15. Magnetism / Superconductivity

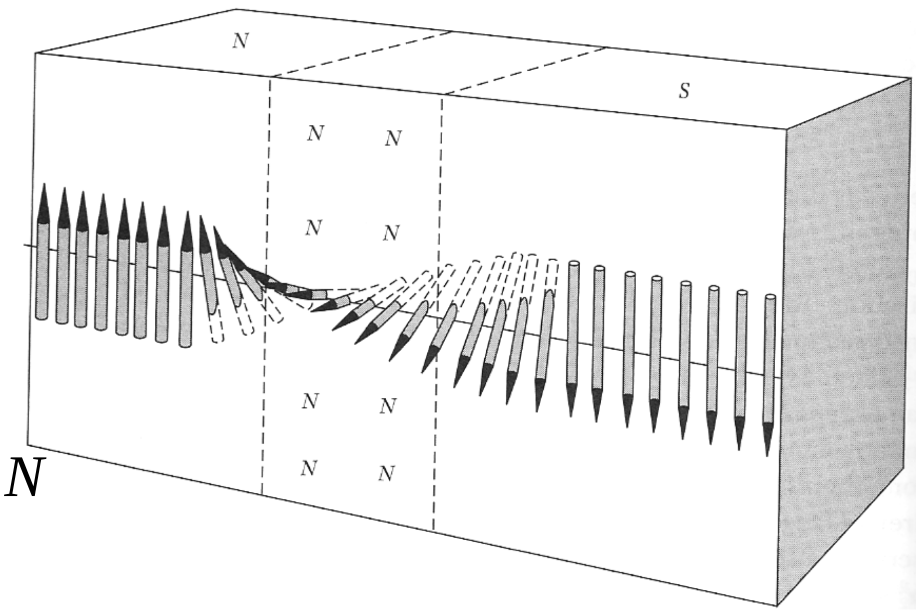
Nov 25, 2019

Bloch wall

Anisotropy energy depends on the number of spins pointing in the hard direction

$\approx KNa$ ← Na = thickness of wall

anisotropy constant J/m^3



Total energy per unit area:

$$E = \frac{JS^2\pi^2}{2Na^2} + KNa \qquad [\text{J/m}^2]$$

smaller for large N smaller for small N

$$\frac{dE}{dN} = 0 \Rightarrow -\frac{JS^2\pi^2}{2N^2a^2} + Ka = 0$$

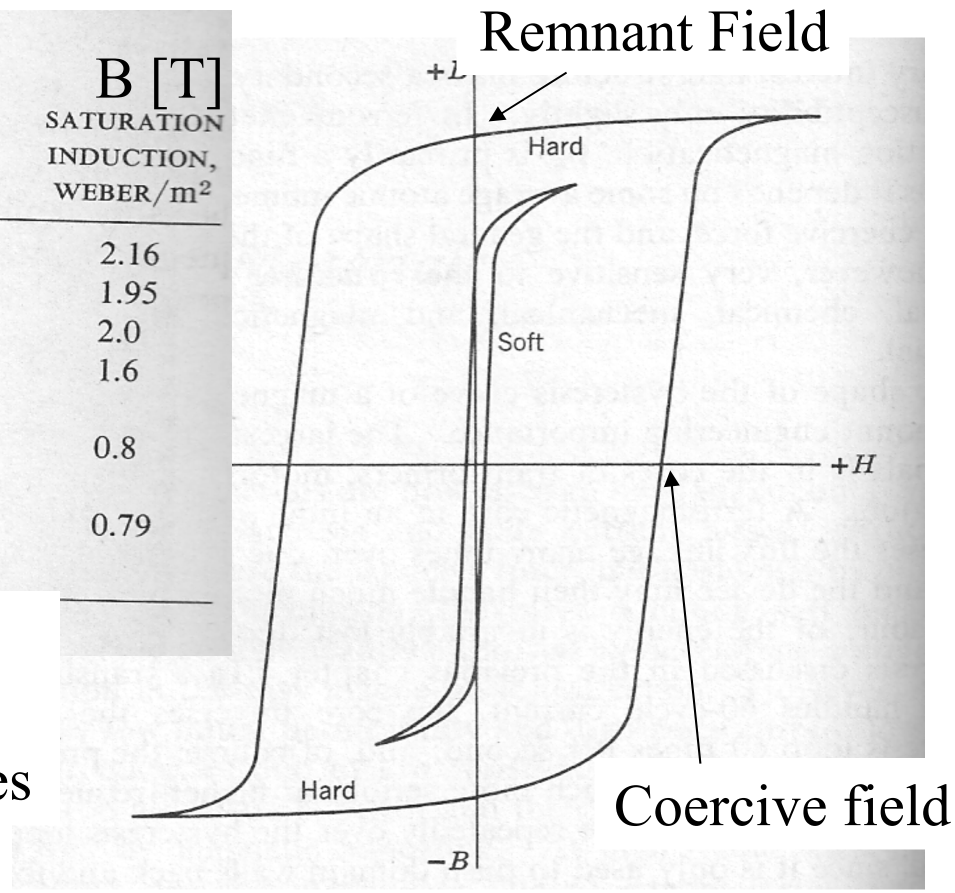
$$N = \sqrt{\frac{JS^2\pi^2}{2Ka^3}}$$

$N \sim 300$ for iron

Soft magnetic materials

soft magnets

MATERIAL	INITIAL RELATIVE PERMEABILITY (μ_r AT $B \sim 0$)	HYSTERESIS LOSS JOULE/m^3 PER CYCLE	B [T] SATURATION INDUCTION, WEBER/m^2
Commercial iron ingot	250	500	2.16
Fe-4% Si, random	500	50–150	1.95
Fe-3% Si, oriented	15,000	35–140	2.0
45 Permalloy (45% Ni-55% Fe)	2,700	120	1.6
Mumetal (75% Ni-5% Cu-2% Cr-18% Fe)	30,000	20	0.8
Supermalloy (79% Ni-15% Fe-5% Mo-0.5% Ma)	100,000	2	0.79



