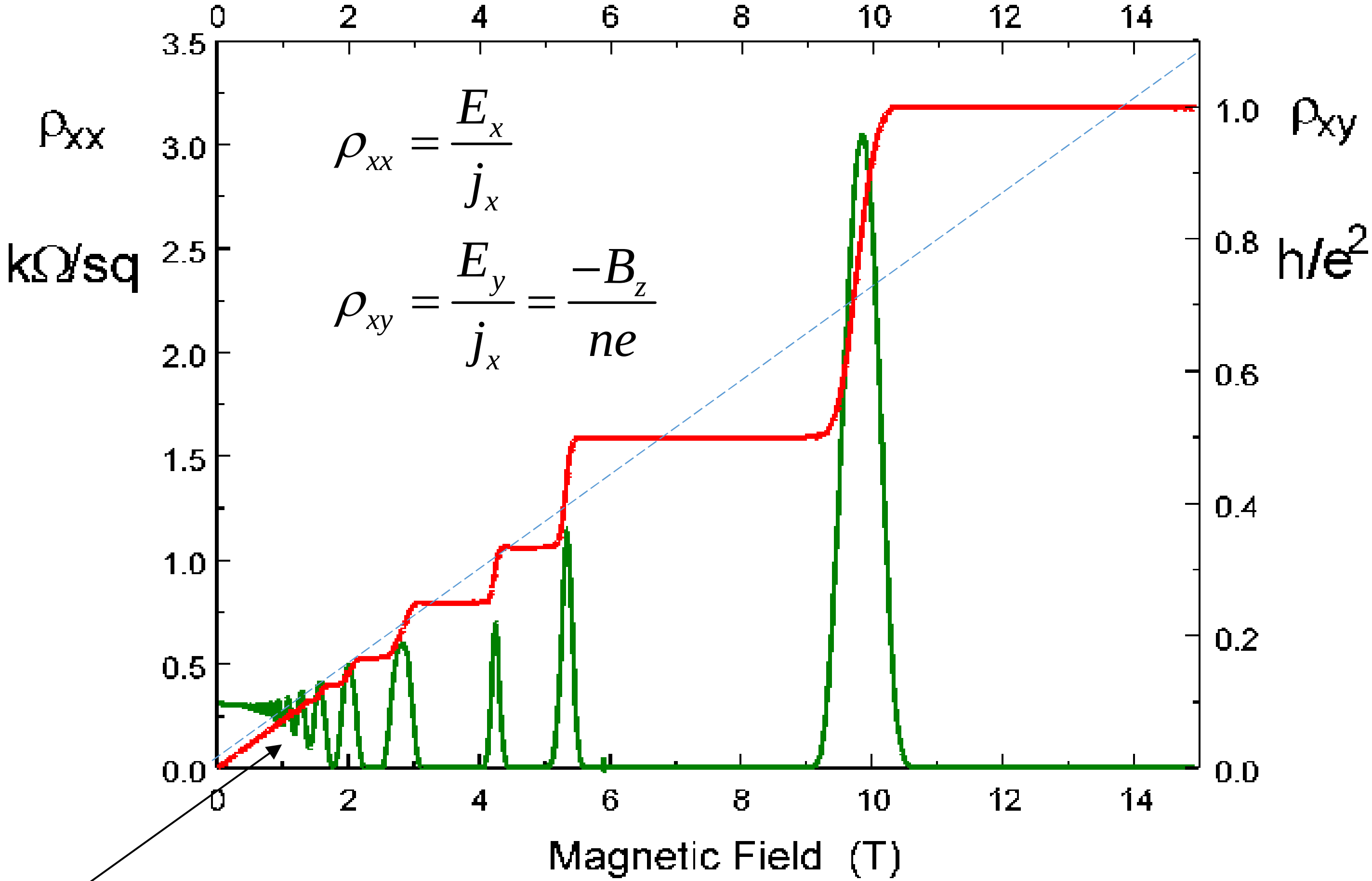
Technische Universität Graz

11. Quantum Hall Effect / Fermi Surfaces

Nov. 8, 2018

Quantum Hall Effect



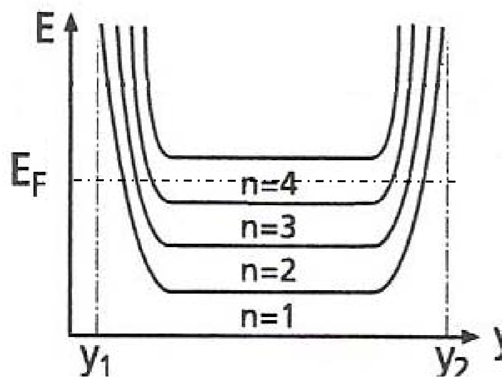
Shubnikov-De Haas oscillations

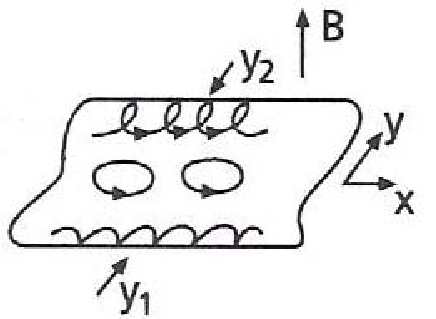
Resistance standard
25812.807557(18) Ω

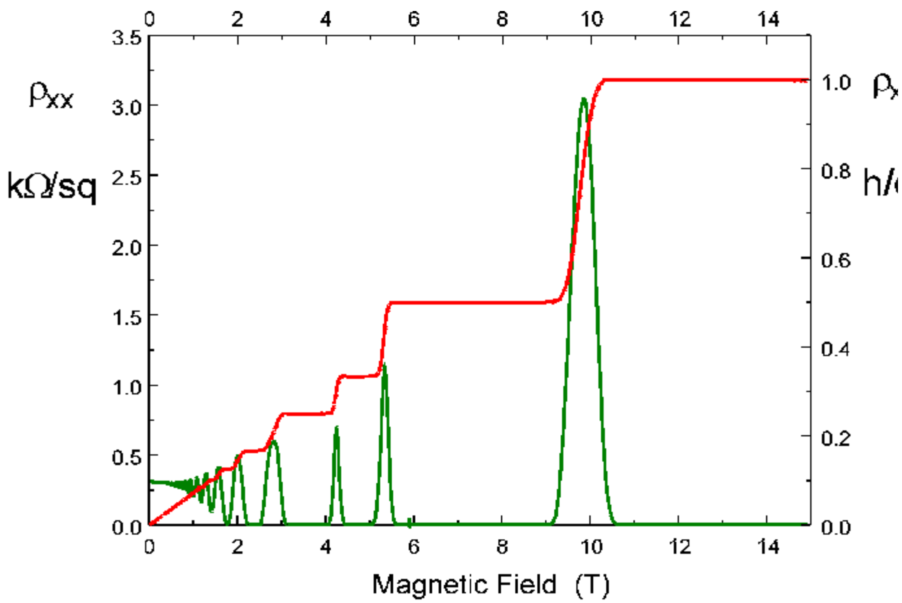
