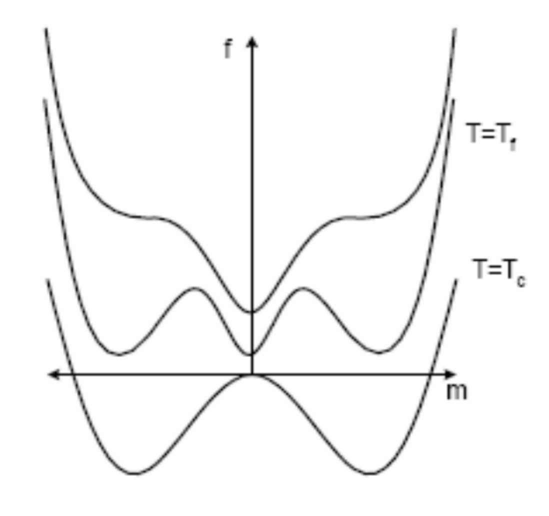
Technische Universität Graz

28. Phase Transistions Qusaiparticles

Jan. 30, 2020

First order transitions

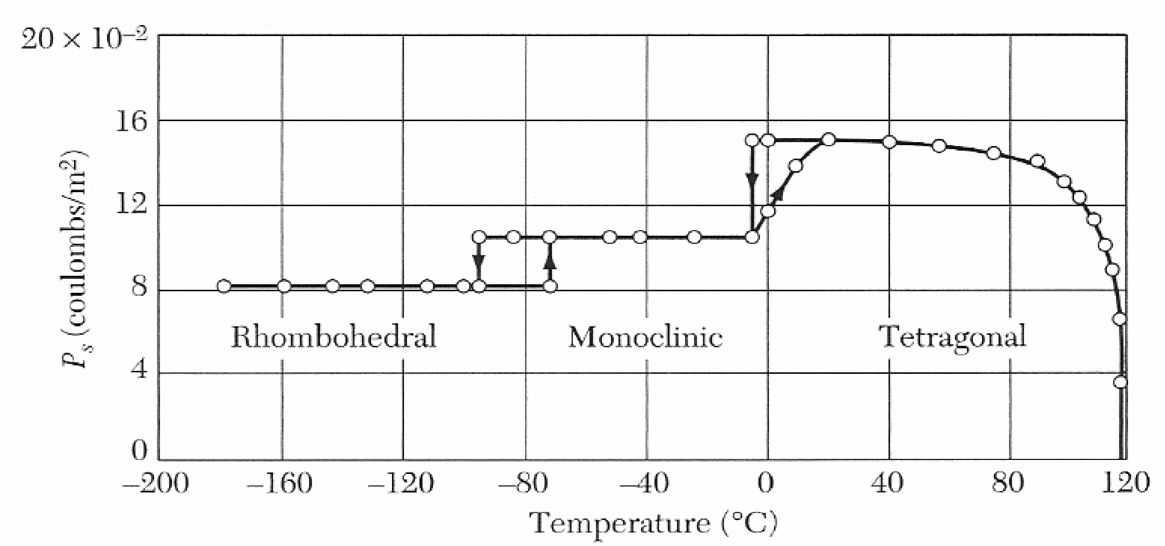
$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0$$

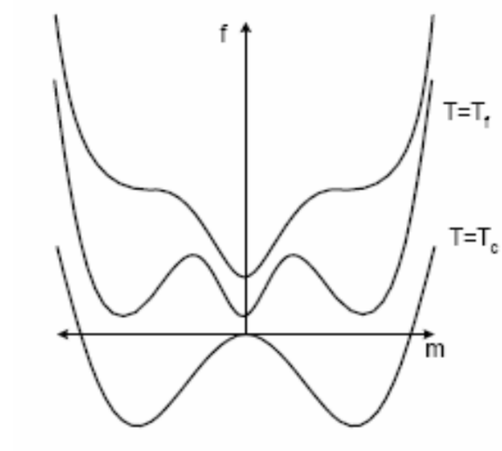


There is a jump in the order parameter at the phase transition.