transformers
magnetic shielding
ferrites have low eddy current losses

$$B = \mu_0 (H + M)$$

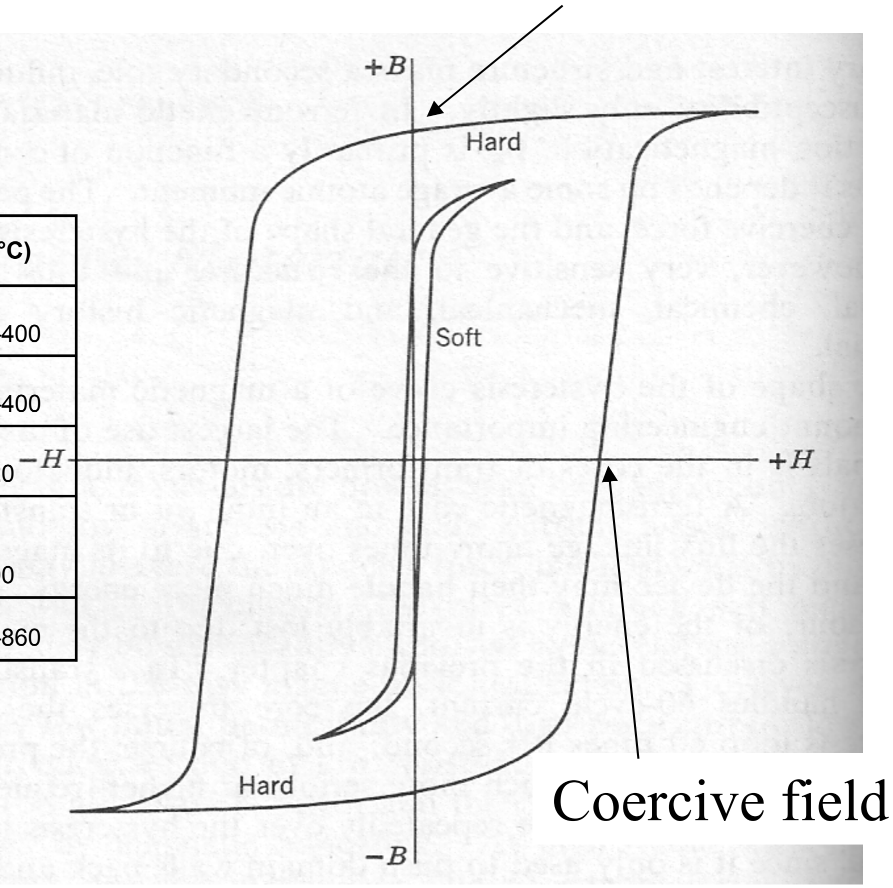
$$B = \mu_r \mu_0 H \qquad M = \chi H$$

$$\mu_r = 1 + \chi$$

Hard magnetic materials

hard magnets

Magnet	B_r (T)	H_{ci} (kA/m)	$(BH)_{max}$ (kJ/m^3)	T_c (°C)
$Nd_2Fe_{14}B$ (sintered)	1.0–1.4	750–2000	200–440	310–400
$Nd_2Fe_{14}B$ (bonded)	0.6–0.7	600–1200	60–100	310–400
$SmCo_5$ (sintered)	0.8–1.1	600–2000	120–200	720
$Sm(Co,Fe,Cu,Zr)_7$ (sintered)	0.9–1.15	450–1300	150–240	800
Alnico (sintered)	0.6–1.4	275	10–88	700–860



Permanent magnets, magnetron, motors, generators
ferrites can also be hard magnets

Defects are introduced to pin the Bloch walls in a hard magnet.

Single domain particles

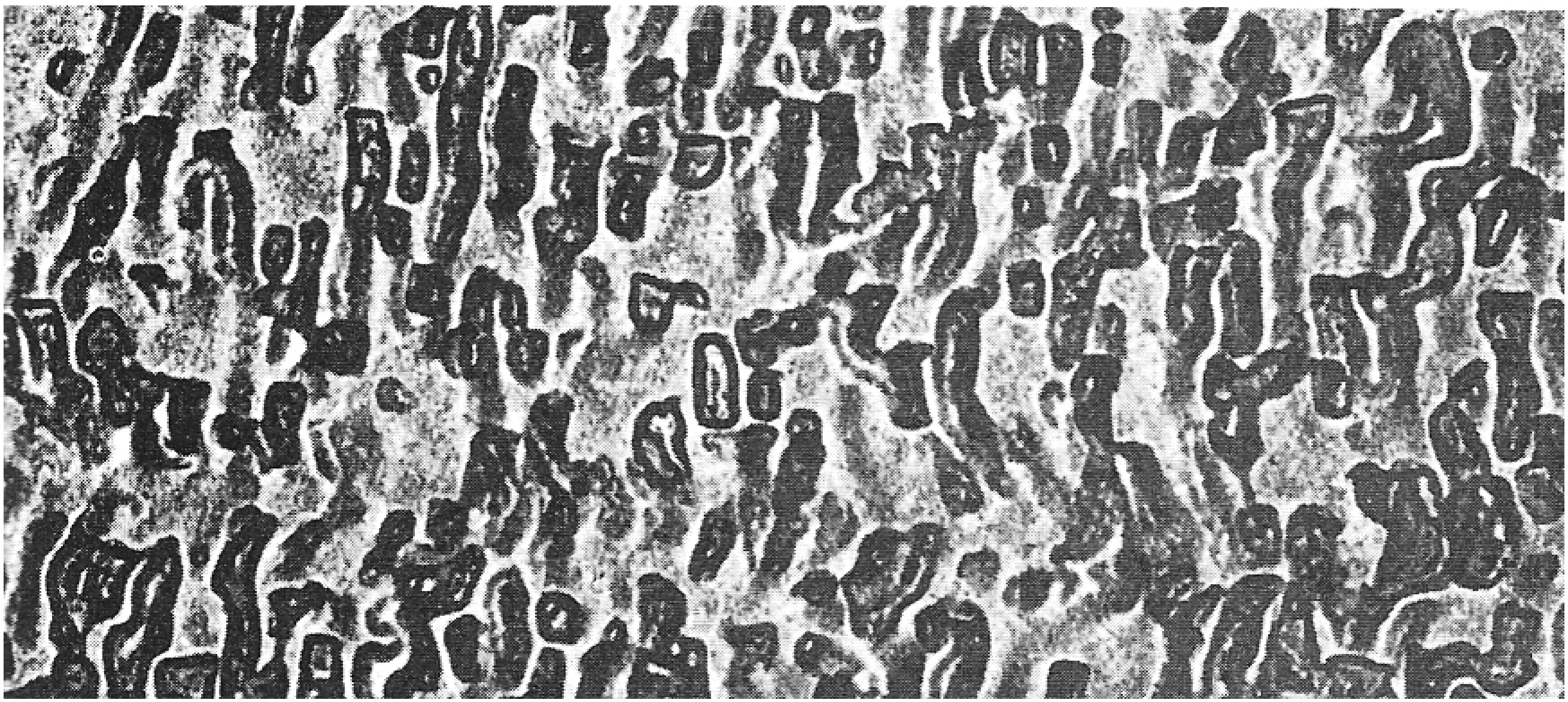
Small 10 - 100 nm particles have single domains.