Quantum Hall effect

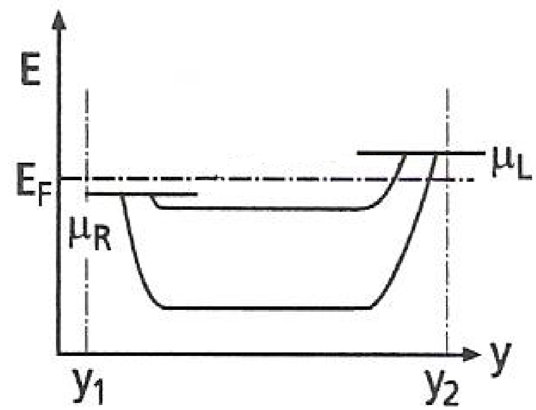
Edge states are responsible for the zero resistance in ρ_{xx}

On the plateaus, resistance goes to zero because there are no states to scatter into.



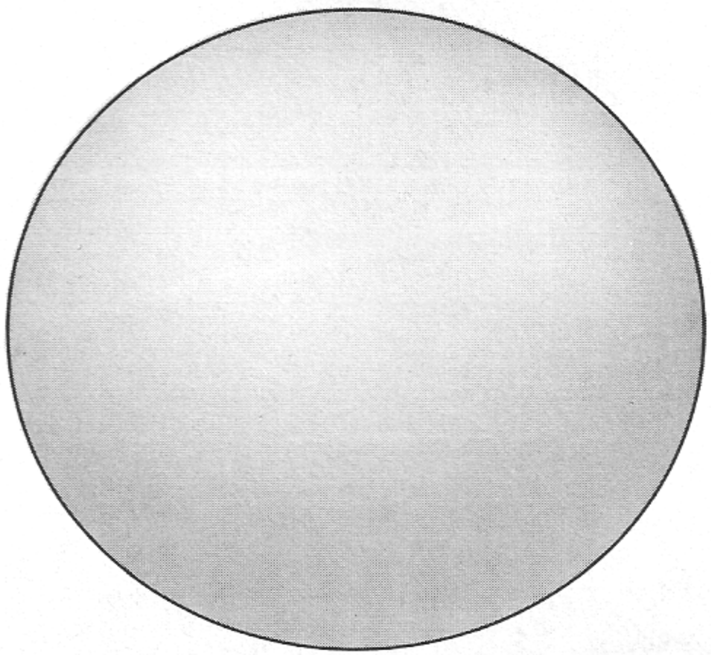




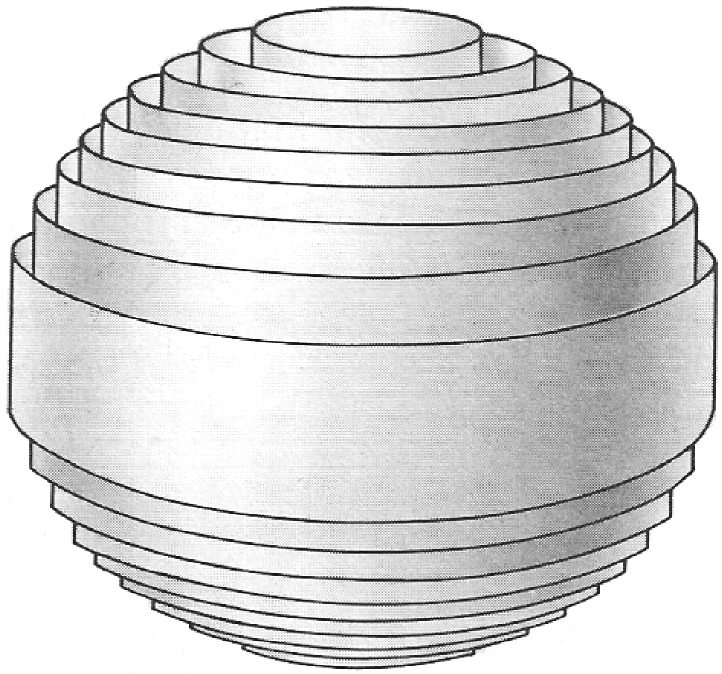


Ibach & Lueth (modified)

