

# 13. Magnetism

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Nov. 15, 2018

# Magnetic ordering

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Ferromagnetism



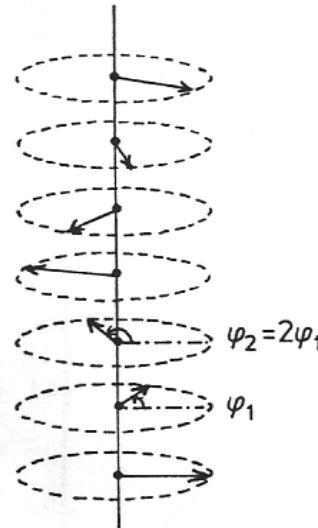
Ferrimagnetism



Antiferromagnetism



Helimagnetism



All ordered magnetic states  
have excitations called  
magnons

# Ferrimagnets

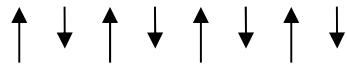
Magnetite  $\text{Fe}_3\text{O}_4$   
(Magneteseisen)



HM: F d -3 m :2  
 $a=8.084\text{\AA}$   
 $b=8.084\text{\AA}$   
 $c=8.084\text{\AA}$   
 $\alpha=90.000^\circ$   
 $\beta=90.000^\circ$   
 $\gamma=90.000^\circ$

Ferrites  $\text{MO}\cdot\text{Fe}_2\text{O}_3$

M = Fe, Zn, Cd, Ni, Cu,  
Co, Mg



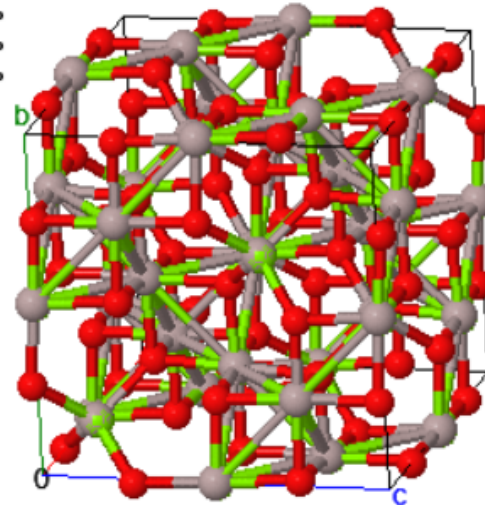
Two sublattices A and B.

Spinel crystal structure  $\text{XY}_2\text{O}_4$

8 tetrahedral sites A (surrounded by 4 O)  $5\mu_B \uparrow$

16 octahedral sites B (surrounded by 6 O)  $9\mu_B \downarrow$

per unit cell



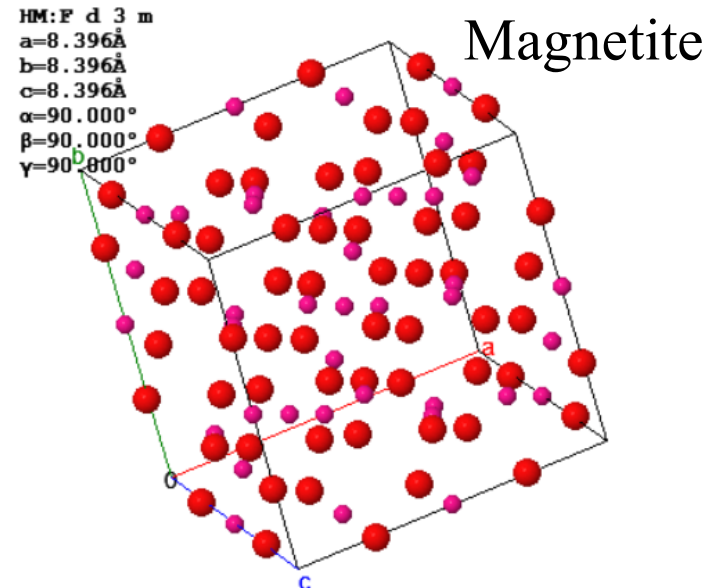
$\text{MgAl}_2\text{O}_4$

# Ferrimagnets

Magnetite  $\text{Fe}_3\text{O}_4$

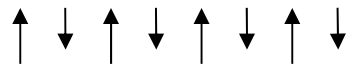
Ferrites  $\text{MO}\cdot\text{Fe}_2\text{O}_3$

M = Fe, Zn, Cd, Ni,  
Cu, Co, Mg



Exchange integrals  $J_{AA}$ ,  $J_{AB}$ , and  $J_{BB}$  are all negative (antiparallel preferred)

