# 17. Microwave engineering / Superconductivity

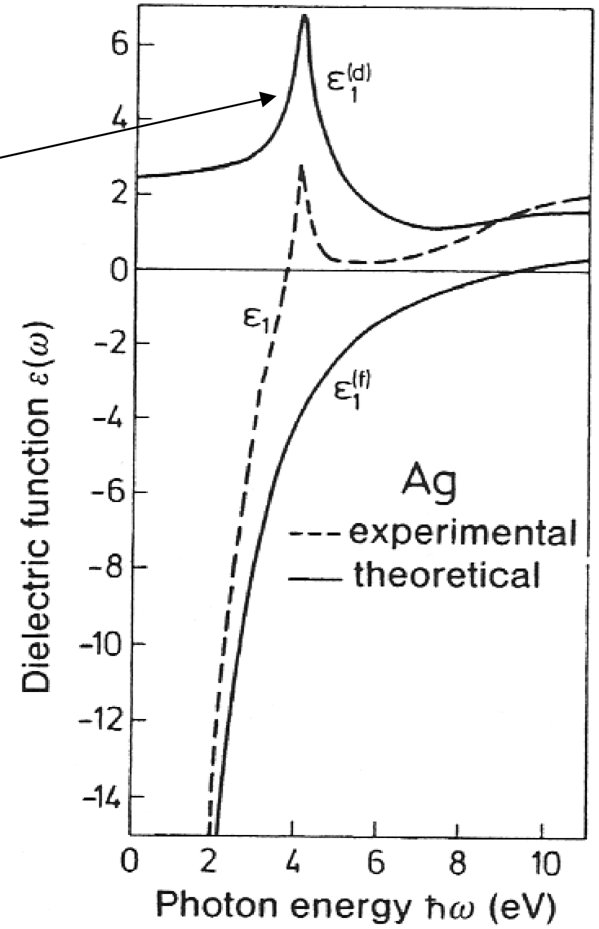
Nov. 29, 2018

# Intraband transitions

When the bands are parallel, there is a peak in the absorption (ε")

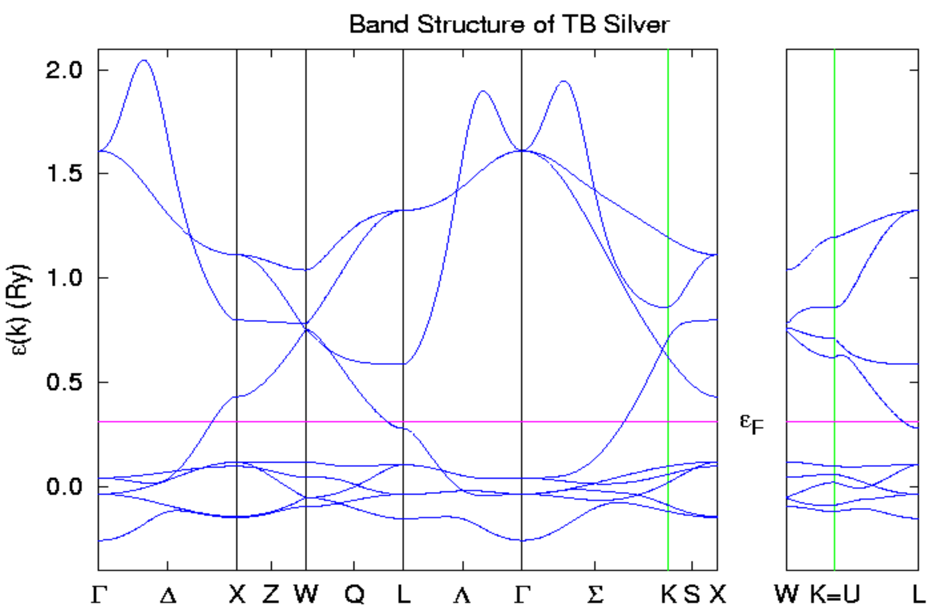
$$\hbar\omega = E_c(\vec{k}) - E_v(\vec{k})$$