Elongated particles have the magnetization along the long axis.

Single domains are used for magnetic recording. Long crystals can be magnetized in either of the two directions along the long axis.

Shape anisotropy.

Hard magnets



Grains too small to contain Bloch walls must be flipped entirely by the field.

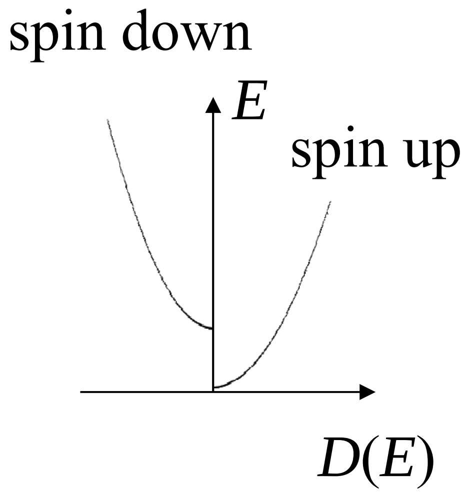
Alnico: 8-12% Al, 15-26% Ni, 5-24% Co, up to 6% Cu, up to 1% Ti, rest is Fe

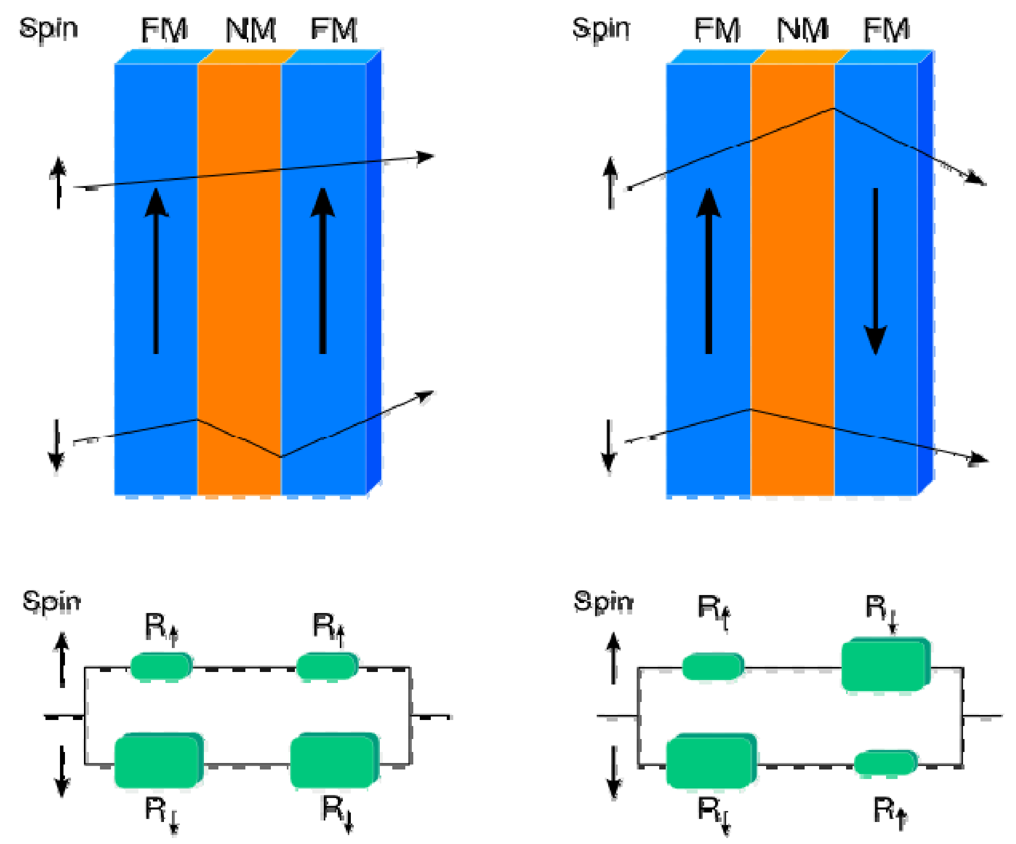
Applications of hard magnets

Motors, generators, speakers, microphone

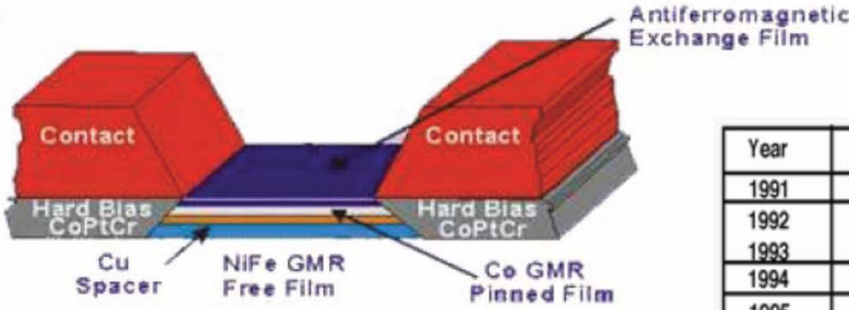
Giant magnetoresistance

spin value



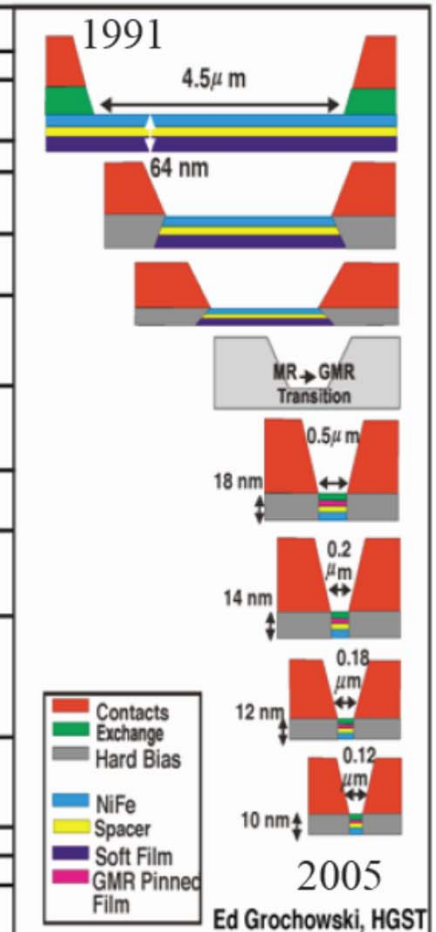


GMR sensors in read-heads for hard-disk drives



Shipment of GMR-read-heads (1997-2007):
5 billion (10^9)