$$|J_{AB}| > |J_{AA}|, |J_{BB}|$$



# Ferrimagnetism

$$\text{gauss} = 10^{-4} \text{ T}$$

$$\text{oersted} = 10^{-4}/4\pi \times 10^{-7} \text{ A/m}$$

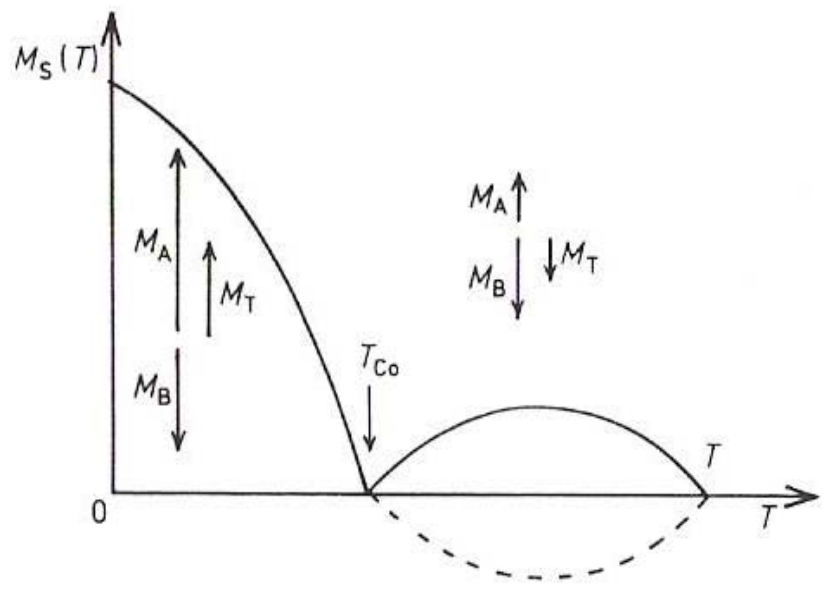


Table 33.3  
**SELECTED FERRIMAGNETS, WITH CRITICAL TEMPERATURES  $T_c$  AND SATURATION MAGNETIZATION  $M_0$**

MATERIAL	$T_c$ (K)	$M_0$ (gauss) <sup>a</sup>
$\text{Fe}_3\text{O}_4$ (magnetite)	858	510
$\text{CoFe}_2\text{O}_4$	793	475
$\text{NiFe}_2\text{O}_4$	858	300
$\text{CuFe}_2\text{O}_4$	728	160
$\text{MnFe}_2\text{O}_4$	573	560
$\text{Y}_3\text{Fe}_5\text{O}_{12}$ (YIG)	560	195

<sup>a</sup> At  $T = 0(\text{K})$ .  
 Source: F. Keffer, *Handbuch der Physik*, vol. 18, pt. 2, Springer, New York, 1966.

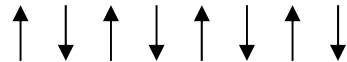
Kittel

D. Gignoux, magnetic properties of Metallic systems

# Antiferromagnetism

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Negative exchange energy  $J_{AB} < 0$ .



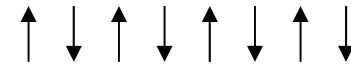
At low temperatures, below the Neel temperature  $T_N$ , the spins are aligned antiparallel and the macroscopic magnetization is zero.

Spin ordering can be observed by neutron scattering.

At high temperature antiferromagnets become paramagnetic. The macroscopic magnetization is zero and the spins are disordered in zero field.

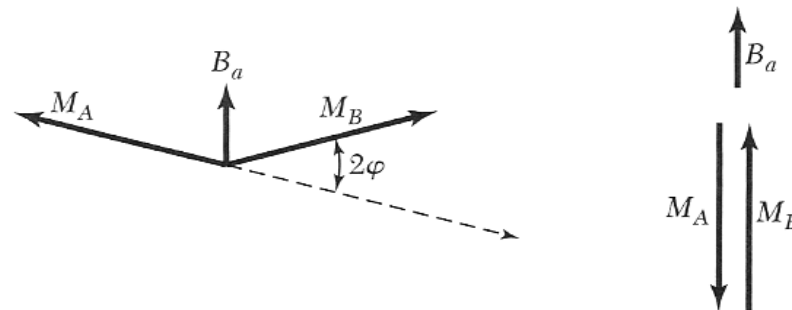
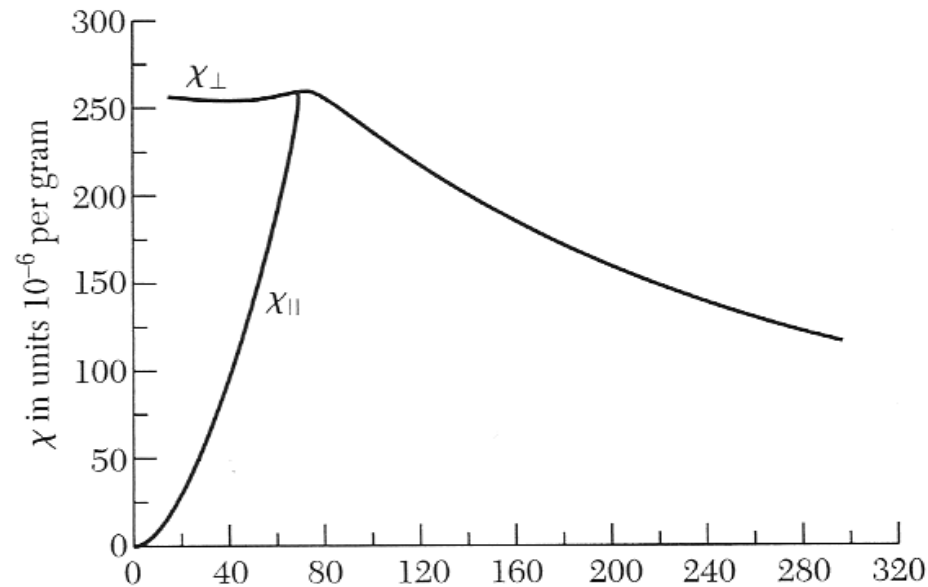
$$\chi = \mu_0 \frac{\vec{M}_A + \vec{M}_B}{\vec{B}_a} = \frac{C}{T + \Theta} \quad \leftarrow \text{Curie-Weiss temperature}$$

# Antiferromagnetism



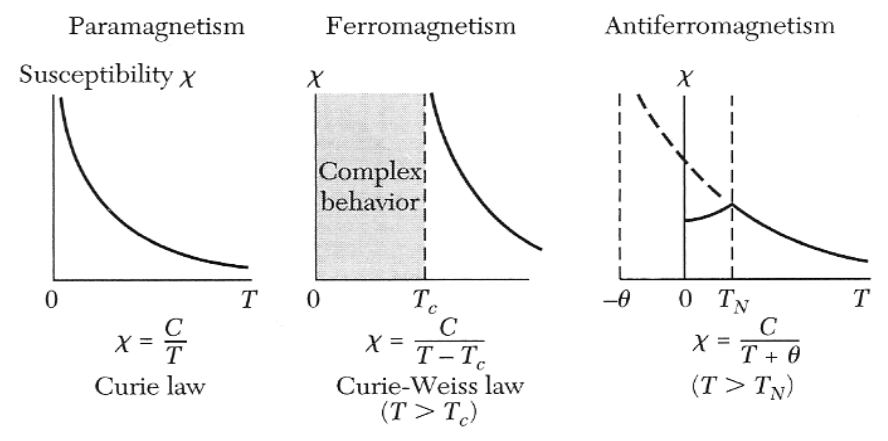
Average spontaneous magnetization is zero at all temperatures.