Intraband (d-band) absorption

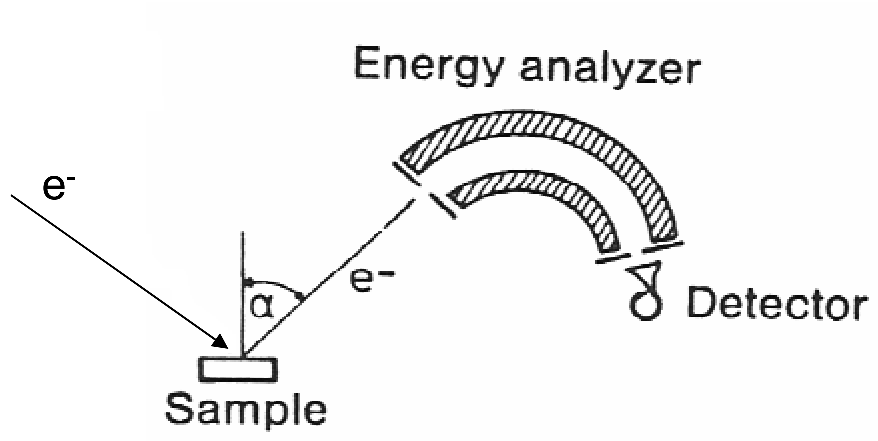



Ibach &Lueth

1 Ry = 13.6 ev

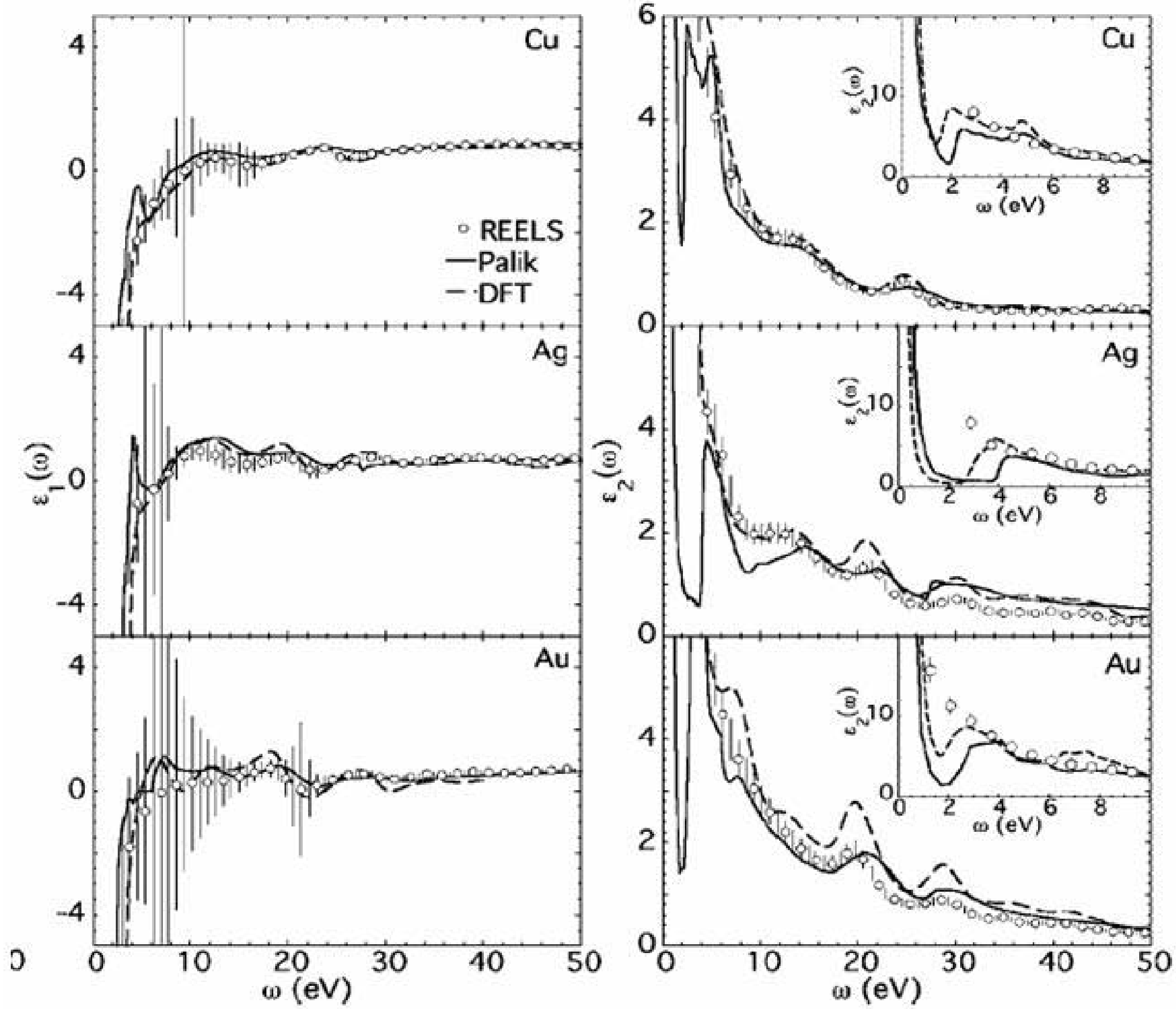


# Reflection Electron Energy Loss Spectroscopy




Fast electrons moving through the solid generate and a time dependent electric field. If the polarization moves out of phase with this field, energy will be lost. This is detected in the reflected electrons.

## Dielectric function of Cu, Ag, and Au obtained from reflection electron energy loss spectra, optical measurements, and density functional theory




Werner (TU Vienna) APL 89 213106 (2006)

# Microwave engineering

Microwave frequencies are a few GHz

The wavelength is smaller than the circuit

Losses in metals increase with increasing frequency

Losses in dielectrics increase with increasing frequency

There is a characteristic length scale called the skin depth which tells us how far into a metal fields penetrate before they are reflected out.

# Skin depth $\omega\tau << 1$

$$\sigma(\omega) = ne\mu\left(\frac{1 - i\omega\tau}{1 + \omega^2\tau^2}\right) \approx ne\mu = \sigma_0$$