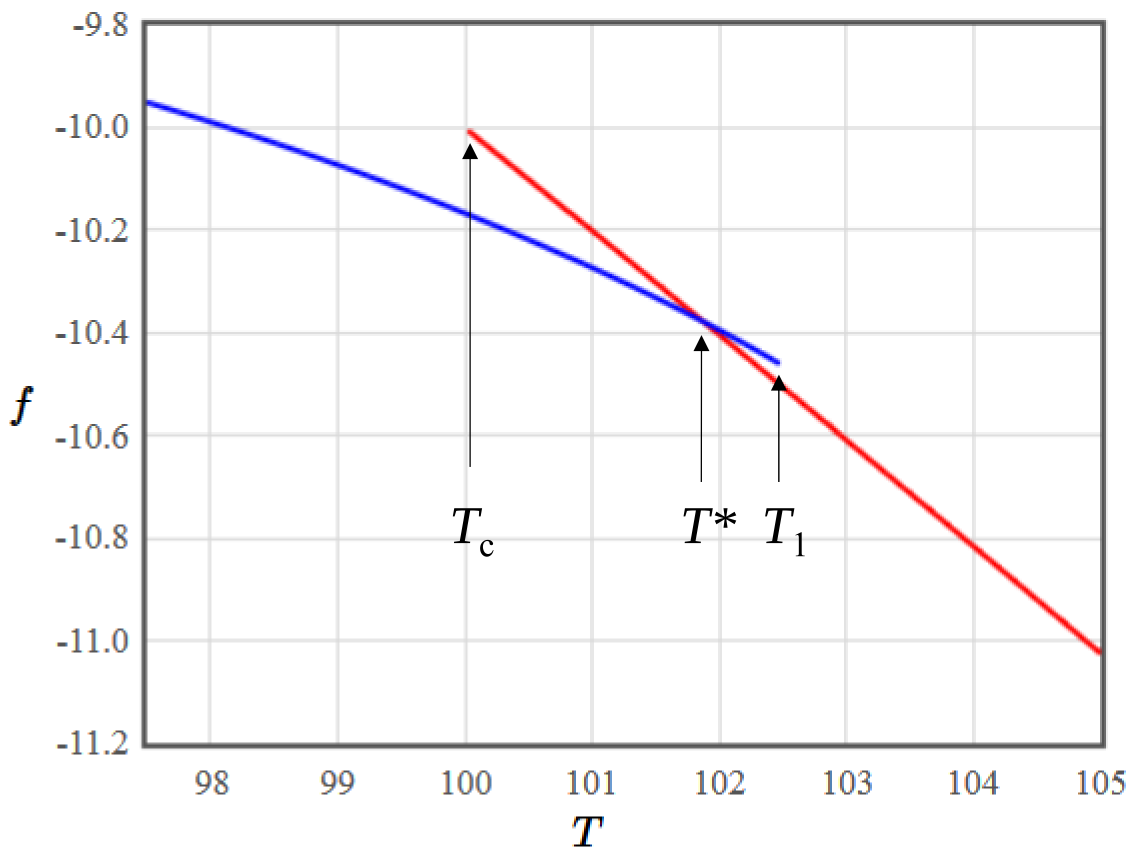
First order transitions

$BaTiO_3$





T_c?



$$T_1 = \frac{\beta^2}{4\alpha_0\gamma} + T_c$$

$$T^* - T_c = \frac{3\beta^2}{16\alpha_0\gamma}$$

First order transitions

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2} \beta m^4 + \tfrac{1}{3} \gamma m^6 + \cdots \qquad \beta < 0 \qquad \gamma > 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 = 0$$

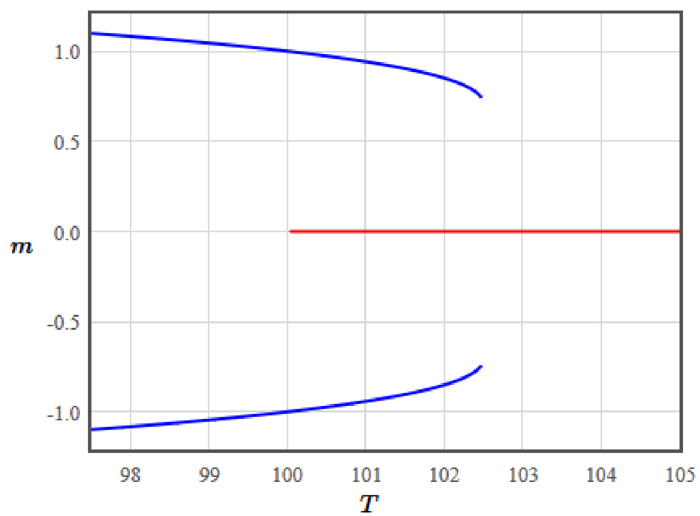
One solution for $m = 0$.