Fermi sphere in a magnetic field



$B = 0$

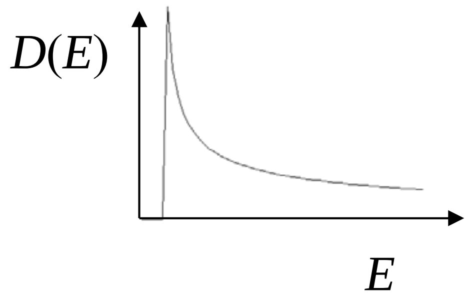


$B \neq 0$

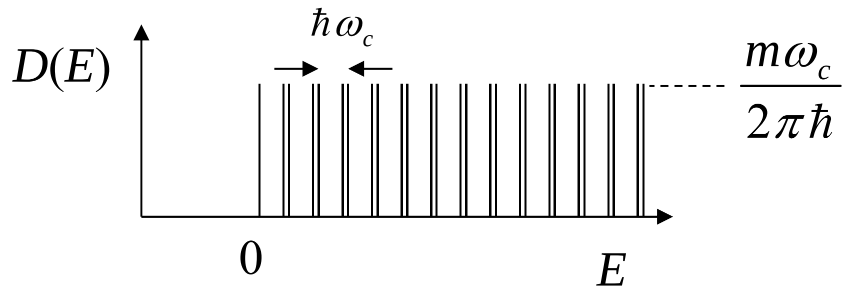
Landau cylinders

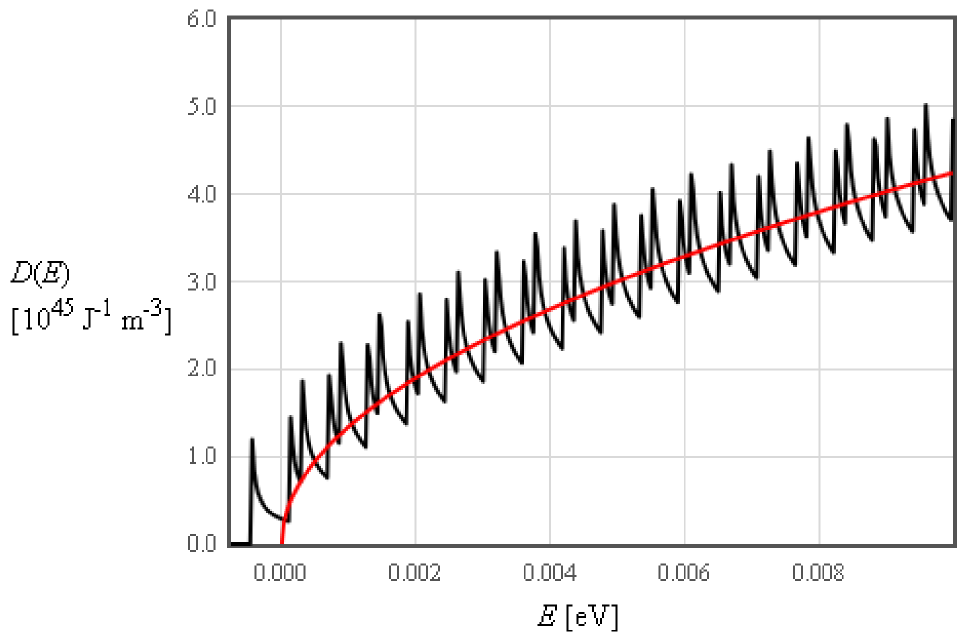
Density of states 3d

convolution of



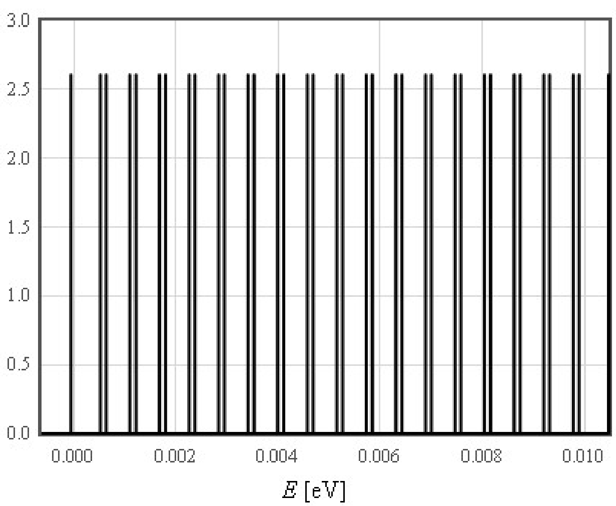
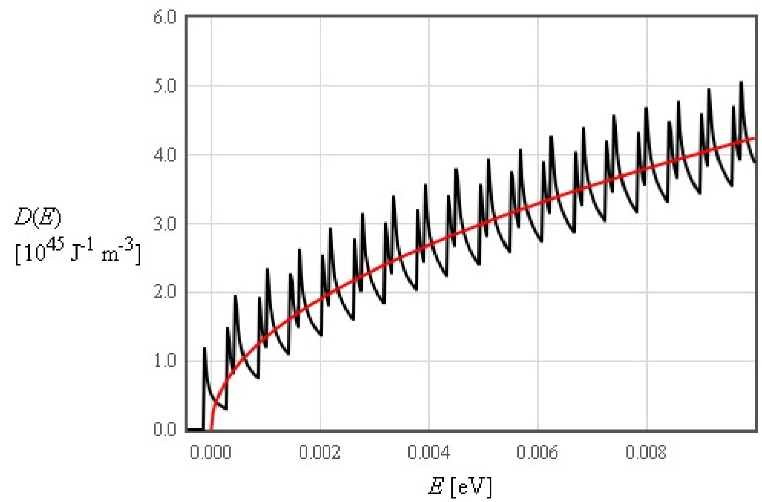
and





$$D(E)=\frac{(2m)^{3/2}\,\omega_c}{8\pi^2\hbar^2}\left(\sum_{\nu=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)}}+\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)}}\right)\qquad \mathrm{J^{-1}m^{-3}}$$

