There is a jump in the order parameter at the phase transition.

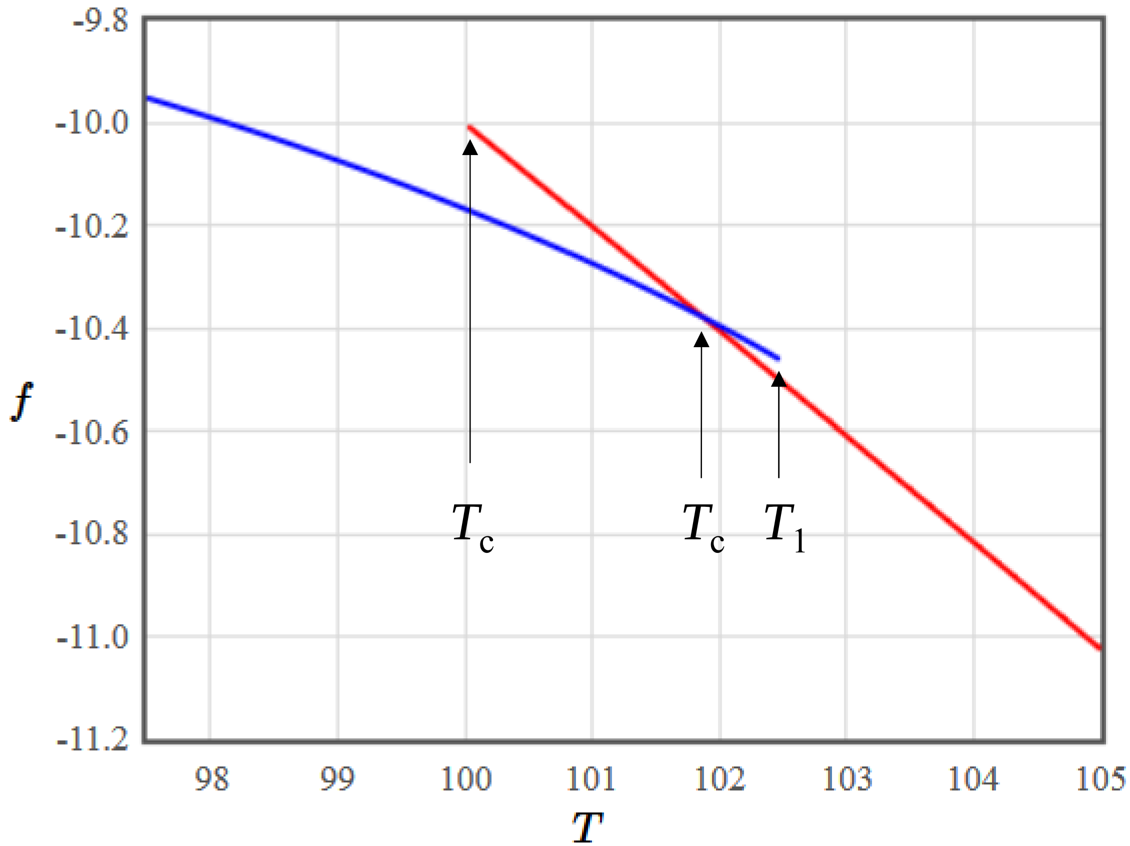
First order transitions

$BaTiO_3$





T_c?


-9.8
-10.0
-10.2
-10.4
-10.6
-10.8
-11.0
-11.2
f
98
99
100
101
102
103
104
105
T
T_c
T_c
T_1

First order transitions

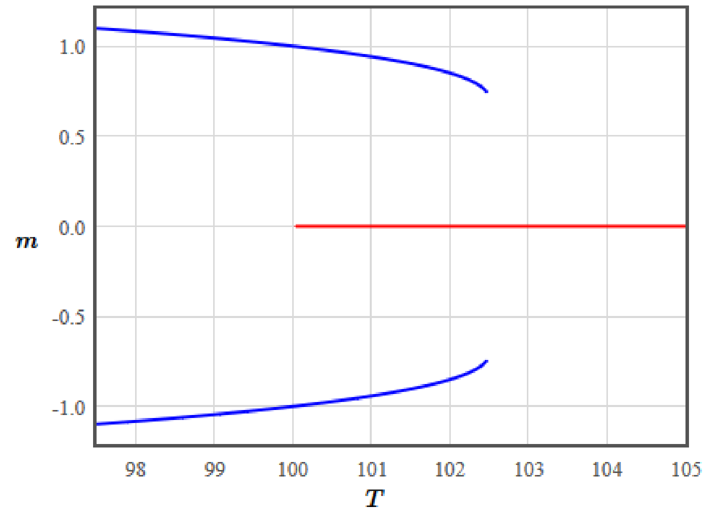
$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0 \qquad \gamma > 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 = 0$$

One solution for $m = 0$.