$$\alpha_0 (T - T_c) + \beta m^2 + \gamma m^4 = 0$$

$$m^2 = 0, \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c) \gamma}}{2\gamma}$$

There will be a minimum at finite m as long as m^2 is real

$$T_1 = \frac{\beta^2}{4\alpha_0 \gamma} + T_c$$



Jump in the order parameter

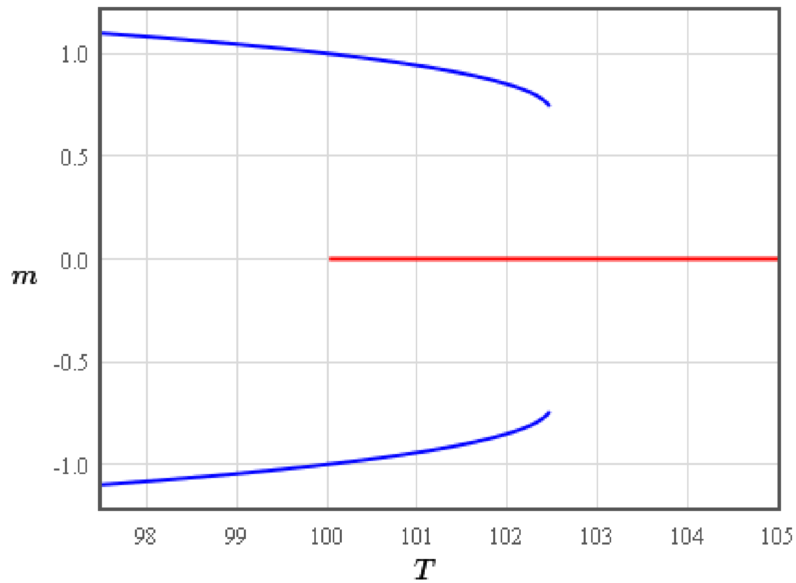
$$m^2 = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 \left(T - T_c\right)\gamma}}{2\gamma}$$

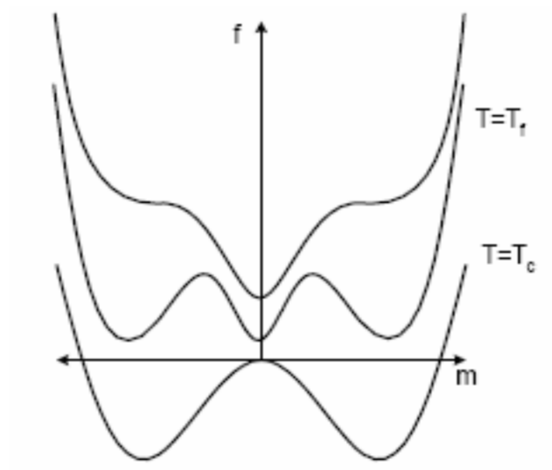
At T_c

$$\Delta m = \sqrt{\frac{-\beta}{\gamma}}$$

At T_1

$$\Delta m = \sqrt{\frac{-\beta}{2\gamma}}$$





First order transitions, entropy, c_v

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 + \cdots \qquad \beta < 0$$

$$m = 0, \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha_0 (T - T_c)\gamma}}{2\gamma}}$$

$$s = -\frac{\partial f}{\partial T} = \frac{\alpha_0}{2\gamma}\left(\beta - \sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}\right)$$

$$c_v = T\frac{\partial s}{\partial T} = \frac{\alpha_0^2 T}{\sqrt{\beta^2 - 4\alpha_0\gamma(T - T_c)}}$$

branch where the order parameter is nonzero

First order transitions, susceptibility

$$f = f_0 + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 - mB \qquad \beta < 0$$

$$\frac{df}{dm} = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5 - B = 0$$

At the minima $$B = 2\alpha_0 (T - T_c) m + 2\beta m^3 + 2\gamma m^5$$

For small m,

$$\chi = \left.\frac{dm}{dB}\right|_{m=0} = \frac{1}{2\alpha_0 (T - T_c)}$$ Curie - Weiss

$$\chi = \left.\frac{dm}{dB}\right|_{m=\sqrt{\frac{-\beta}{2\gamma}}} = \frac{1}{2\alpha_0 (T - T_1)}$$