Source: Kittel



**Table 2 Antiferromagnetic crystals**    ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Substance	Paramagnetic ion lattice	Transition temperature, $T_N$ , in K	Curie-Weiss $\theta$ , in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF <sub>2</sub>	bc tetr.	67	82	1.24	0.76
FeF <sub>2</sub>	bc tetr.	79	117	1.48	0.72
FeCl <sub>2</sub>	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
CoCl <sub>2</sub>	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
NiCl <sub>2</sub>	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			



from Kittel



# Applications of magnetism

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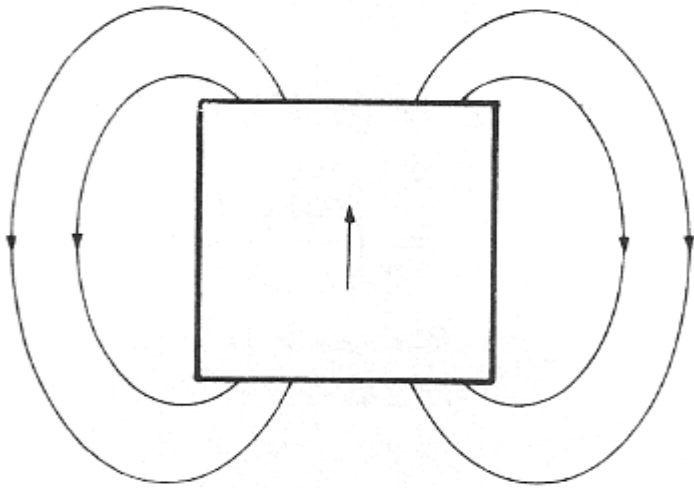
Hard magnets: permanent magnets, motors, generators, microphones

Soft magnets: transformers

Magnetic recording

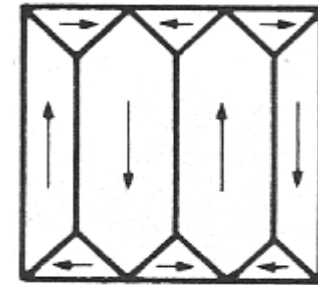
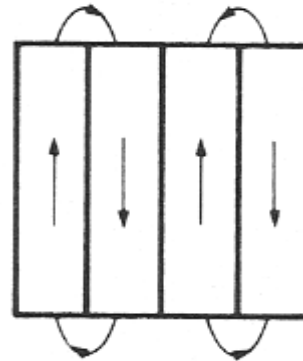
# Magnetic domains (weissche Bezirke )

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Magnetic energy density

$$\frac{B^2}{2\mu_0}$$



Costs energy to introduce domain walls where spin up regions are adjacent to spin down regions.