$$\alpha_0 (T - T_c) + \beta m^2 + \gamma m^4 = 0$$

$$m^2 = 0, \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}$$



There will be a minimum at finite m as long as m^2 is real

$$T_1 = \frac{\beta^2}{4\alpha_0\gamma} + T_c$$

Jump in the order parameter

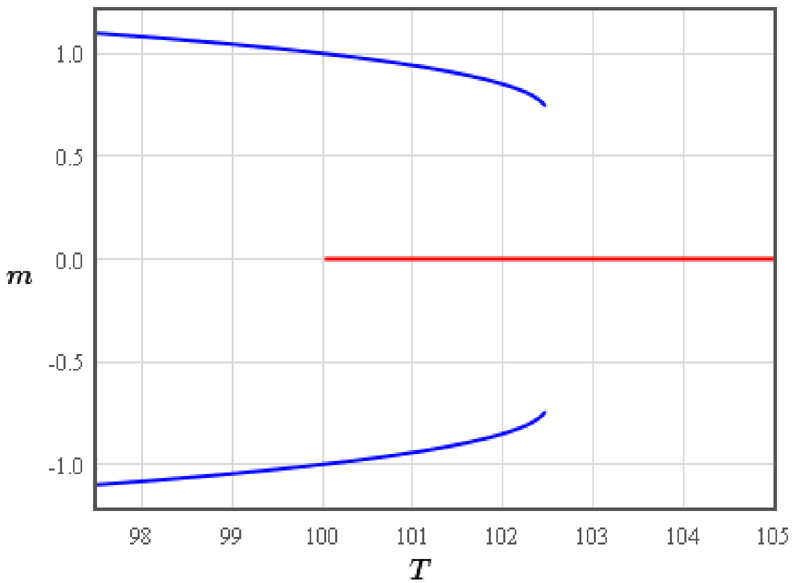
$$m^2 = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 \left(T - T_c\right)\gamma}}{2\gamma}$$

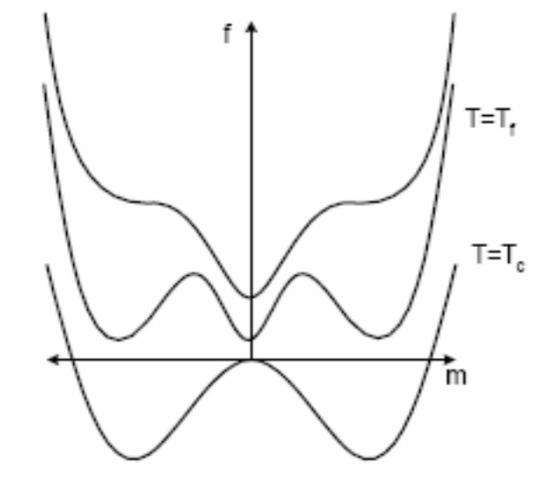
At T_c

$$\Delta m = \sqrt{\frac{-\beta}{\gamma}}$$

At T_1

$$\Delta m = \sqrt{\frac{-\beta}{2\gamma}}$$





First order transitions, entropy, c_v

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0$$

$$m = 0, \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}}$$

$$s = -\frac{\partial f}{\partial T} = \frac{\alpha_0}{2\gamma}\left(\beta - \sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}\right)$$

$$c_v = T\frac{\partial s}{\partial T} = \frac{\alpha_0^2 T}{\sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}}$$

branch where the order parameter is nonzero

First order transitions, susceptibility

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 - mB \qquad \beta < 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 - B = 0$$

At the minima $$B = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5$$

For small m,

$$\chi = \left.\frac{dm}{dB}\right|_{m=0} = \frac{1}{2\alpha_0 (T - T_c)} \qquad \text{Curie - Weiss}$$

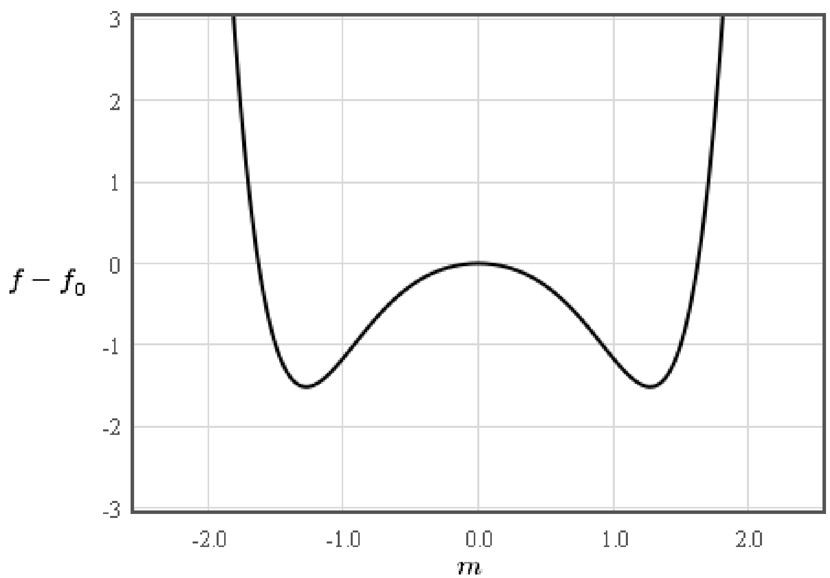
$$\chi = \left.\frac{dm}{dB}\right|_{m=\sqrt{\frac{-\beta}{2\gamma}}} = \frac{1}{2\alpha_0 (T - T_1)}$$

Landau theory of a first order phase transition

The free energy for a first order transition in Landau theory is,

$$f(T) = f_0(T) + \alpha_0(T - T_c)m^2 + \frac{1}{2}\beta m^4 + \frac{1}{3}\gamma m^6 \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0.$$

Here $f_0(T)$ describes the temperature dependence of the high temperature phase near the phase transition.



$\alpha_0 =$ 0.1
$\beta =$ -1
$\gamma =$ 1
$T =$ 90
$T_c =$ 100
$f_0(T) =$ -0.01*T*T
submit

Order parameter