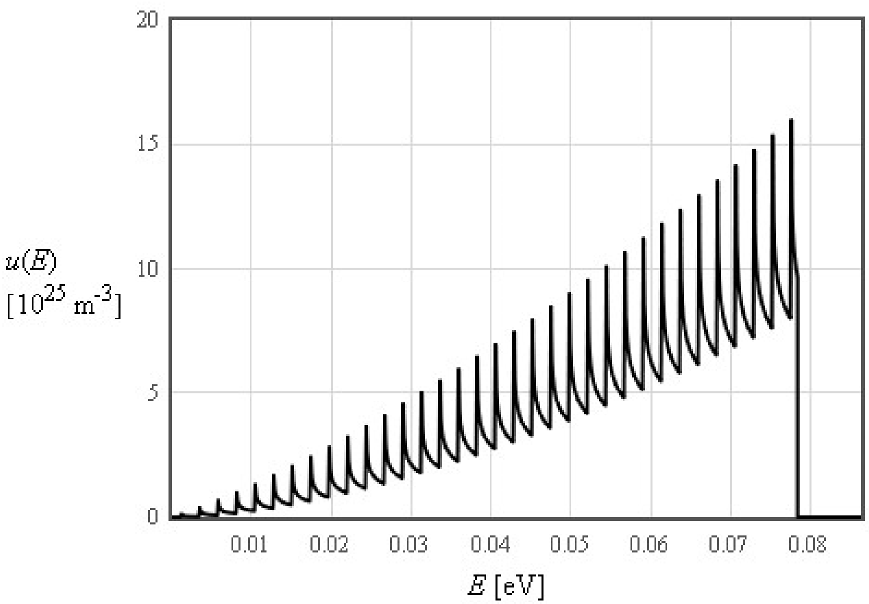
quation for free electrons a magnetic field in 2 and 3 dimensions.

2-D Schrödinger equation	3-D Schrödinger equation
$i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$	$i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$
$\psi=g_v(x)\exp\left(ik_yy\right)$ $g_v(x)$ is a harmonic oscillator wavefunction	$\psi=g_v(x)\exp\left(ik_yy\right)\exp\left(ik_zz\right)$ $g_v(x)$ is a harmonic oscillator wavefunction
$E=\hbar\omega_c\left(v+\frac{1}{2}\right)$ J $v=0,1,2,\cdots \qquad \omega_c=\frac{\lvert eB_z\rvert}{m}$	$E=\frac{\hbar^2k_z^2}{2m}+\hbar\omega_c\left(v+\frac{1}{2}\right)$ J $v=0,1,2,\cdots \qquad \omega_c=\frac{\lvert eB_z\rvert}{m}$
$\sum_{v=0}^{\infty}\delta\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)-\frac{g\mu_B}{2}B\right)+\delta\left(E-\hbar\omega_c\left(v+\frac{1}{2}\right)+\frac{g\mu_B}{2}B\right)$ J^{-1}m^{-2}  Calculate DoS	$D(E)=\frac{(2m)^{3/2}\omega_c}{8\pi^2\hbar^2}\left(\sum_{v=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(v+\frac{1}{2}-g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(v+\frac{1}{2}-g/4\right)}}+\frac{H\left(E-\hbar\omega_c\left(v+\frac{1}{2}+g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(v+\frac{1}{2}+g/4\right)}}\right)$ J^{-1}m^{-3}  Calculate DoS
$E_F=\hbar\omega\left(\text{Int}\left(\frac{\pi\hbar n}{\ldots}\right)+\frac{1}{\ldots}\right)$	