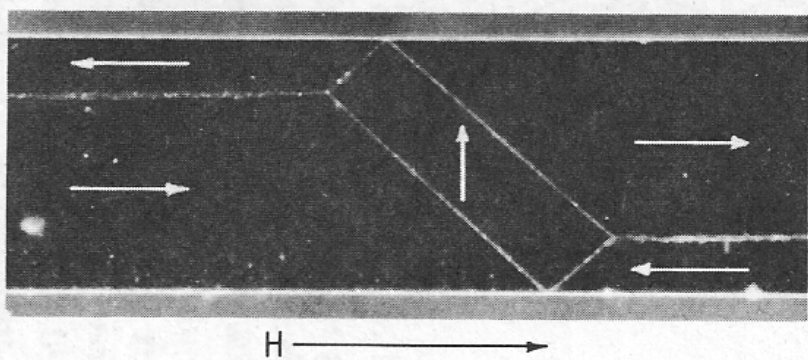
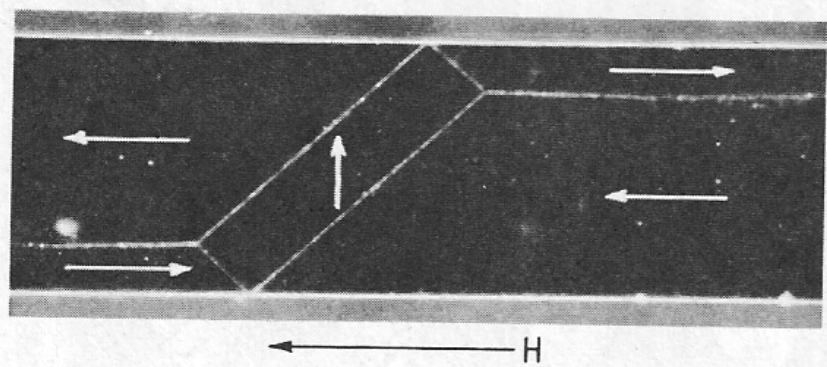
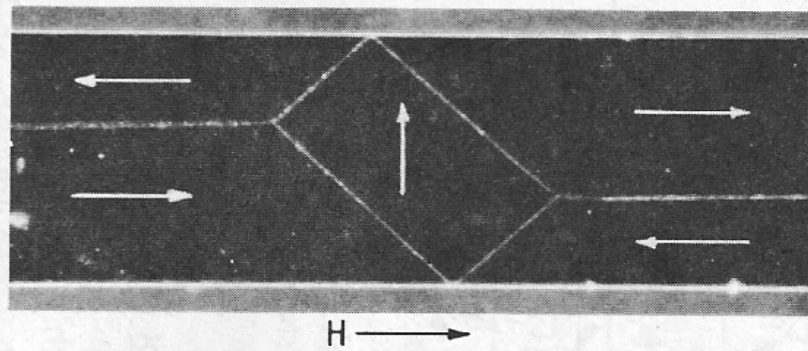
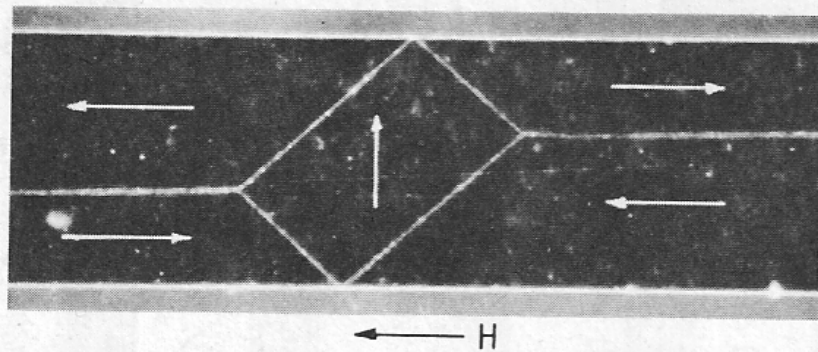
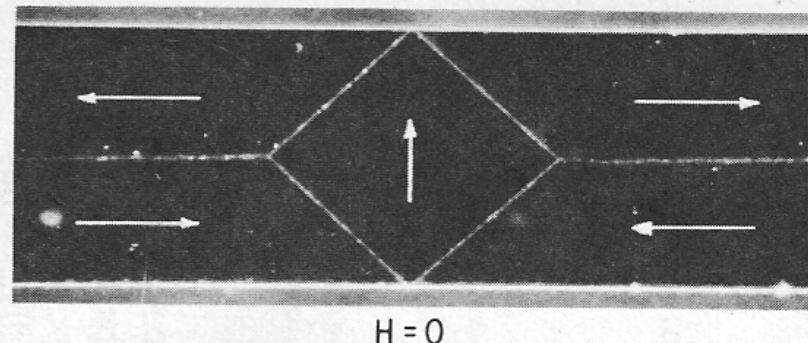
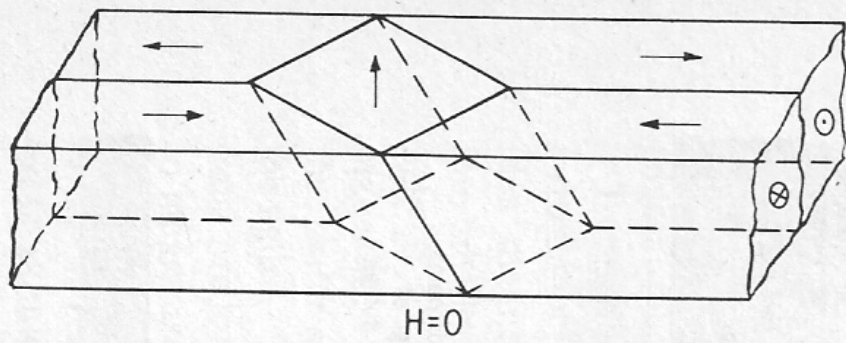
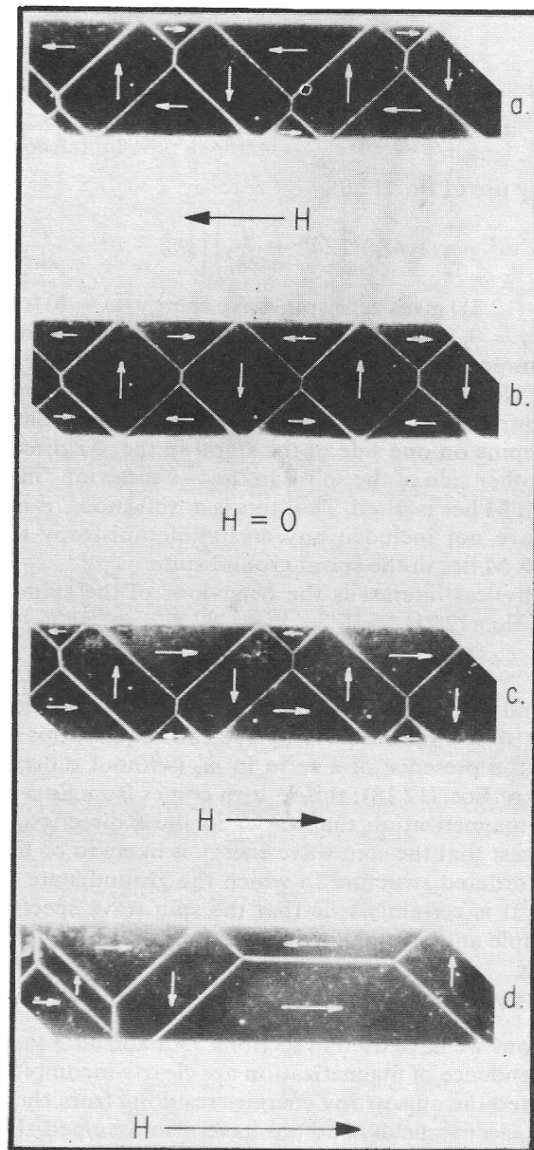


Figure 27. Photographs of diamond-shaped vortices in a channel with a magnetic field  $H$ .