Outline
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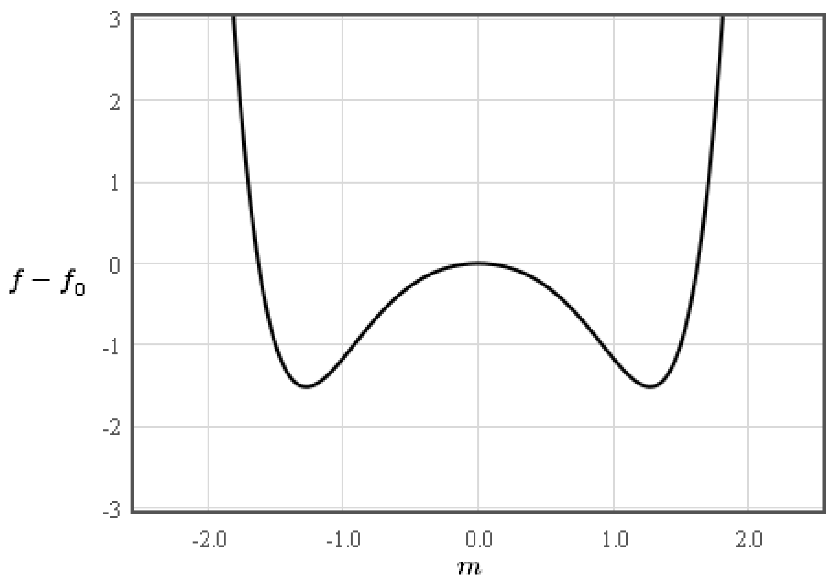
Landau theory of a first order phase transition

The free energy for a first order transition in Landau theory is,

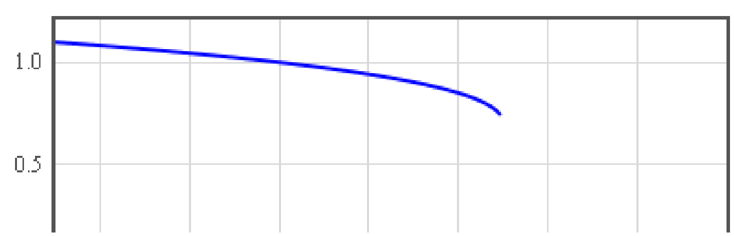
$$f(T) = f_0(T) + \alpha_0 (T - T_c) m^2 + \tfrac{1}{2}\beta m^4 + \tfrac{1}{3}\gamma m^6 \qquad \alpha_0 > 0, \quad \beta < 0, \quad \gamma > 0.$$

Here $f_0(T)$ describes the temperature dependence of the high temperature phase near the phase transition.



$\alpha_0 =$ 0.1
$\beta =$ -1
$\gamma =$ 1
$T =$ 90
$T_c =$ 100
$f_0(T) =$ -0.01*T*T
submit

Order parameter



Quasiparticles

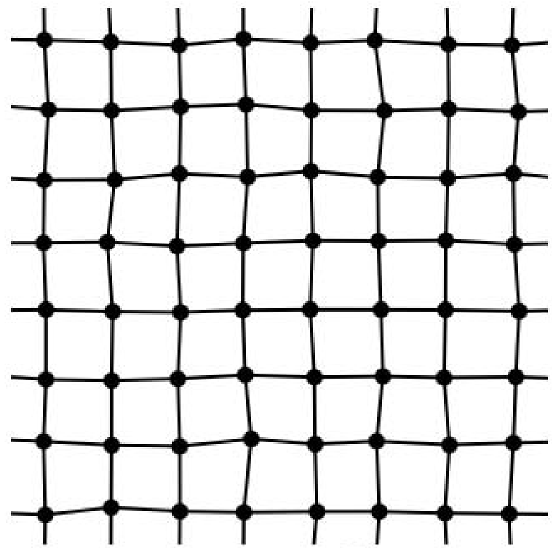
Excitations from equilibrium

phonons
plasmons
magnons
polaritons
(electrons)
Landau's theory of a Fermi liquid

polarons
excitons

Normal Modes and Phonons

At finite temperatures, the atoms in a crystal vibrate. In the simulation below, the atoms move randomly around their equilibrium positions.