Energy spectral density 3d

$$u(E) = ED(E)f(E)$$

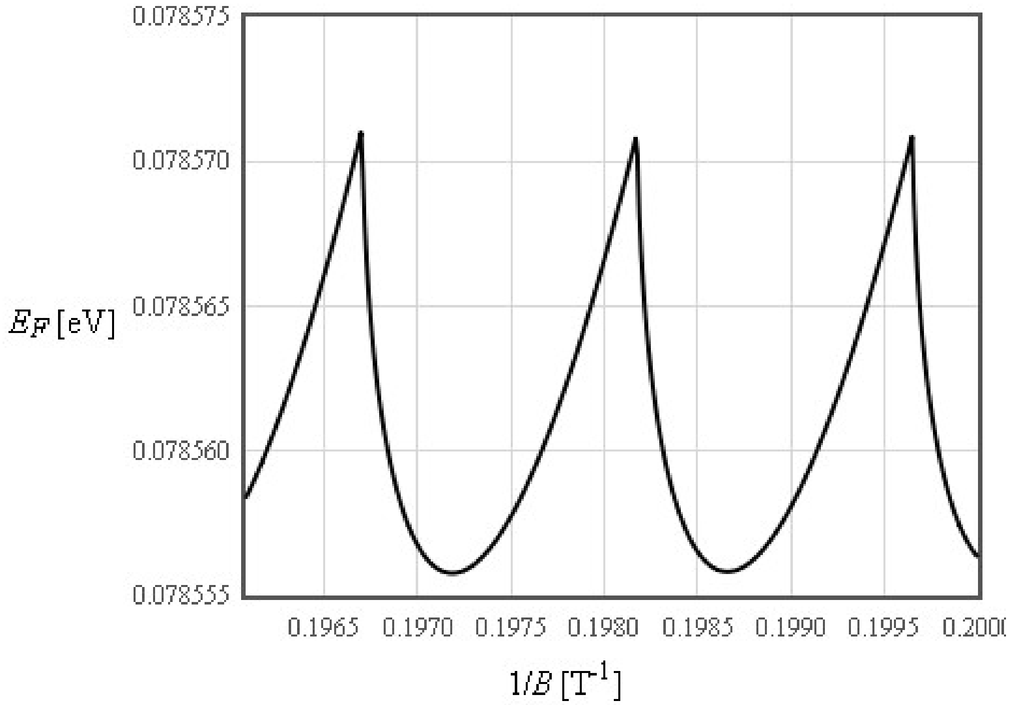
At $T = 0$



$$u(T=0) = \int_{-\infty}^{E_F} ED(E)dE$$

Fermi energy 3d

$$n = \int_{-\infty}^{E_F} D(E)dE$$

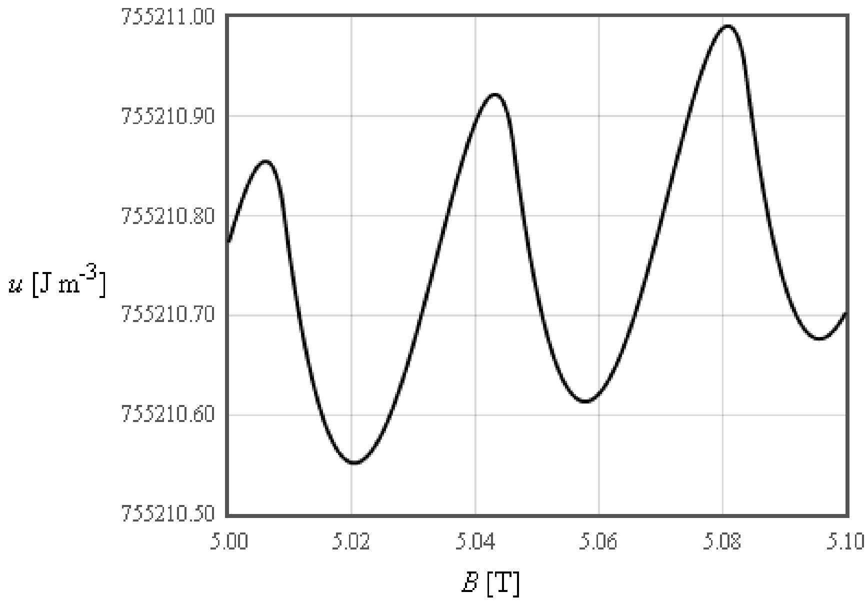


Periodic in 1/B

Internal energy 3d

$$u = \int_{-\infty}^{E_F} ED(E)dE$$

At $T = 0$

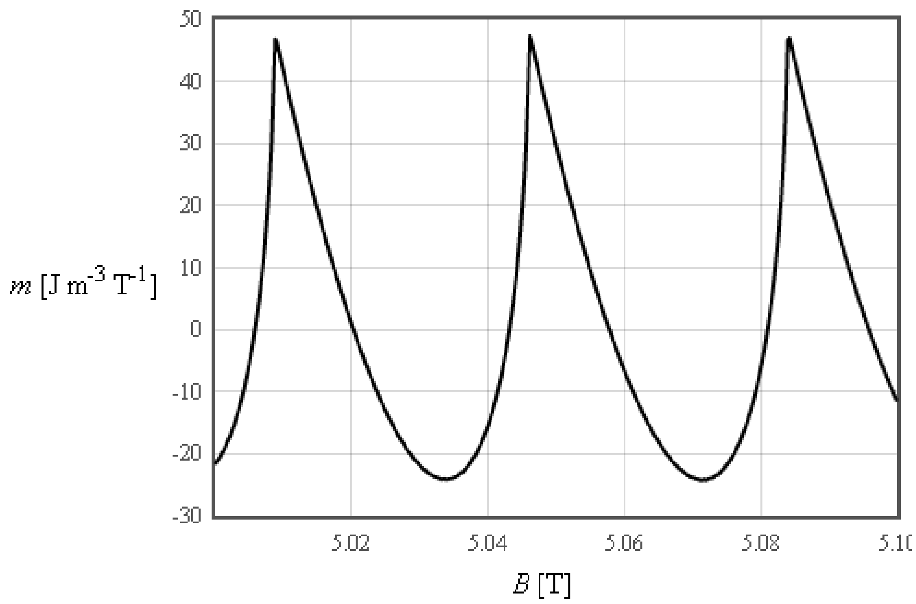


$$u = \frac{(2m)^{3/2}\omega_c}{4\pi^2\hbar^2}\sum_{\nu=0}^{\nu<\frac{E_F}{\hbar\omega_c}-\frac{1}{2}}\int_{\hbar\omega_c(\nu+\frac{1}{2})}^{E_F}\frac{EdE}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}\right)}} \quad \text{J m}^{-3}$$

$$u = \frac{(2m)^{3/2}\omega_c}{6\pi^2\hbar^2}\sum_{\nu=0}^{\nu<\frac{E_F}{\hbar\omega_c}-\frac{1}{2}}\left(2\hbar\omega_c\left(\nu+\frac{1}{2}\right)+E_F\right)\sqrt{E_F-\hbar\omega_c\left(\nu+\frac{1}{2}\right)} \quad \text{J m}^{-3}$$