Year	Areal Density Gbits/in2	Product
1991	0.132	Corsair
1992	0.260	Allicat
1993	0.354	Spitfire
1994	0.578	Ultrastar XP
1995	0.829	Ultrastar 2XP
	0.923	Travelstar 2LP
1996	1.32	Travelstar 2XP
	1.45	Travelstar VP
1997	2.64	Travelstar 5GS
	2.68	Deskstar 16GP
	3.12	Travelstar 6GN
1998	3.74	Travelstar 6GT
	4.1	Deskstar 25GP
	5.7	Travelstar 6GN
1999	5.3	Deskstar 37GP
	10.1	Travelstar 18GT
2000	7.04	Ultrastar 36LZX
	14.5	Deskstar 40GV
	17.1	Travelstar 30GT
2001	13.2	Ultrastar 73LZX
	25.7	Travelstarr 30GN
	29.7	Deskstar 120GXP
	34.0	Travelstar 40GN
2002	26.3	Ultrastar 146Z10
	45.5	Deskstar 180GXP
	29.7	Deskstar 120GXP
2003	70.0	Travelstar 80GN
2004	>100	
2005	>200	



Peter Gruenberg Nobel Lecture 2007:
From Spinwaves to Giant Magnetoresistance (GMR) and Beyond

Superconductivity

Primary characteristic: zero resistance at dc

There is a critical temperature T_c above which superconductivity disappears

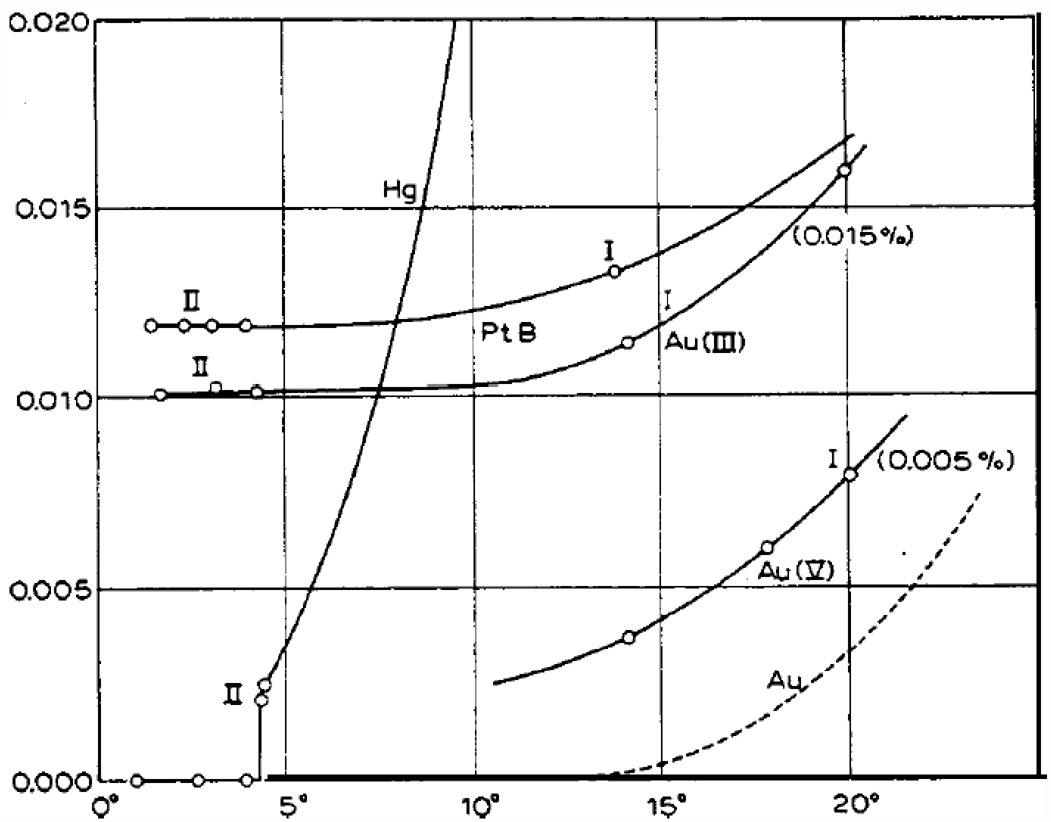
About 1/3 of all metals are superconductors