T = 300 K - +

Longitudinal plasma waves

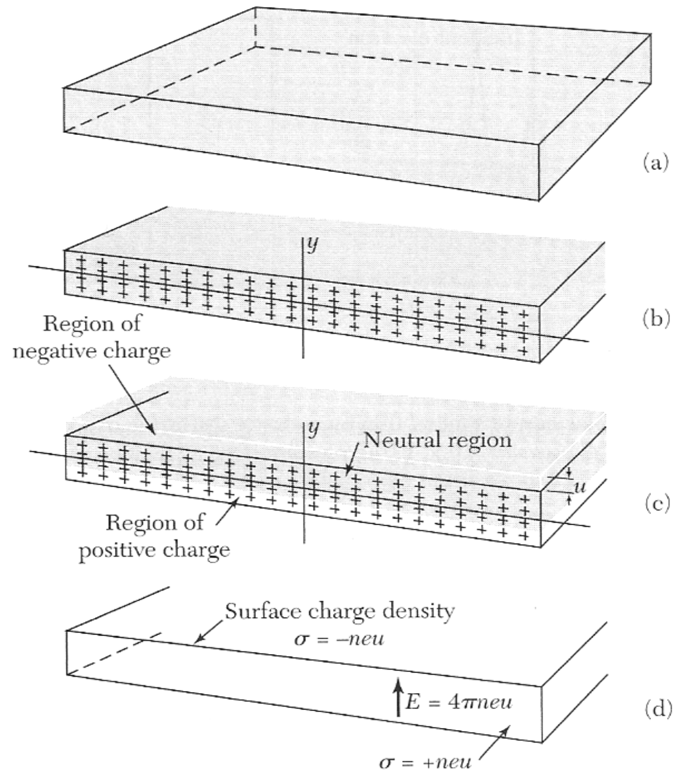
$$nm\frac{d^2y}{dt^2} = -neE$$

$$E = \frac{ney}{\varepsilon_0}$$

$$nm\frac{d^2y}{dt^2} = -\frac{n^2e^2y}{\varepsilon_0}$$

$$\frac{d^2y}{dt^2} + \omega_p^2 y = 0$$

Plasma frequency $$\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$$



Kittel

There is no magnetic component of the wave.

Plasma waves can be quantized like any other wave

Transverse optical plasma waves

The dispersion relation for light

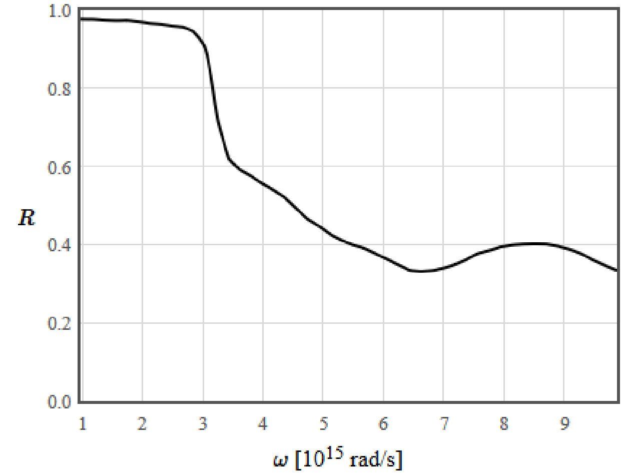
$$\varepsilon\varepsilon_0\mu_0\omega^2 = \frac{\varepsilon\omega^2}{c^2} = k^2$$