Magnetization 3d

$$m = -\frac{du}{dB}$$



Periodic in $1/B$

At finite temperatures this function would be smoother

de Haas - van Alphen oscillations

Practically all properties are periodic in 1/*B*

Internal energy

$$u = \int_{-\infty}^{\infty} E D(E) f(E) dE$$

Specific heat

$$c_v = \left(\frac{\partial u}{\partial T}\right)_{V=const}$$

Entropy

$$s = \int \frac{c_v}{T} dT$$

Helmholtz free energy

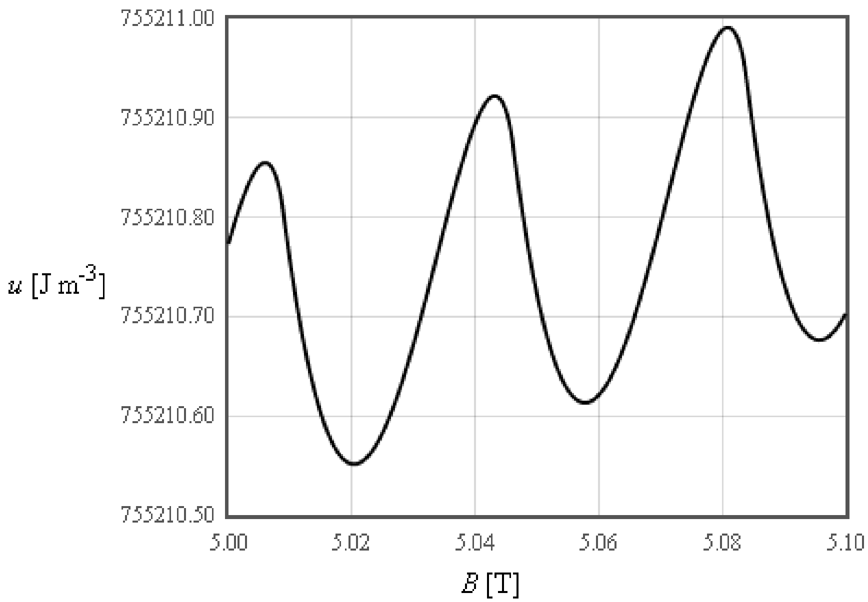
$$f = u - Ts$$

Pressure

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T=const}$$

Bulk modulus

$$B = -V \frac{\partial P}{\partial V}$$

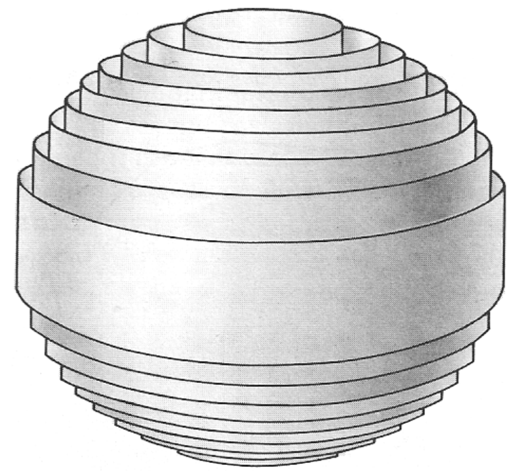


Magnetization

$$M = -\frac{dU}{dH}$$

Fermi sphere in a magnetic field

Cross sectional area $S = \pi k_F^2$



$$\hbar\omega_c\left(\nu+\tfrac{1}{2}\right)=\frac{\hbar^2 k_F^2}{2m}$$

$$\hbar\frac{eB_\nu}{m}\left(\nu+\tfrac{1}{2}\right)=\frac{\hbar^2 k_F^2}{2m}$$

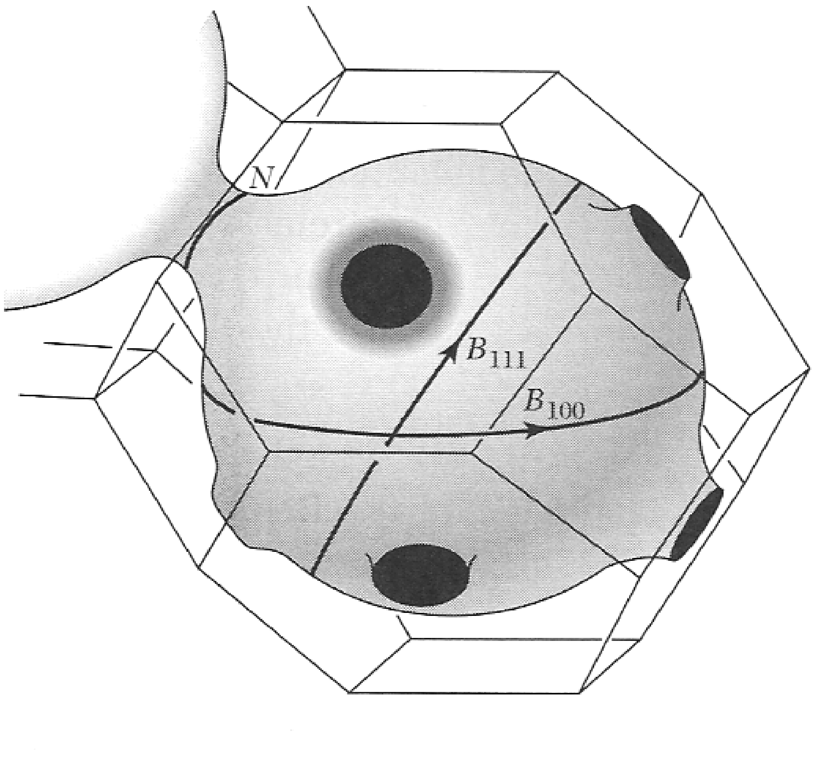
$$\frac{2\pi e}{\hbar}\left(\nu+1+\tfrac{1}{2}\right)=\frac{S}{B_{\nu+1}} \qquad \frac{2\pi e}{\hbar}\left(\nu+\tfrac{1}{2}\right)=\frac{S}{B_\nu}$$