Metals are usually superconductors OR magnetic, not both

Good conductors are bad superconductors

Kittel chapter 10

Superconductivity

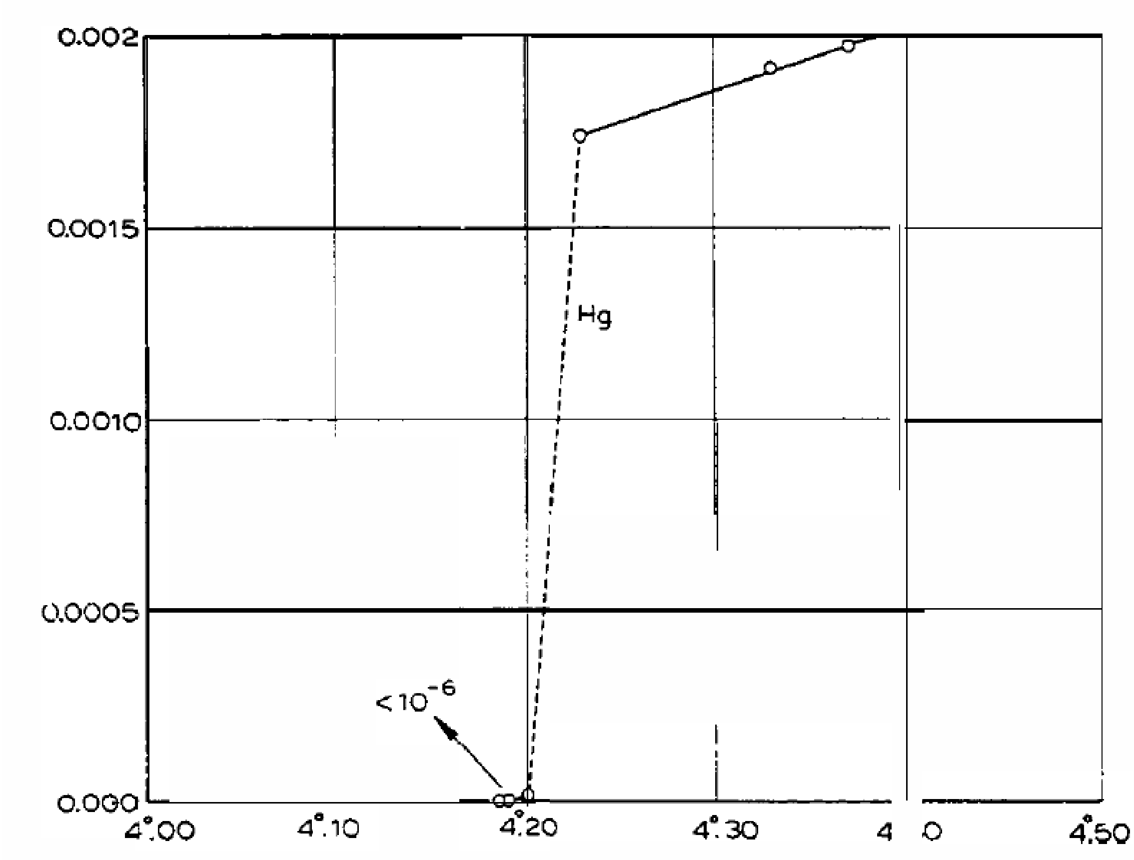


Nobel Lecture 1913

Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Superconductivity



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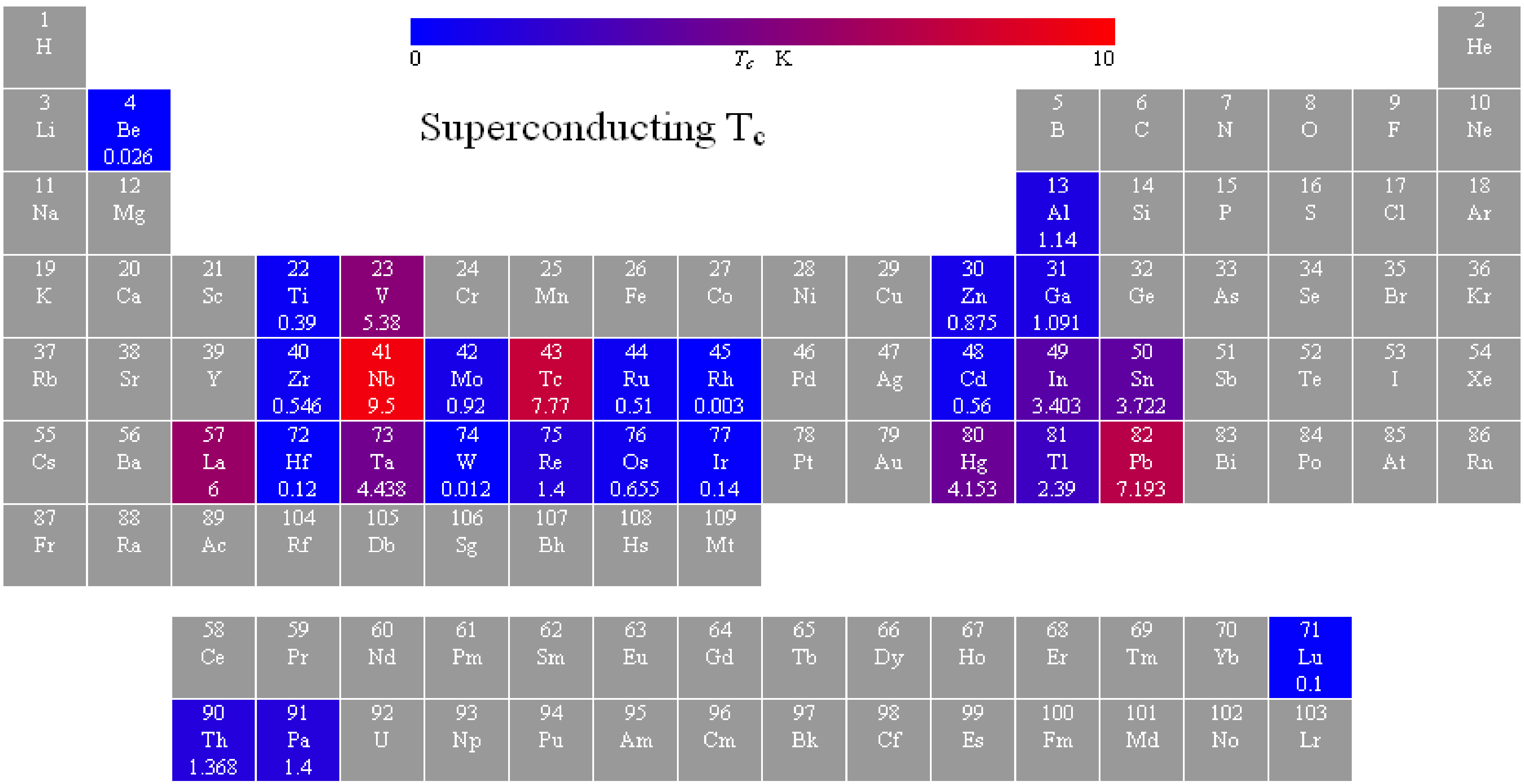
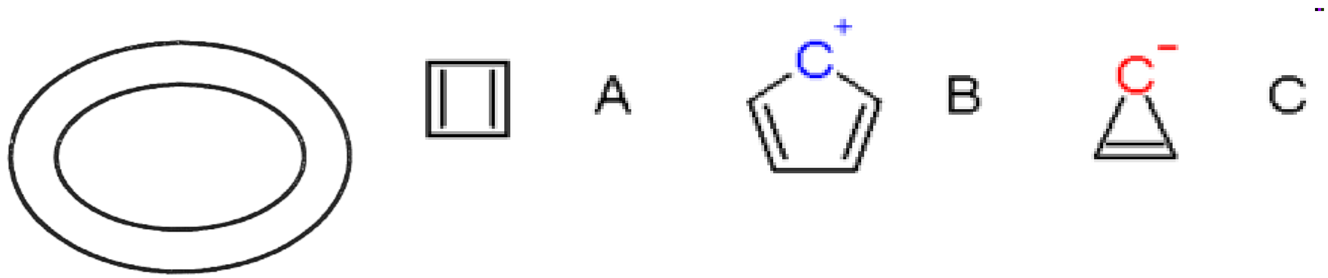
Critical temperature



