

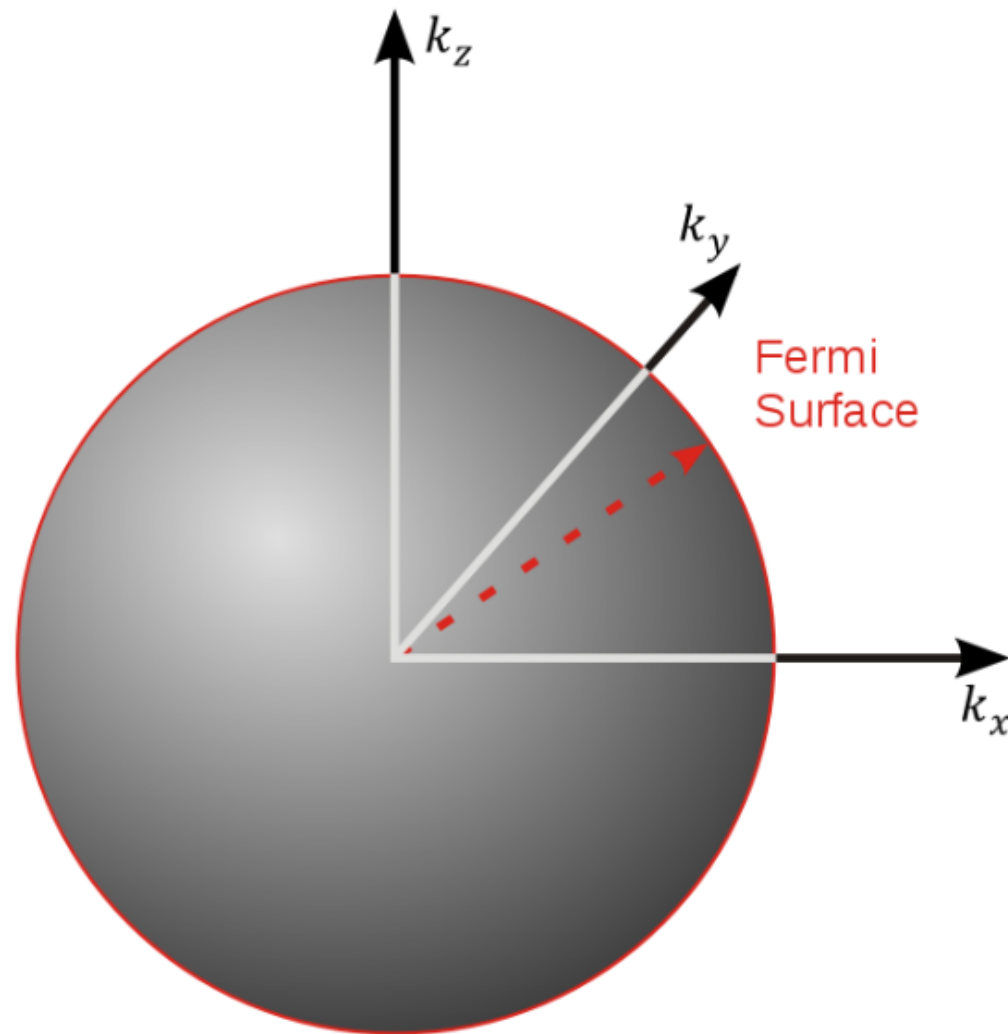
# 3. Metals

---

Oct 10, 2019

# Fermi surface for free electrons

---

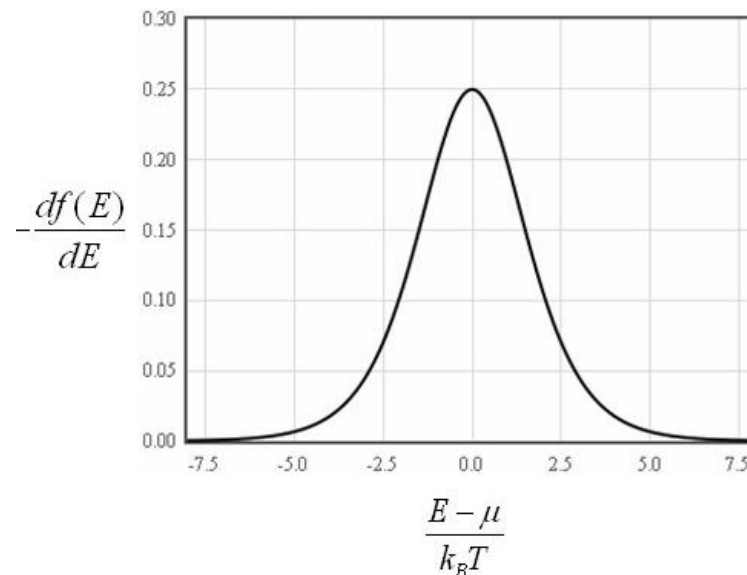


# Properties of metals depend mostly on the electron states at the Fermi surface

---

$$n = \int_{-\infty}^{\infty} D(E) f(E) dE = \int_{-\infty}^{\infty} \frac{D(E) dE}{\exp\left(\frac{E - \mu}{k_B T}\right) + 1}$$

$$n = \int_{-\infty}^{\infty} D(E) f(E) dE = K(\infty) f(\infty) - K(-\infty) f(-\infty) - \int_{-\infty}^{\infty} K(E) \frac{f(E)}{dE} dE.$$



# Thermodynamic properties

---

Chemical potential  
(implicitly defined by):