$\omega\tau << 1$

Ohm's law

$$\vec{J} = \sigma_0 \vec{E}$$

Take the curl

$$\frac{1}{\sigma_0}\nabla \times \vec{J} = \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

Faraday's law

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Ampere's law

$$\frac{1}{\sigma_0 \mu_0}\nabla \times \nabla \times \vec{B} = -\frac{d\vec{B}}{dt}$$

Vector identity

$$\nabla \times \nabla \times \vec{B} = \nabla\left(\overset{0}{\cancel{\nabla \cdot \vec{B}}}\right) - \nabla^2 \vec{B}$$

$$\frac{1}{\sigma_0 \mu_0}\nabla^2 \vec{B} = \frac{d\vec{B}}{dt}$$

# Skin depth

$$\frac{1}{\sigma_0 \mu_0} \nabla^2 \vec{B} = \frac{d\vec{B}}{dt}$$

Assume harmonic dependence $\quad B_0 e^{i(kx - \omega t)}$

$$\frac{k^2}{\sigma_0 \mu_0} = i\omega$$

$$k = \sqrt{i\omega\sigma_0\mu_0} = \sqrt{\frac{\omega\sigma_0\mu_0}{2}} + i\underbrace{\sqrt{\frac{\omega\sigma_0\mu_0}{2}}}_{\text{Exponential decay}}$$

Skin depth $\quad \delta = \sqrt{\frac{2}{\mu_0 \sigma_0 \omega}}$

Light $\omega < \omega_p$ is reflected out of a metal. The waves penetrate a length $\delta$.