Superconductivity

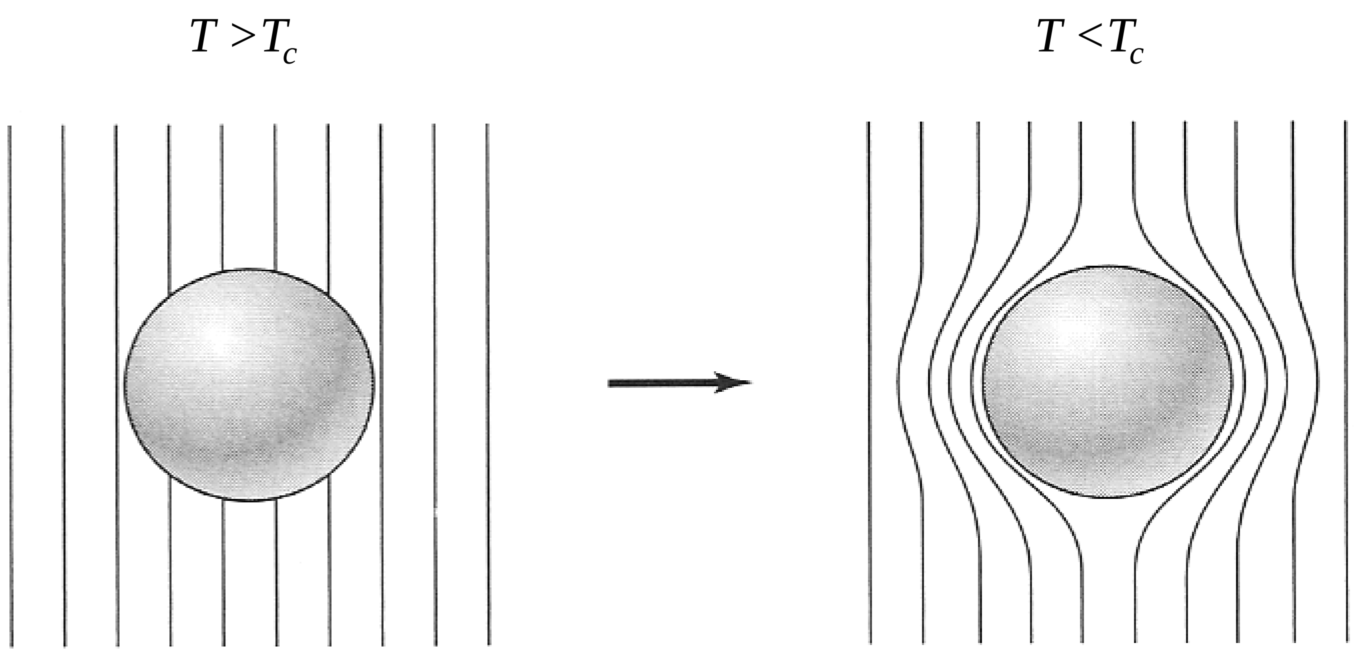


Superconducting ring   Molecule with magnetic moment

Antiaromatic molecules are unstable and highly reactive

No measurable decay in current after 2.5 years. $\rho < 10^{-25}\ \Omega\text{m}$.

Meissner effect



Superconductors are perfect diamagnets at low fields.
$B = 0$ inside a bulk superconductor.

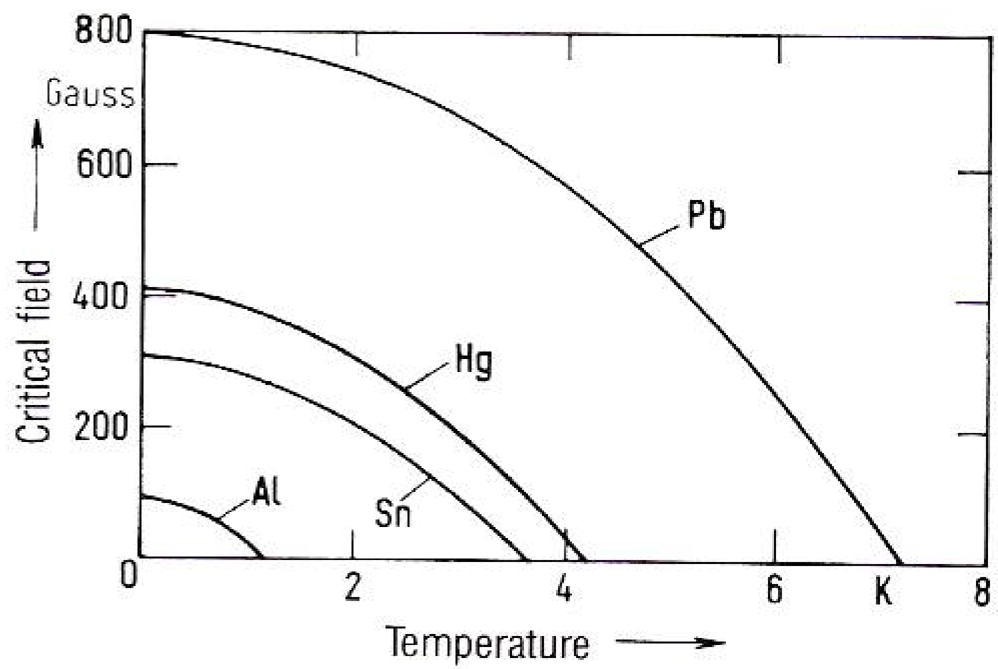
Superconductors are used for magnetic shielding.