$$n = \int_{-\infty}^{\infty} \frac{D(E)}{1 + \exp\left(\frac{E - \mu}{k_B T}\right)} dE$$

DoS →  
μ

Internal energy density:

$$u = \phi + Ts + \mu n = \int_{-\infty}^{\infty} \frac{ED(E)}{1 + \exp\left(\frac{E - \mu}{k_B T}\right)} dE$$

DoS →  
u(T)

Energy spectral density:

$$u(E, T) = \frac{ED(E)}{1 + \exp\left(\frac{E - \mu}{k_B T}\right)}$$

DoS →  
u(E)

Specific heat:

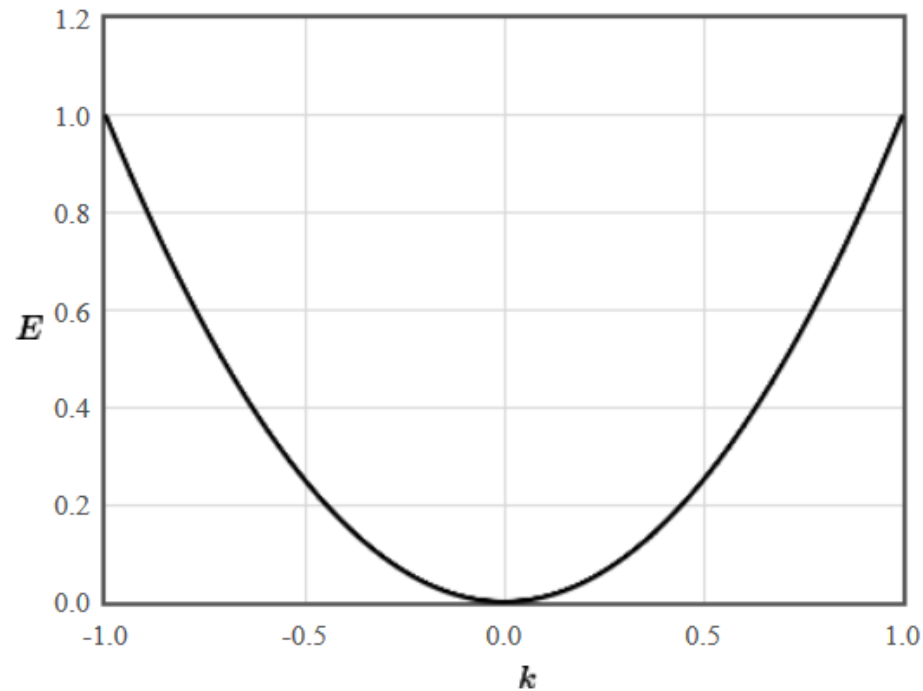
$$c_v = \frac{\partial u}{\partial T} = \int_{-\infty}^{\infty} \frac{ED(E)(E - \mu) \exp\left(\frac{E - \mu}{k_B T}\right)}{k_B T^2 \left(1 + \exp\left(\frac{E - \mu}{k_B T}\right)\right)^2} dE$$

DoS →  
cv(T)