# Skin depth

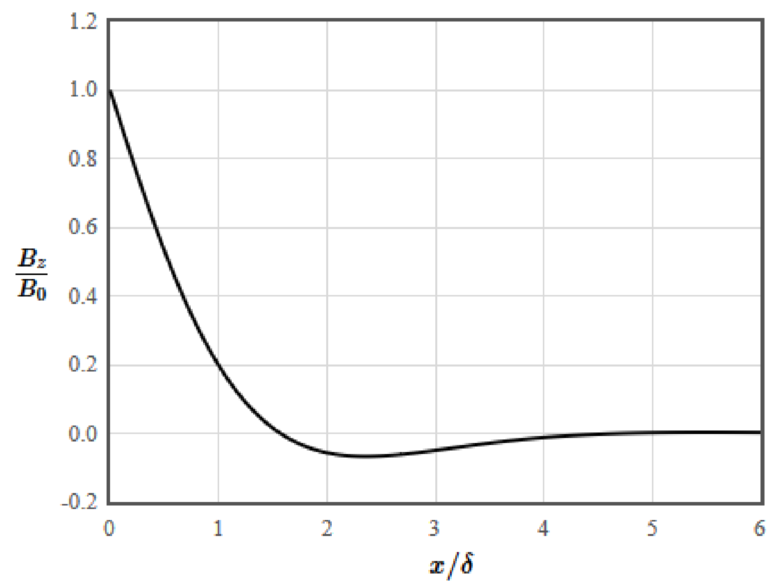
$$\vec{B} = B_0 e^{-x/\delta} e^{i(x/\delta - \omega t)} \hat{z}$$

$$\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B} = \vec{B} = \frac{B_0(1-i)}{\mu_0 \delta} e^{-x/\delta} e^{i(x/\delta - \omega t)} \hat{y}$$

$$1 - i = \sqrt{2} e^{-i\pi/4}$$

$$\vec{J} = \frac{\sqrt{2} B_0}{\mu_0 \delta} e^{-x/\delta} e^{i(x/\delta - \omega t - \pi/4)} \hat{y}$$

$$\vec{E} = \frac{\vec{J}}{\sigma_0} = \frac{\sqrt{2} B_0}{\mu_0 \delta \sigma_0} e^{-x/\delta} e^{i(x/\delta - \omega t - \pi/4)} \hat{y}$$

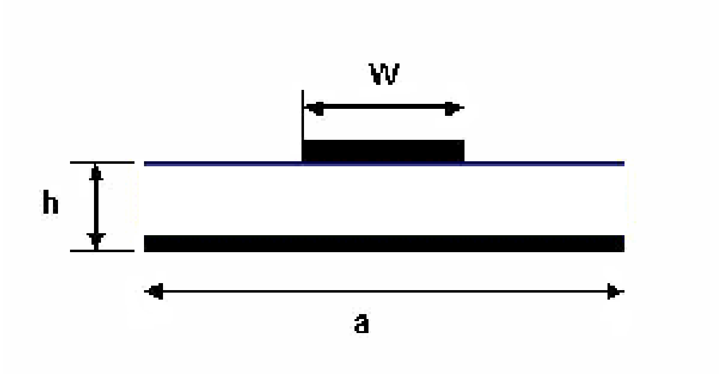



The electric field lags behind the magnetic field by 45 degrees.