# Ferromagnetic domains

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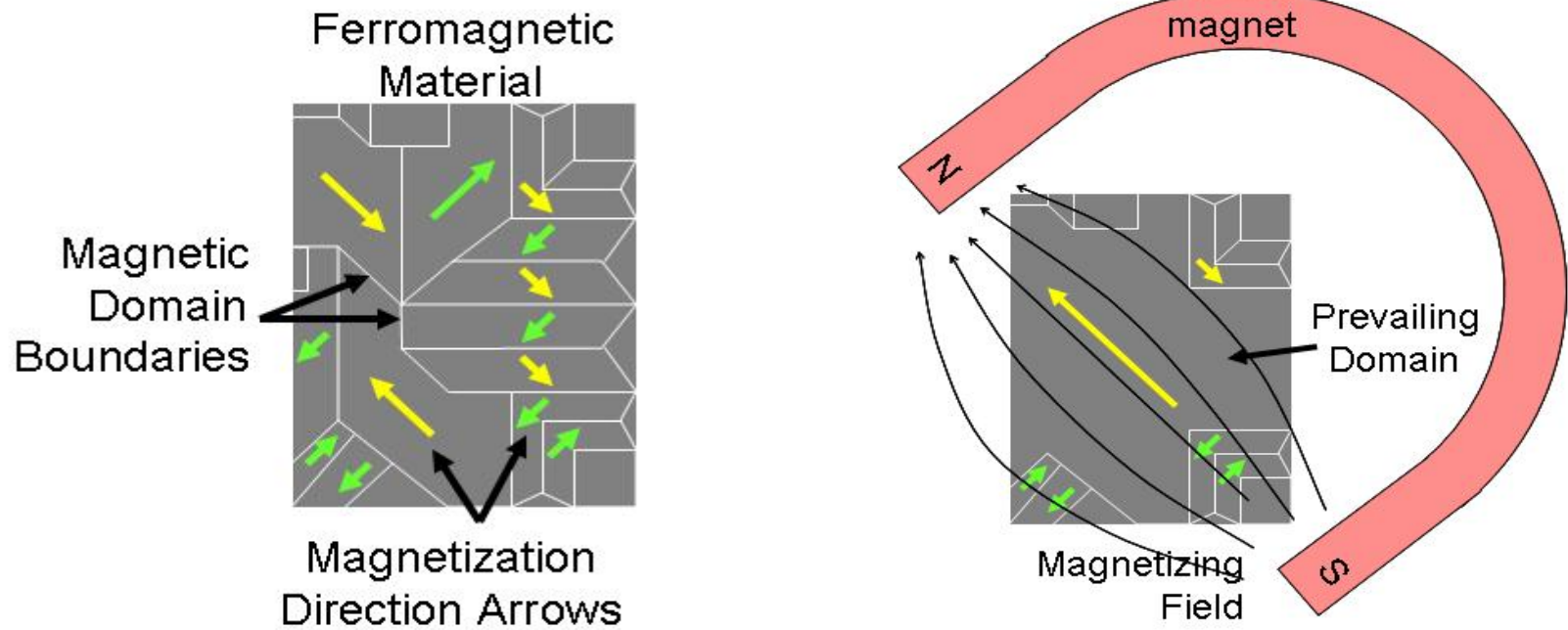
Weak fields: favorable domains expand  
Strong fields: domains rotate to align with field

Irreproducible jump between c and d.

Fig. 12.5. Photographs showing reversible domain wall motion in a  $50\ \mu\text{m}$  whisker from (a) to (b) to (c), with an irreversible jump from (c) to (d).  
{R. W. de Blois and C. D. Graham, *J. Appl. Phys.*, **29**, 931 (1958)}.

# Magnetizing a magnet

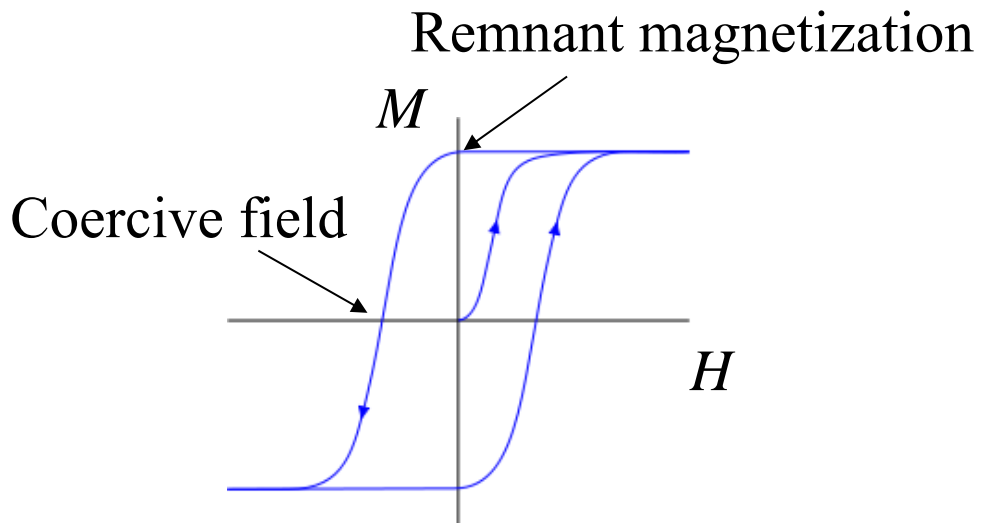
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Weak fields: favorable domains expand  
Strong fields: domains rotate to align with field

# Hysteresis

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$$B = \mu_0 (H + M)$$

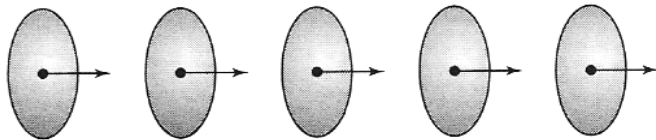
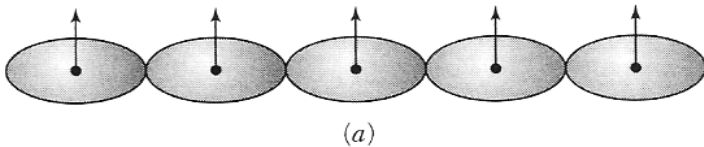
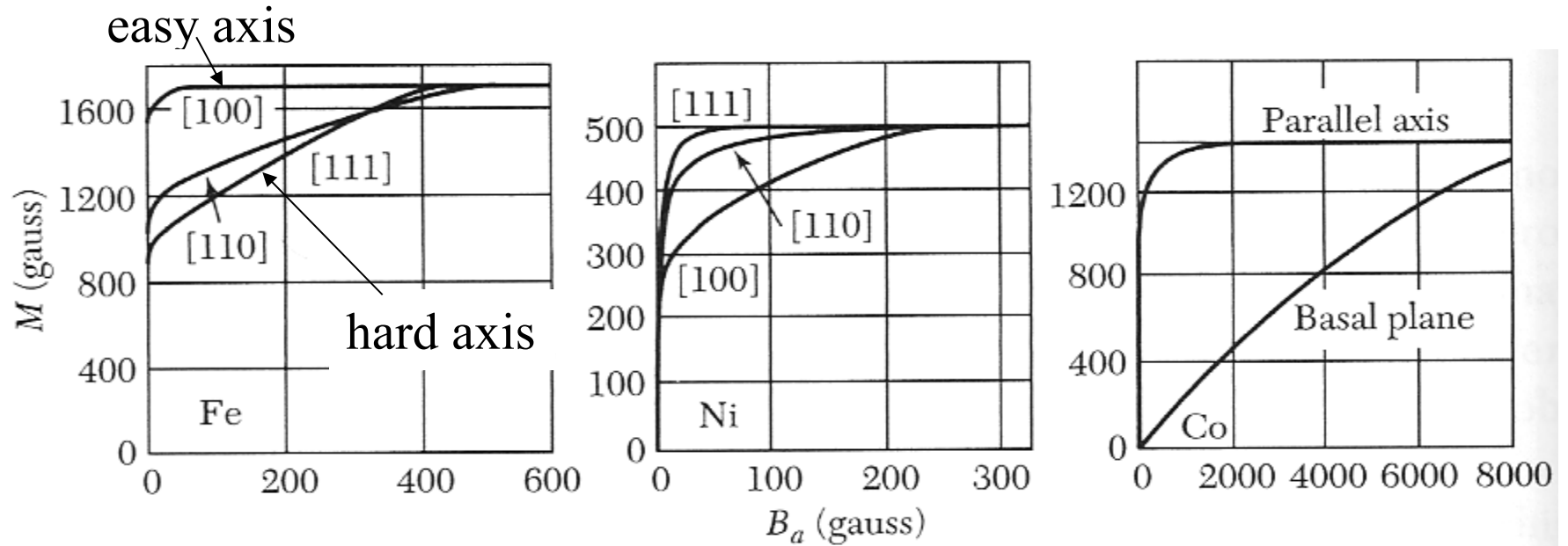
$$M = \chi H$$