Superconductivity

Critical temperature T_c

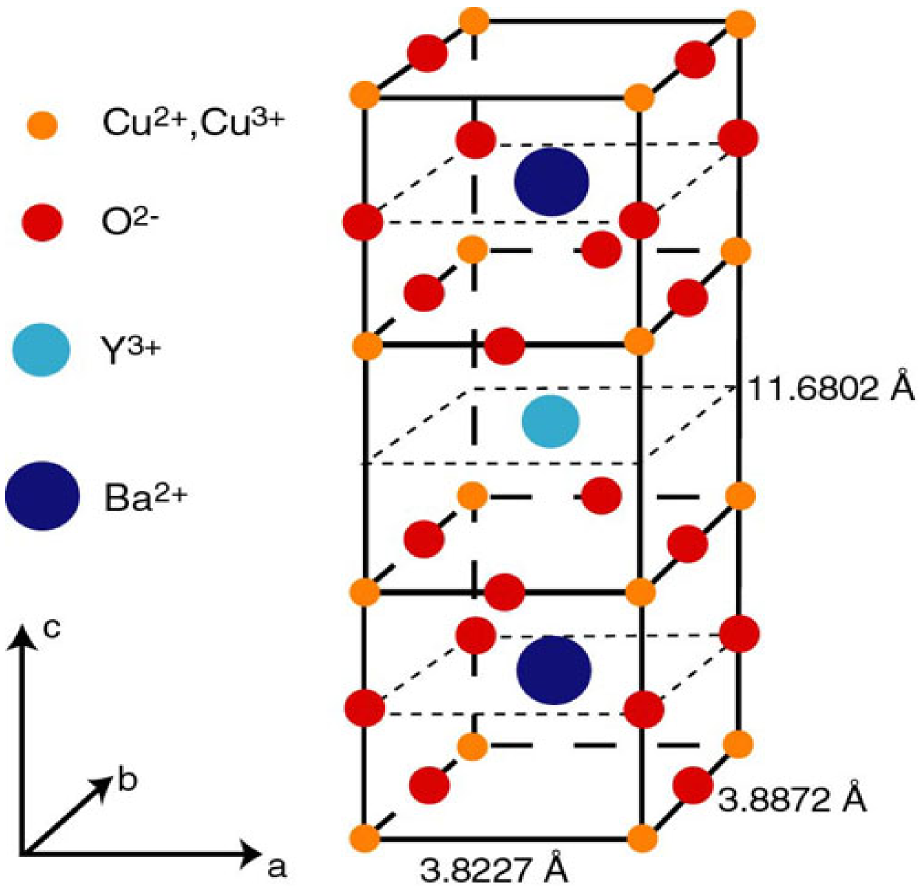
Critical current density J_c

Critical field H_c

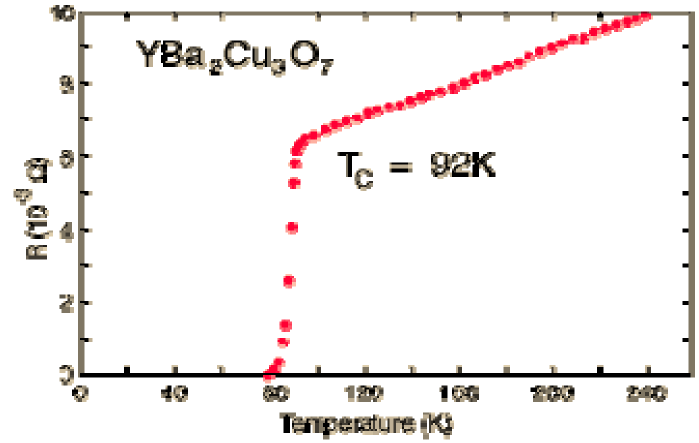


$$n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \tfrac{1}{2} nmv^2 = \frac{m}{2ne^2} J_c^2$$

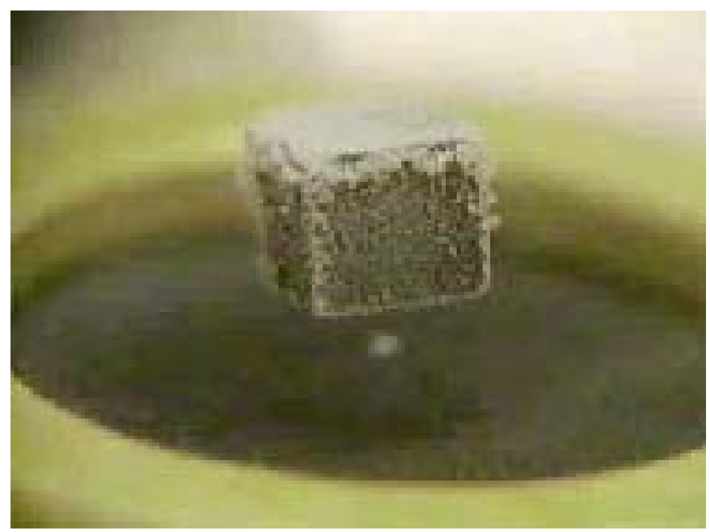
$YBa_2Cu_3O_x$



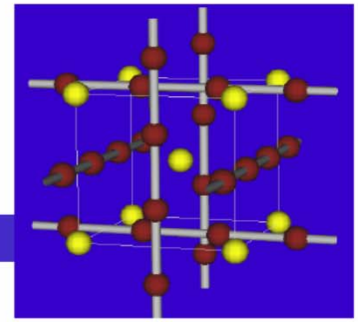
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

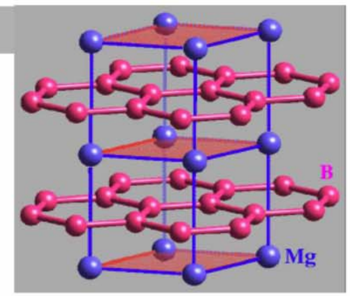


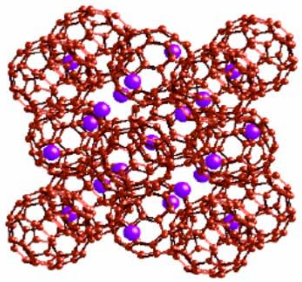
YBa2Cu3O7
Tc = 92K
Temperature (K)



	Material	T_c
• Legierung:	NbTi	9,6 K
• Verbindungen:	NbN	16,0 K
Borocarbide:	$(Lu/Y)Ni_2B_2C$	16,0 K
"A15"-Strukturen:	Nb_3Sn	18,0 K
(= β-Wolfram-Struktur)	Nb_3Al	18,7 K
	Nb_3Ge	22,5 K
neu:	MgB_2	39 K
Fullerene:	Cs_2RbC_{60}	33 K
+ Druck 15 kbar:	Cs_3C_{60}	40 K

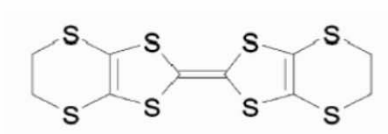


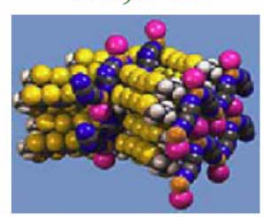




Organische Supraleiter:

$(BEDT\text{-}TTF)_2Cu[N(CN)_2]Br$ 11,2 K





Polymere

hochdotierte Halbleiter

http://www.wmi.badw.de/teaching/Lecturenotes/index.html

Compound	T_c, in K	Compound	T_c, in K
Nb_3Sn	18.05	V_3Ga	16.5
Nb_3Ge	23.2	V_3Si	17.1
Nb_3Al	17.5	$YBa_2Cu_3O_{6.9}$	90.0
NbN	16.0	Rb_2CsC_{60}	31.3
K_3C_{60}	19.2	MgB_2	39.0