# Surface resistance

At low frequencies: $$R = \frac{\rho\ell}{wt} = \frac{\ell}{\sigma_0 wt}$$

When $\delta < t$ : $$R = \frac{\ell}{\sigma_0 w\delta}$$



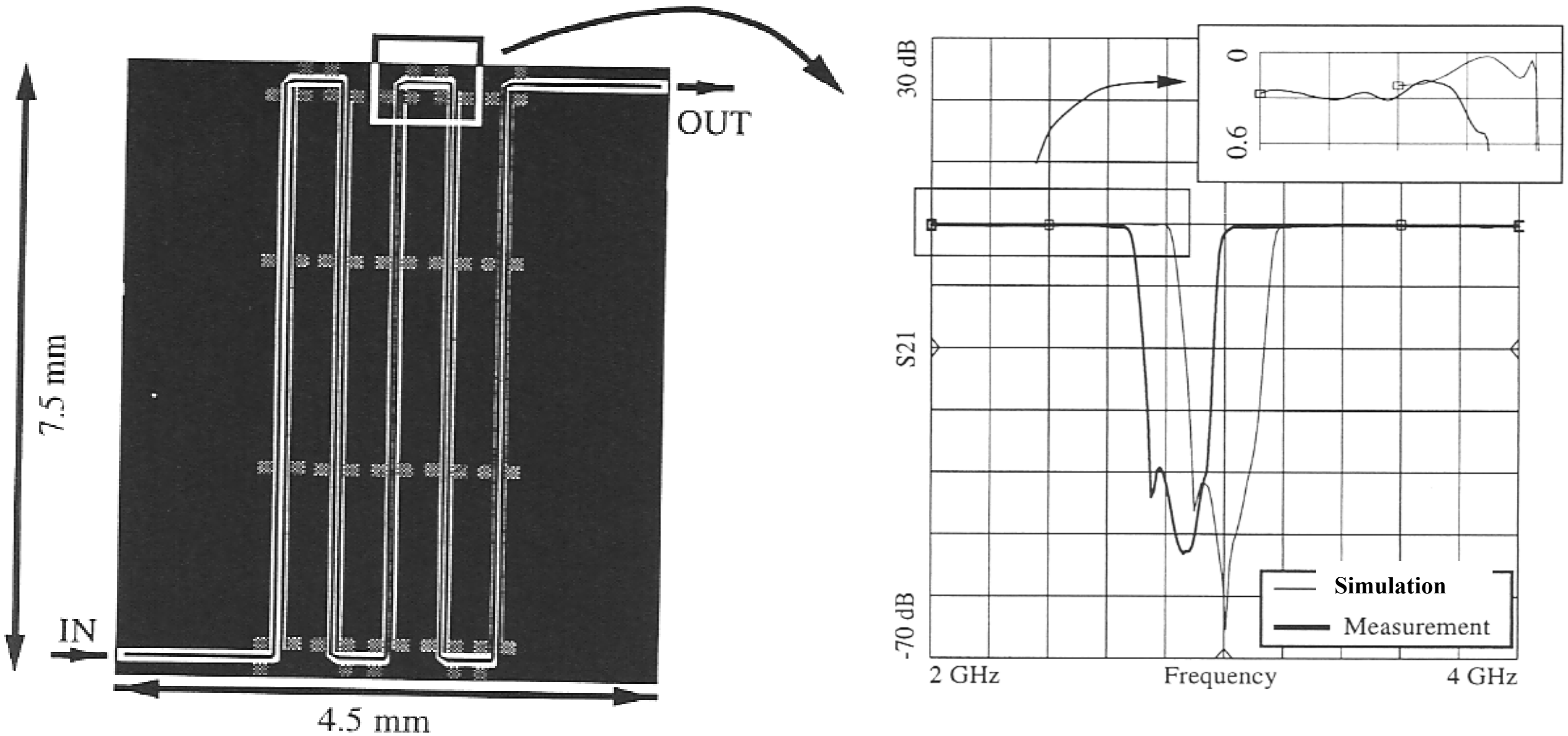

for $$\ell = w$$

$$R_s = \frac{1}{\sigma_0\delta} \propto \sqrt{\omega}$$

Complex signal processing at high frequencies > 1 GHz is difficult because the losses increase with frequency.

Usually you mix down to a lower frequency as soon as possible.

# Superconducting filter




Complex signal processing at high frequencies > 1 GHz is difficult because the losses increase with frequency.