$$B = \mu_0 (1 + \chi) H = \mu_r \mu_0 H$$

$$\mu_r = (1 + \chi)$$

Area of the loop is proportional to energy dissipated in traversing the loop.

# Anisotropy energy



Spin-orbit coupling couples the shape of the wavefunction to the spin. The exchange energy depends on the overlap of the wavefunctions and thus on spin direction.

# Bloch wall

energy of two spins

$$w = -J\vec{S}_i \cdot \vec{S}_j = -JS^2 \cos \varphi \approx -JS^2 \left(1 - \frac{\varphi^2}{2}\right)$$

neglecting the constant part

$$w \approx JS^2 \frac{\varphi^2}{2}$$

$$\varphi = \frac{\pi}{N}$$

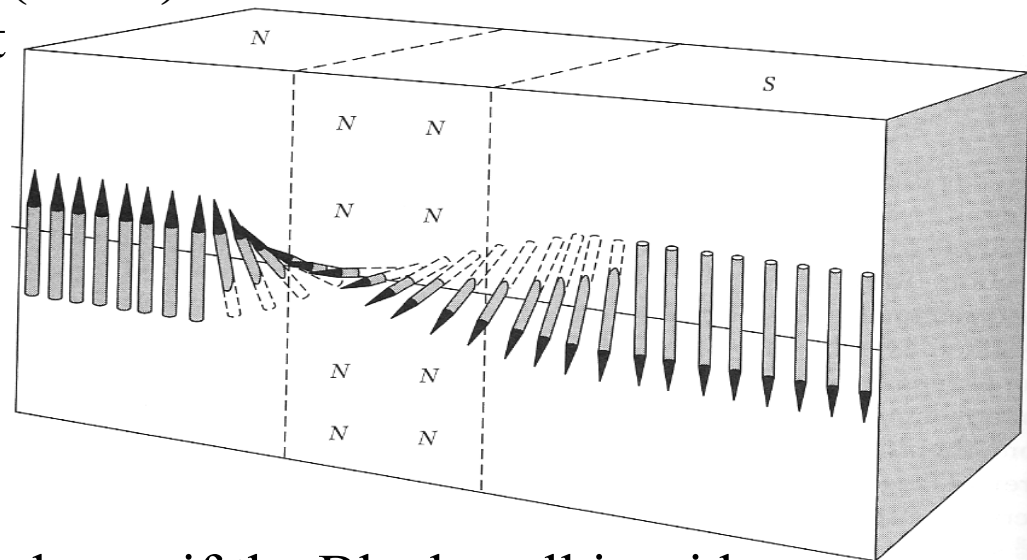
energy of a line of spins

$$Nw \approx \frac{JS^2 \pi^2}{2N}$$

energy is lower if the Bloch wall is wide

$$\text{energy per unit area} \approx \frac{JS^2 \pi^2}{2Na^2}$$

$a$  is the lattice constant





# Bloch wall

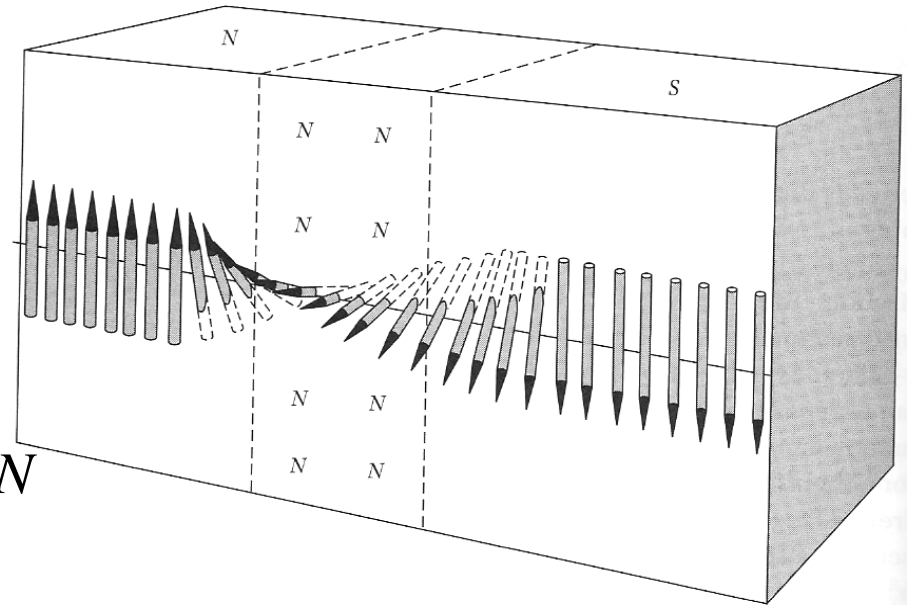
Anisotropy energy depends on the number of spins pointing in the hard direction