Subtract right from left

$$S\left(\frac{1}{B_{\nu+1}}-\frac{1}{B_\nu}\right)=\frac{2\pi e}{\hbar}$$

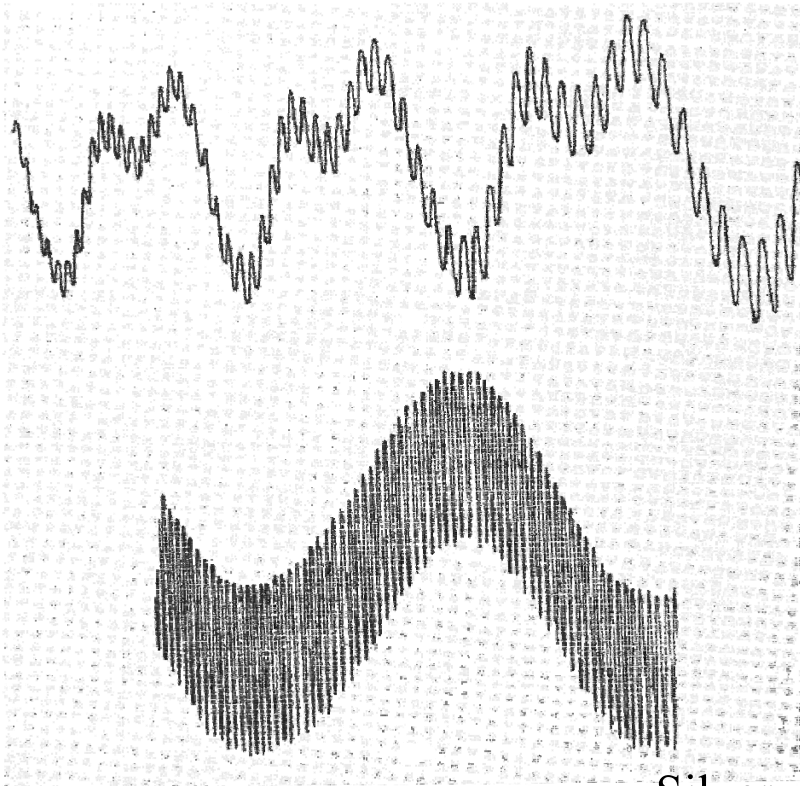
From the period of the oscillations, you can determine the cross sectional area *S*.