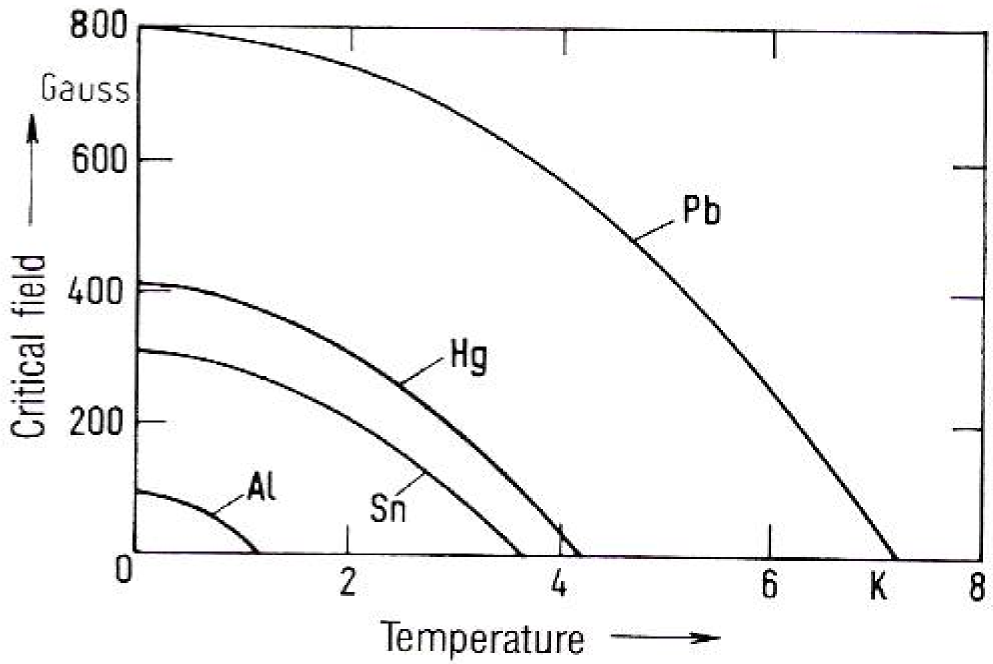
$BaPb_{0.75}Bi_{0.25}O_3$	$T_c = 12$ K	[BPBO]
$La_{1.85}Ba_{0.15}CuO_4$	$T_c = 36$ K	[LBCO]
$YBa_2Cu_3O_7$	$T_c = 90$ K	[YBCO]
$Tl_2Ba_2Ca_2Cu_3O_{10}$	$T_c = 120$ K	[TBCO]
$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	$T_c = 138$ K	
$(Sn_5In)Ba_4Ca_2Cu_{10}O_y$	$T_c = 212$ K	

Superconductivity

Critical temperature T_c

Critical current density J_c

Critical field H_c



$$n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \tfrac{1}{2} nmv^2 = \frac{m}{2ne^2} J_c^2$$

Superconductivity

Perfect diamagnetism

Jump in the specific heat like a 2nd order phase transition, not a structural transition

Superconductors are good electrical conductors but poor thermal conductors, electrons no longer conduct heat

There is a dramatic decrease of acoustic attenuation at the phase transition, no electron-phonon scattering

Dissipationless currents - quantum effect

Electrons condense into a single quantum state - low entropy.

Electron decrease their energy by Δ but loose their entropy.

Probability current

Schrödinger equation for a charged particle in an electric and magnetic field is

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}(-i\hbar\nabla-qA)^2\psi+V\psi$$

write out the $(-i\hbar\nabla-qA)^2\psi$ term

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}\left(-\hbar^2\nabla^2+i\hbar qA\nabla+ihq\nabla A+q^2A^2\right)\psi+V\psi$$

write the wave function in polar form

$$\psi=|\psi|e^{i\theta}$$

$$\nabla\psi=\nabla|\psi|e^{i\theta}+i\nabla\theta|\psi|e^{i\theta}$$

$$\nabla^2\psi=\nabla^2|\psi|e^{i\theta}+2i\nabla\theta\nabla|\psi|e^{i\theta}+i\nabla^2\theta|\psi|e^{i\theta}-\left(\nabla\theta\right)^2|\psi|e^{i\theta}$$

Probability current

Schrödinger equation becomes:

$$i\hbar\frac{\partial|\psi|}{\partial t}-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(\nabla^2|\psi|+2i\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)$$

$$+i\hbar qA\left(\nabla|\psi|+i\nabla\theta|\psi|\right)+ihq\nabla A|\psi|+q^2A^2|\psi|\Big]+V|\psi|$$

Real part:

$$-\hbar|\psi|\frac{\partial\theta}{\partial t}=\frac{-\hbar^2}{2m}\left(\nabla^2-\left(\nabla\theta-\frac{q}{\hbar}\vec{A}\right)^2\right)|\psi|+V|\psi|$$

Imaginary part:

$$\hbar\frac{\partial|\psi|}{\partial t}=\frac{1}{2m}\Big[-\hbar^2\left(2\nabla\theta\nabla|\psi|+i\nabla^2\theta|\psi|-(\nabla\theta)^2|\psi|\right)+2\hbar qA\nabla|\psi|+\hbar q|\psi|\nabla A\Big]$$

Probability current

Imaginary part:

$$\hbar \frac{\partial |\psi|}{\partial t} = \frac{1}{2m}\left[-\hbar^2 \left(2\nabla\theta\nabla|\psi| + i\nabla^2\theta|\psi| - (\nabla\theta)^2|\psi| \right) + 2\hbar qA\nabla|\psi| + \hbar q|\psi|\nabla A \right]$$

Multiply by $|\psi|$ and rearrange

$$\frac{\partial}{\partial t}|\psi|^2 + \nabla\cdot\left[\frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)\right] = 0$$

This is a continuity equation for probability

$$\frac{\partial P}{\partial t} + \nabla\cdot\vec{S} = 0$$

The probability current: $\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$

Probability current / supercurrent

The probability current: $$\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$$

This result holds for all charged particles in a magnetic field.

In superconductivity the particles are Cooper pairs $q = -2e$, $m = 2m_e$, $|\psi|^2 = n_{cp}$.

All superconducting electrons are in the same state so

$$\vec{j} = -2en_{cp}\vec{S}$$

$$\boxed{\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)}$$

London gauge $\nabla\theta = 0$

$$\vec{j} = \frac{-2n_{cp}e^2}{m_e}\vec{A} = \frac{-n_s e^2}{m_e}\vec{A} \qquad n_s = 2n_{cp}$$

1st London equation


Heinz & Fritz

$$\vec{j} = \frac{-n_s e^2}{m_e}\vec{A}$$

$$\frac{d\vec{j}}{dt} = \frac{-n_s e^2}{m_e}\frac{d\vec{A}}{dt} = \frac{n_s e^2}{m_e}\vec{E}$$

$$\frac{d\vec{A}}{dt} = -\vec{E}$$

First London equation:

$$\boxed{\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}}$$

Classical derivation:

$$-e\vec{E} = m\frac{d\vec{v}}{dt} = -\frac{m}{n_s e}\frac{d\vec{j}}{dt}$$

$$\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}$$

2nd London equation

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \nabla \times \vec{A}$$

Second London equation:

$$\boxed{\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B}}$$