# Superconductivity

Primary characteristic: zero resistance at dc

There is a critical temperature $T_c$ above which superconductivity disappears

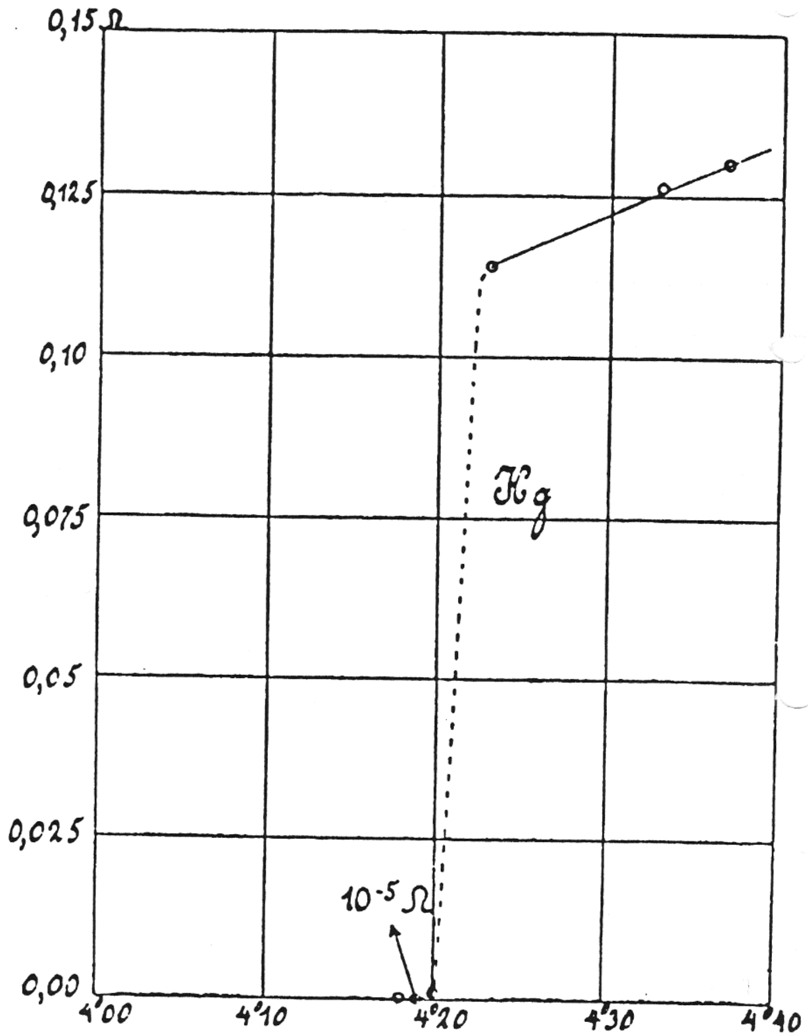
About 1/3 of all metals are superconductors