Experimental determination of the Fermi surface



Kittel

Rhenium

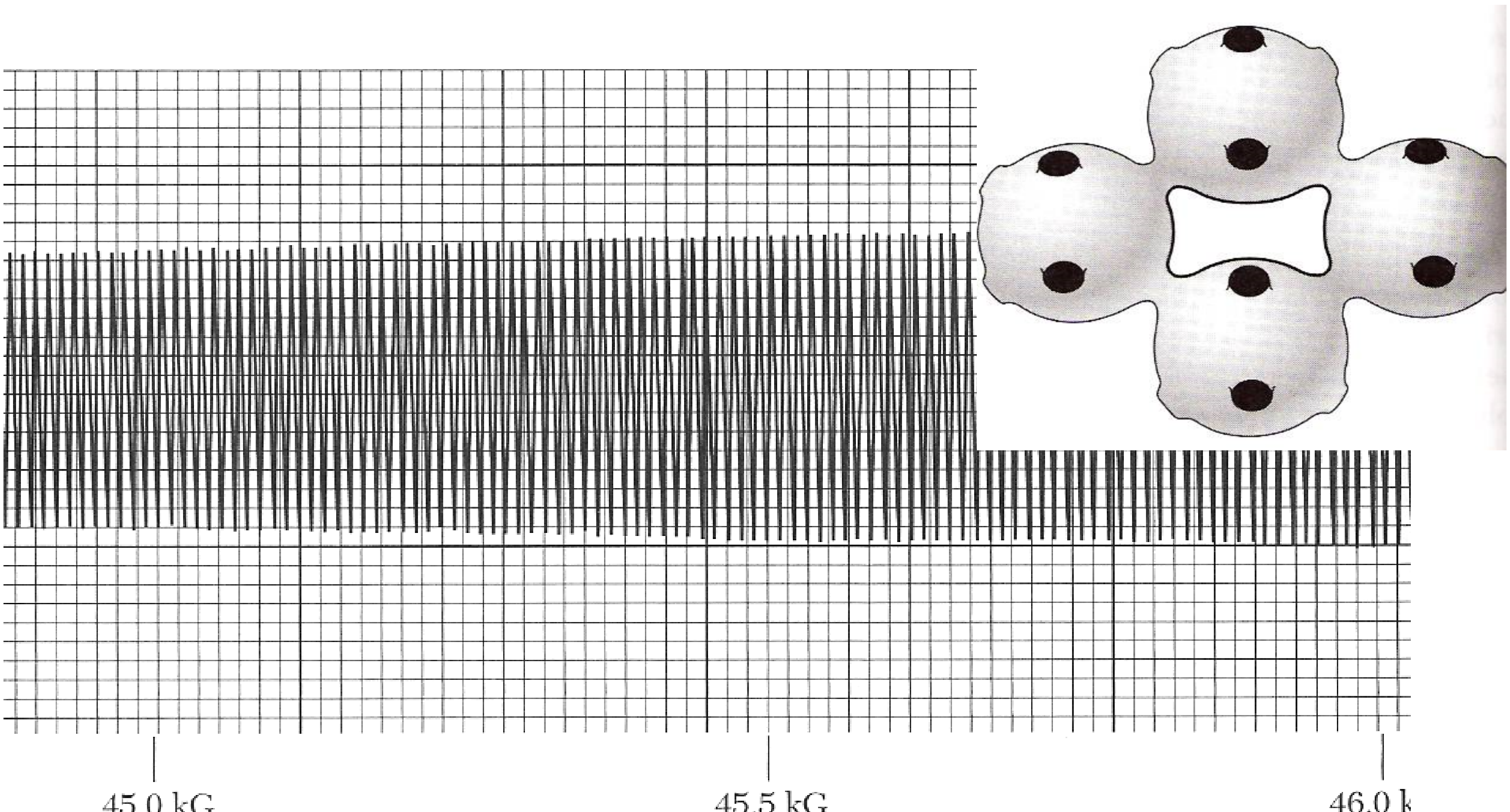


Silver

de Haas - van Alphen

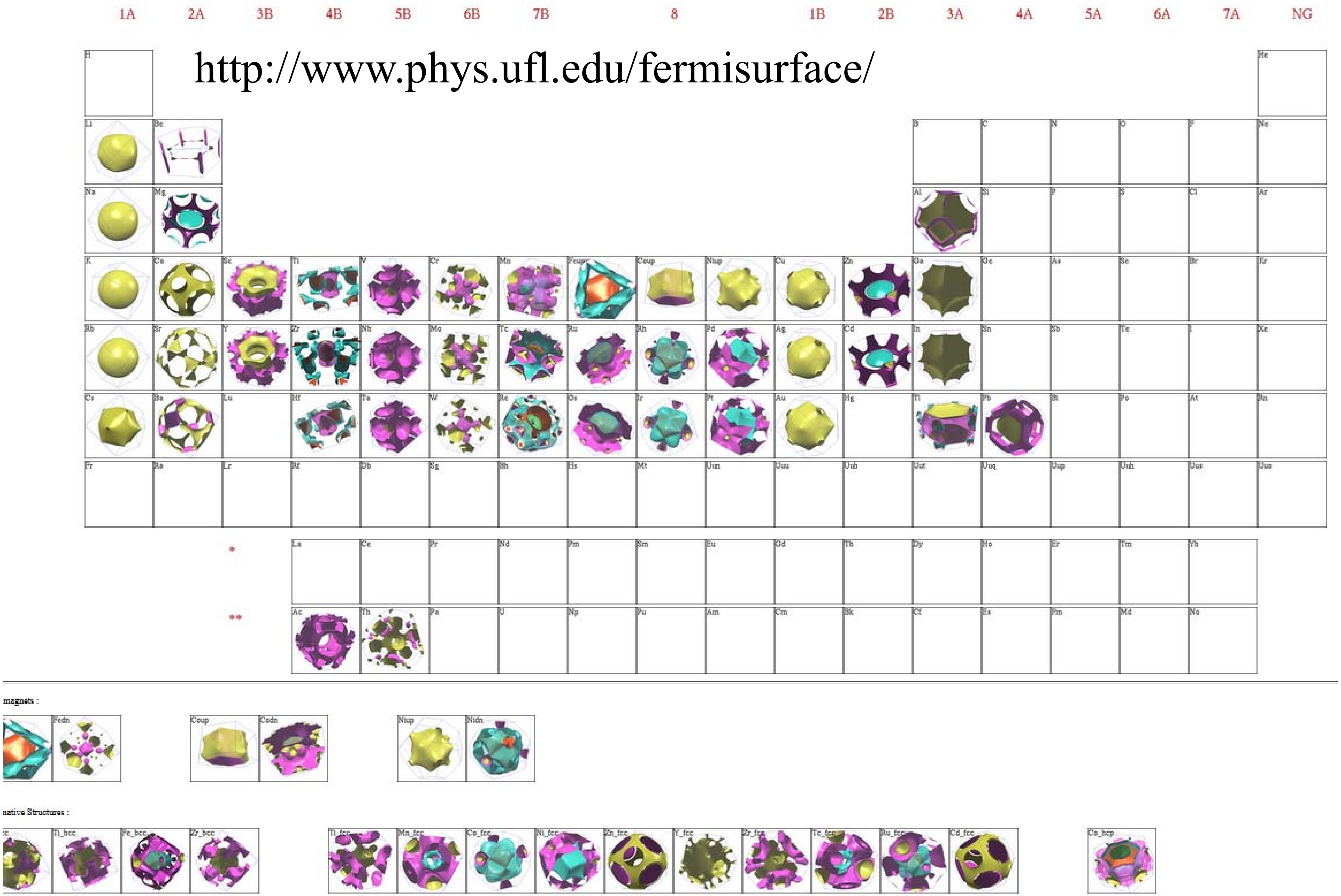
De Haas - van Alphen effect

The magnetic moment of gold oscillates periodically with $1/B$



Kittel

http://www.phys.ufl.edu/fermisurface/



Magnetism

diamagnetism
paramagnetism
ferromagnetism (Fe, Ni, Co)
ferrimagnetism (Magneteisenstein)
antiferromagnetism

$$H = -\sum_i \frac{\hbar^2}{2m_e}\nabla_i^2 - \sum_A \frac{\hbar^2}{2m_A}\nabla_A^2 - \sum_{i,A}\frac{Z_A e^2}{4\pi\varepsilon_0 r_{iA}} + \sum_{i<j}\frac{e^2}{4\pi\varepsilon_0 r_{ij}} + \sum_{A<B}\frac{Z_A Z_B e^2}{4\pi\varepsilon_0 r_{AB}}$$

Coulomb interactions cause ferromagnetism not magnetic interactions.

Magnetism

magnetic intensity

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right)$$

magnetic induction field

magnetization

$$\vec{M} = \chi \vec{H}$$

χ is the magnetic susceptibility

$\chi < 0$ diamagnetic

$\chi > 0$ paramagnetic

χ is typically small (10^{-5}) so $B \approx \mu_0 H$