$\approx KNa$  ←  $Na = \text{thickness of wall}$   
 anisotropy constant  $\text{J/m}^3$

Total energy per unit area:

$$E = \frac{JS^2\pi^2}{2Na^2} + KNa \quad [\text{J/m}^2]$$

smaller for large  $N$     smaller for small  $N$



$$\frac{dE}{dN} = 0 \Rightarrow -\frac{JS^2\pi^2}{2N^2a^2} + Ka = 0$$

$$N = \sqrt{\frac{JS^2\pi^2}{2Ka^3}}$$

$N \sim 300$  for iron

# Soft magnetic materials

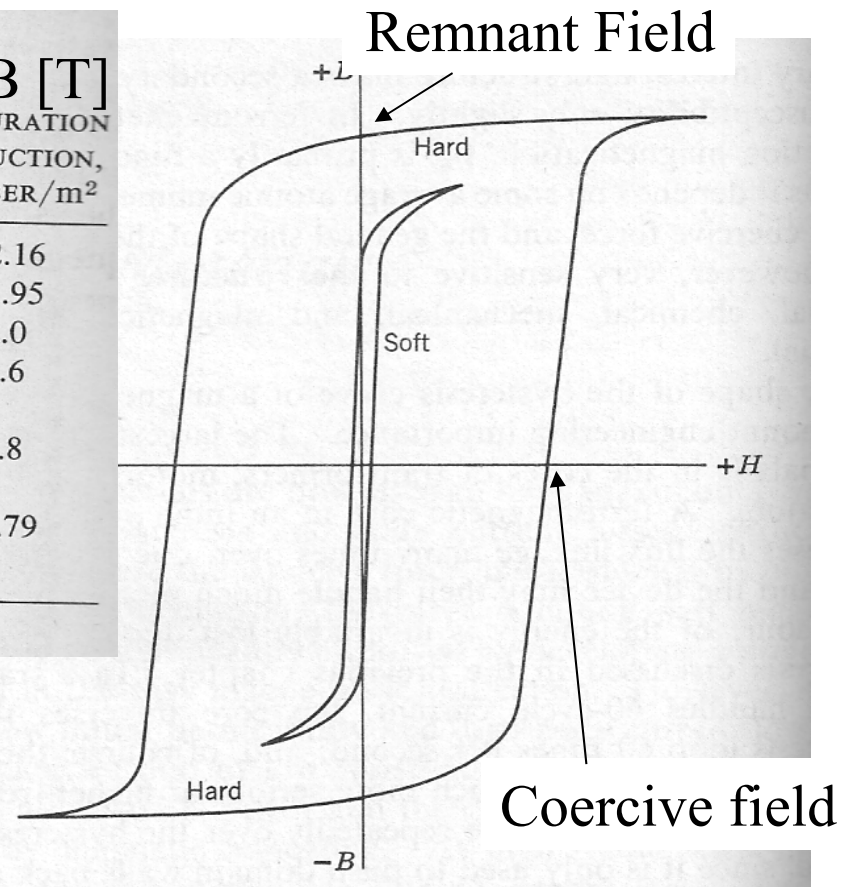
## soft magnets

MATERIAL	INITIAL RELATIVE PERMEABILITY ( $\mu_r$ AT $B \sim 0$ )	HYSTERESIS LOSS JOULE/m <sup>3</sup> PER CYCLE	B [T] SATURATION INDUCTION, WEBER/m <sup>2</sup>
Commercial iron ingot	250	500	2.16
Fe-4% Si, random	500	50-150	1.95
Fe-3% Si, oriented	15,000	35-140	2.0
45 Permalloy (45% Ni-55% Fe)	2,700	120	1.6
Mumetal (75% Ni-5% Cu-2% Cr-18% Fe)	30,000	20	0.8
Supermalloy (79% Ni-15% Fe-5% Mo-0.5% Ma)	100,000	2	0.79

transformers

magnetic shielding

ferrites have low eddy current losses



$$B = \mu_0 (H + M)$$

$$B = \mu_r \mu_0 H$$

$$M = \chi H$$

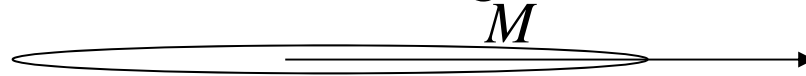
$$\mu_r = 1 + \chi$$

# Single domain particles

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Small 10 - 100 nm particles have single domains.

Elongated particles have the magnetization along the long axis.