Metals are usually superconductors OR magnetic, not both

Good conductors are bad superconductors

Kittel chapter 10

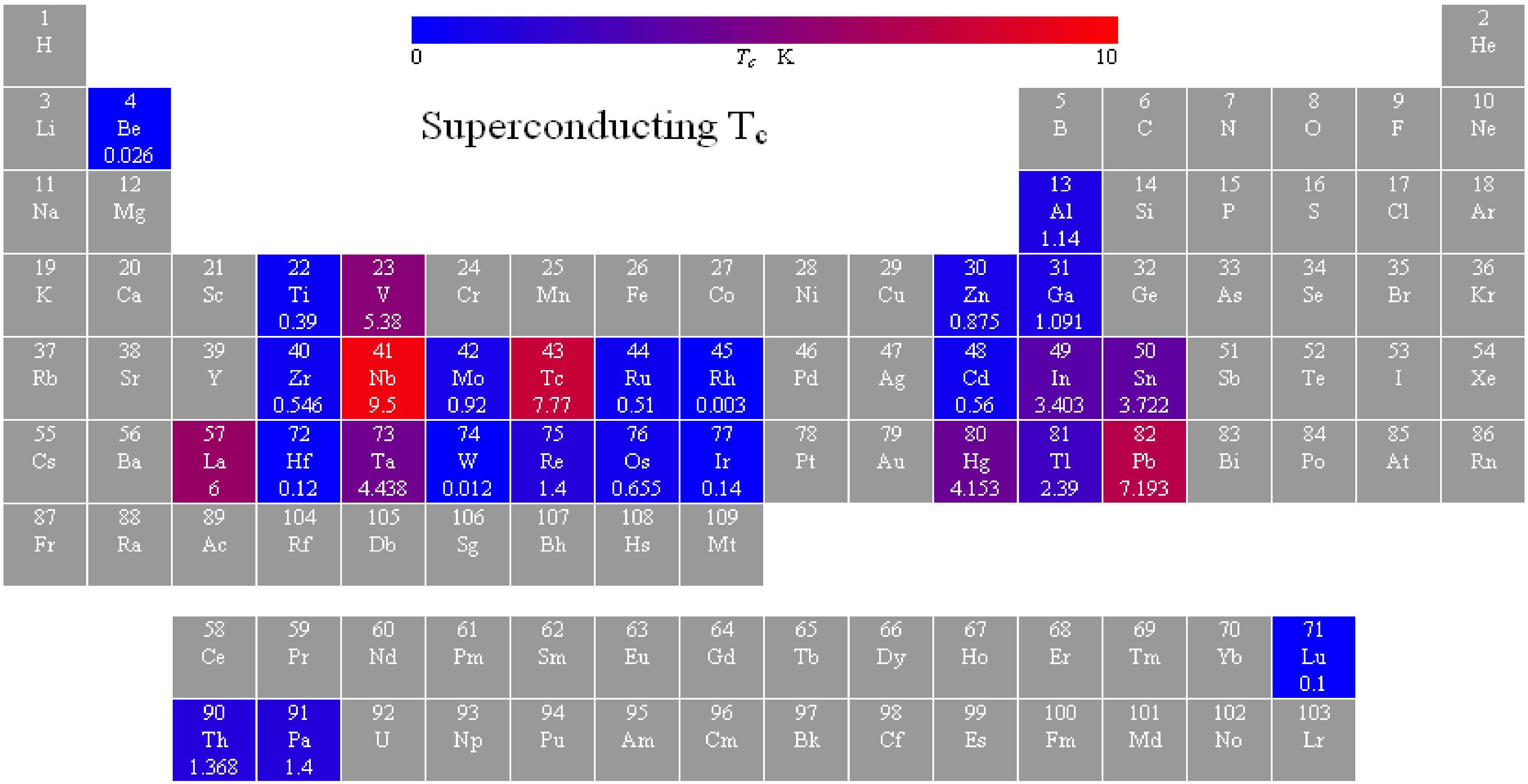
# Superconductivity



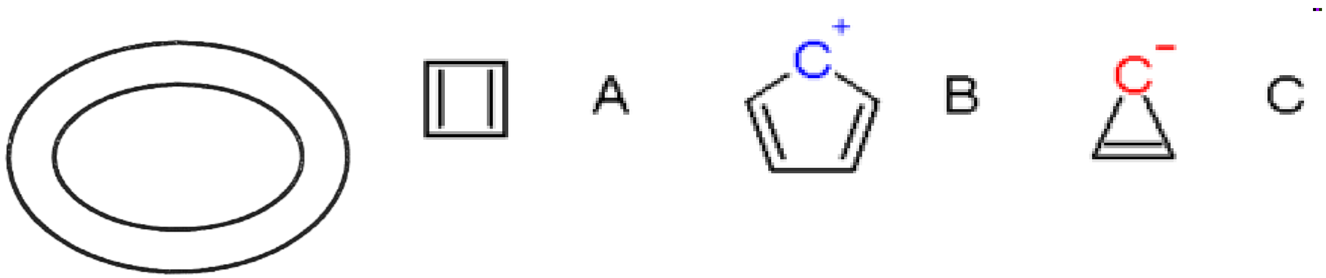

Heike Kammerling-Onnes

Superconductivity was discovered in 1911

# Critical temperature



# Superconductivity

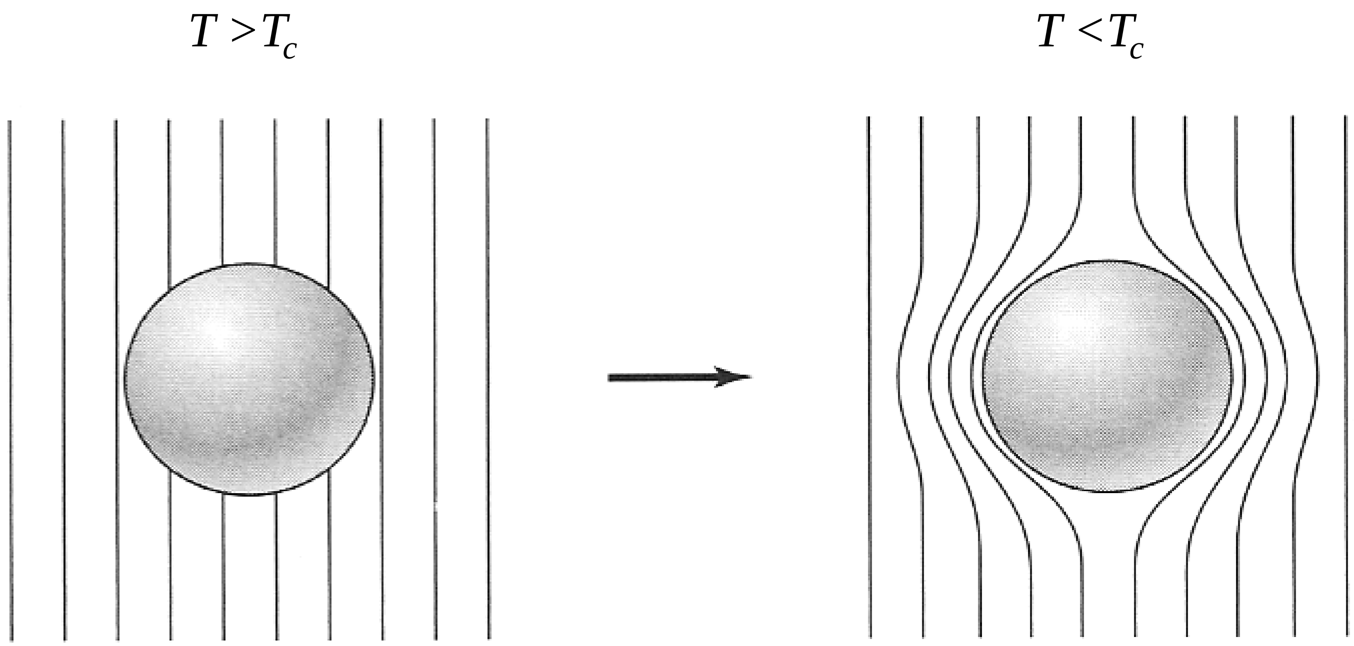



Superconducting ring     Molecule with magnetic moment

Antiaromatic molecules are unstable and highly reactive