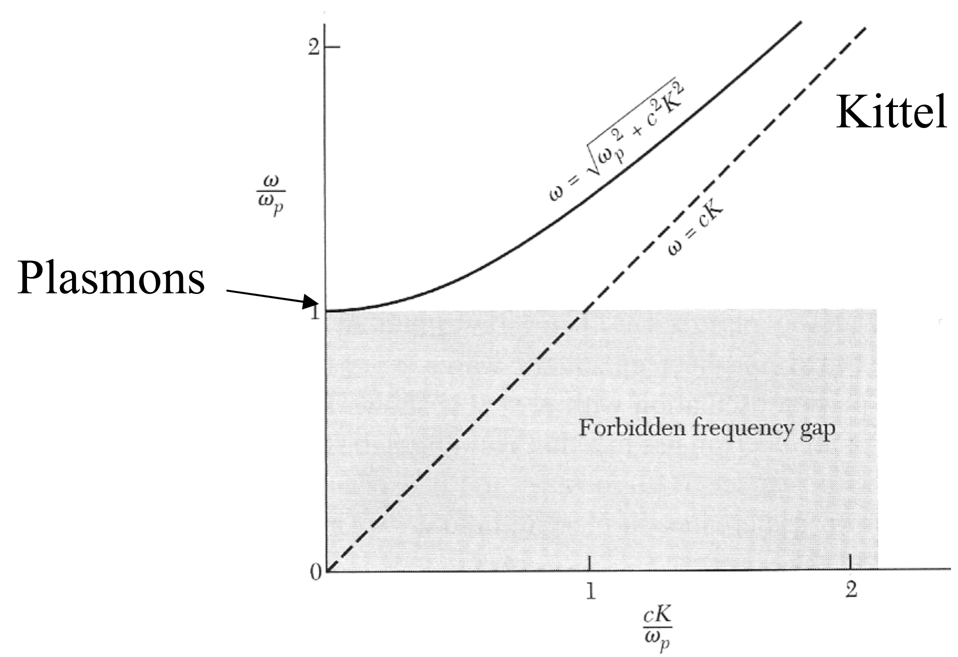
For a free electron gas

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\left(1 - \frac{\omega_p^2}{\omega^2}\right)\omega^2 = c^2k^2$$

$$\omega^2 = \omega_p^2 + c^2k^2$$





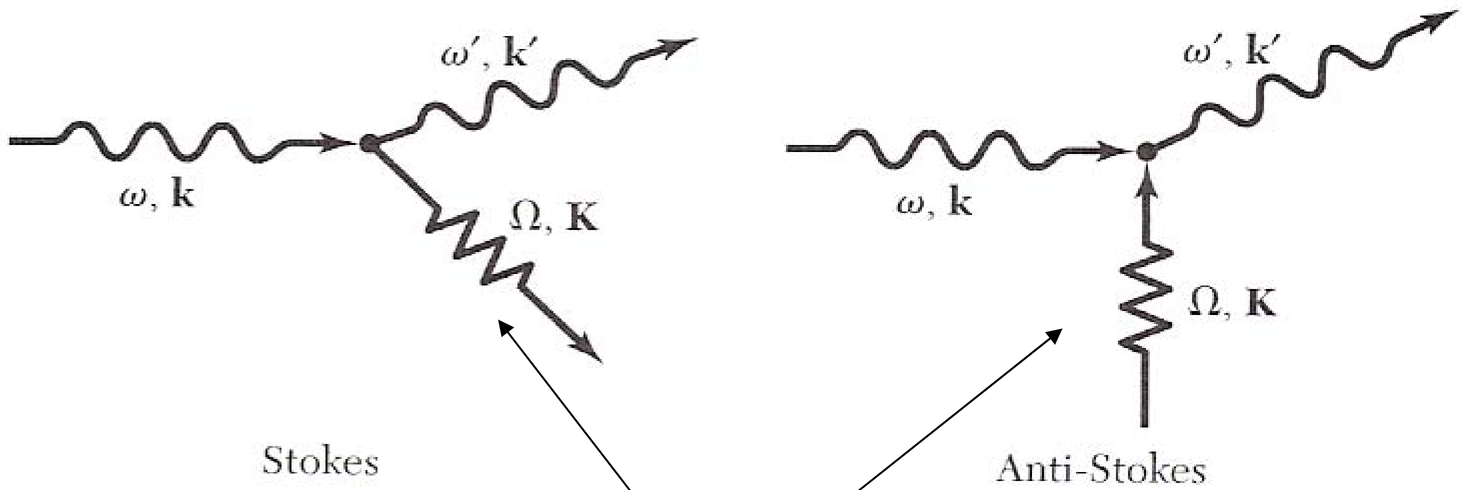
Raman Spectroscopy

C. V. Raman

Inelastic light scattering

$$\omega = \omega' \pm \Omega$$

$$\vec{k} = \vec{k}' \pm \vec{K} \pm \vec{G}$$



Phonons, magnons, plasmons, polaritons, ...

$$\vec{K} \approx 0$$

Raman Spectroscopy

$$\chi = \chi_0 + \frac{\partial \chi}{\partial X} X \cos(\Omega t)$$

$$\vec{P} = \varepsilon_0 \chi \vec{E} \cos(\omega t) + \varepsilon_0 \frac{\partial \chi}{\partial X} X \cos(\Omega t) \vec{E} \cos(\omega t)$$

There are components of the polarization that oscillate at $\omega \pm \Omega$.