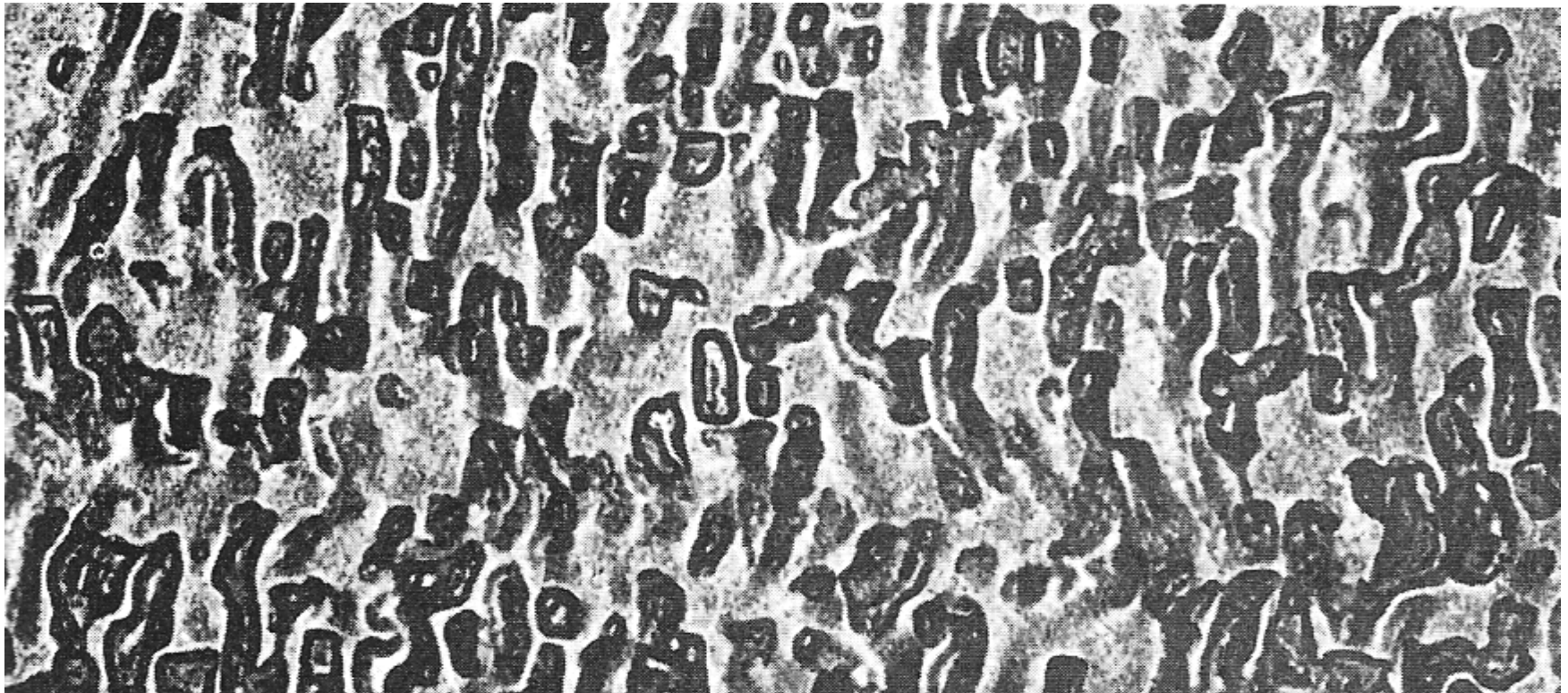


Single domains are used for magnetic recording. Long crystals can be magnetized in either of the two directions along the long axis.

Shape anisotropy.

# Hard magnets

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Grains too small to contain Bloch walls must be flipped entirely by the field.

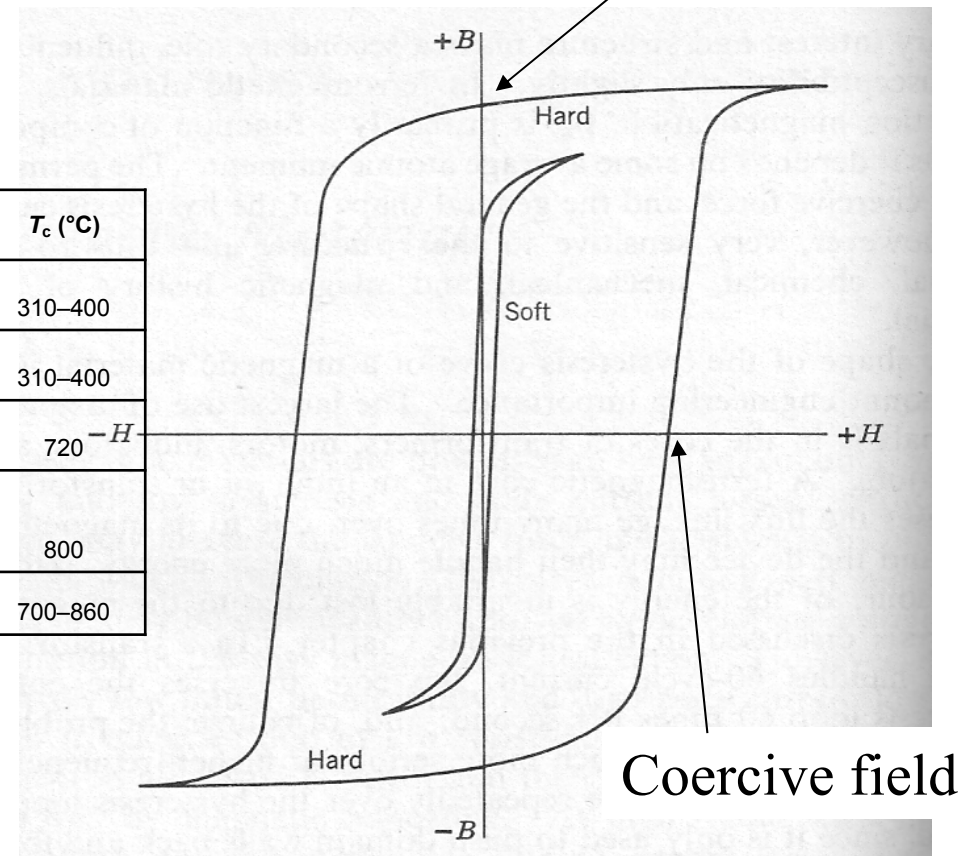
Alnico: 8-12% Al, 15-26% Ni, 5-24% Co, up to 6% Cu, up to 1% Ti, rest is Fe

# Hard magnetic materials

Remnant Field

hard magnets

Magnet	$B_r$ (T)	$H_{ci}$ (kA/m)	$(BH)_{max}$ (kJ/m <sup>3</sup> )	$T_c$ (°C)
Nd <sub>2</sub> Fe <sub>14</sub> B (sintered)	1.0–1.4	750–2000	200–440	310–400
Nd <sub>2</sub> Fe <sub>14</sub> B (bonded)	0.6–0.7	600–1200	60–100	310–400
SmCo <sub>5</sub> (sintered)	0.8–1.1	600–2000	120–200	720 <sup>-H</sup>
Sm(Co,Fe,Cu,Zr) <sub>7</sub> (sintered)	0.9–1.15	450–1300	150–240	800
Alnico (sintered)	0.6–1.4	275	10–88	700–860



Permanent magnets, magnetron,  
motors, generators  
ferrites can also be hard magnets

Defects are introduced to pin the Bloch walls in a hard magnet.