No measurable decay in current after 2.5 years. $\rho < 10^{-25}\ \Omega\mathrm{m}$.

# Meissner effect




Superconductors are perfect diamagnets at low fields.
$B = 0$ inside a bulk superconductor.

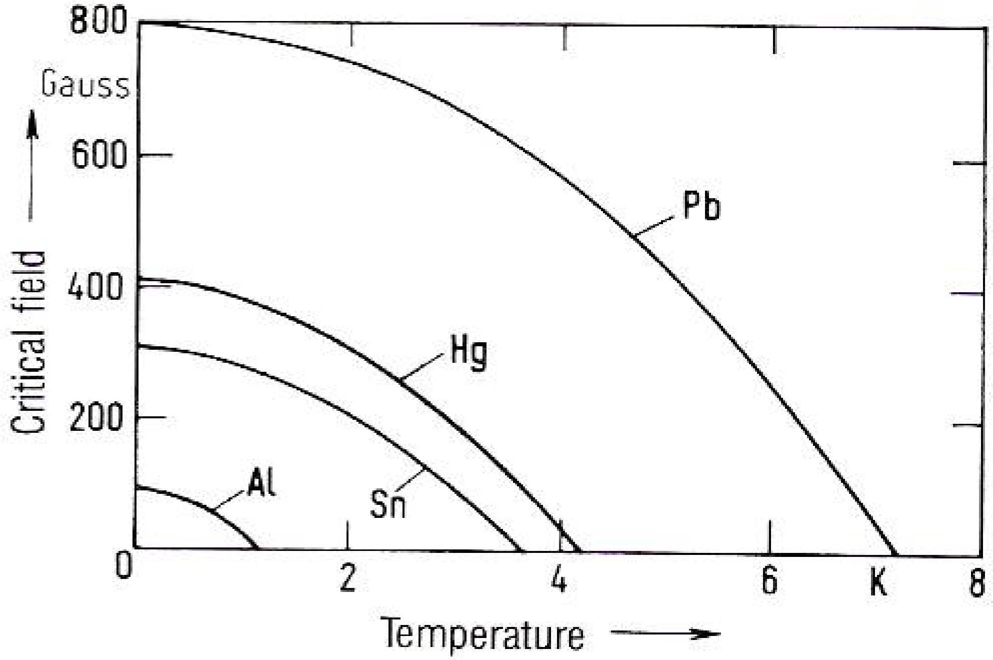
Superconductors are used for magnetic shielding.

# Superconductivity

Critical temperature $T_c$

Critical current density $J_c$

Critical field $H_c$




$$n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \tfrac{1}{2} nmv^2 = \frac{m}{2ne^2} J_c^2$$