# Empty lattice approximation

---

Free electrons cannot absorb photons



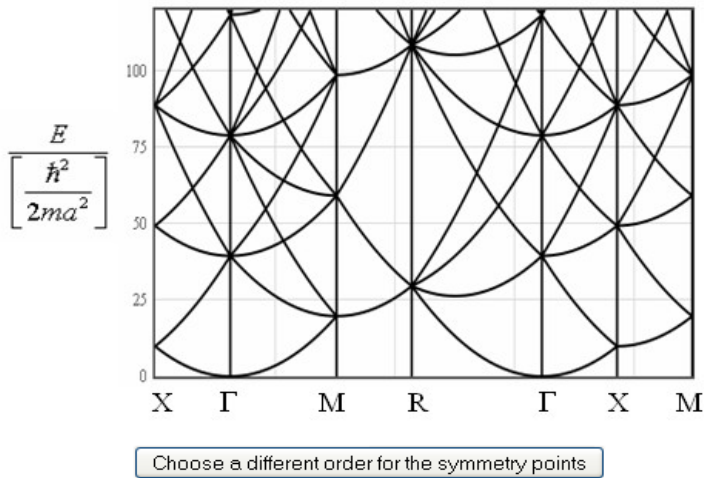
$$\hbar\vec{k}_f = \hbar\vec{k}_i + \hbar\vec{G}$$

$$\vec{k}_f - \vec{k}_i = \vec{G}$$

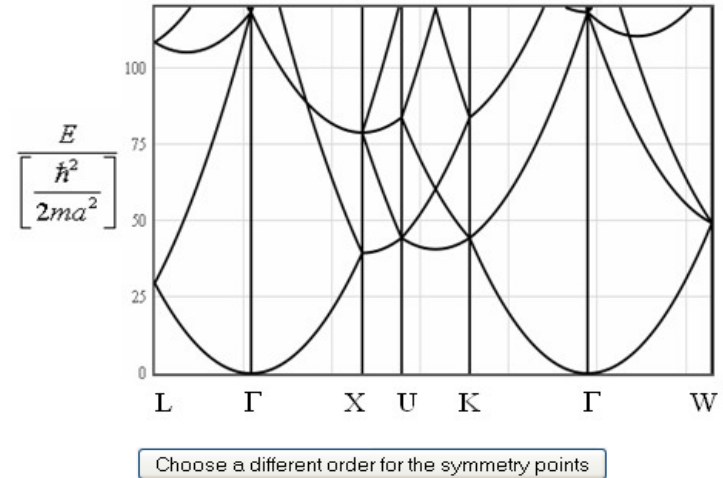
Laue condition

# Empty lattice approximation

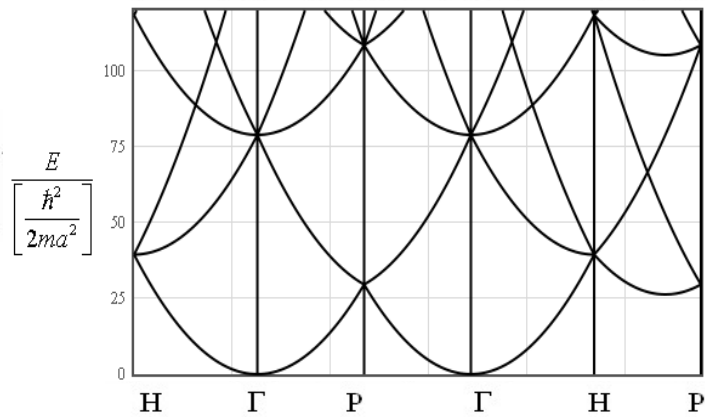
## Simple cubic



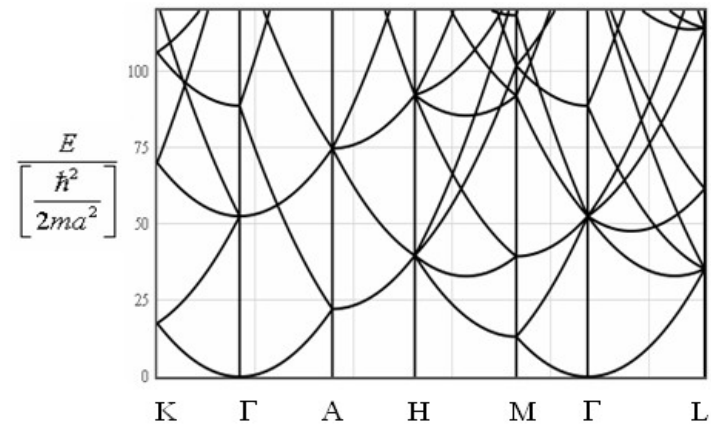
## Face centered cubic



## Body centered cubic

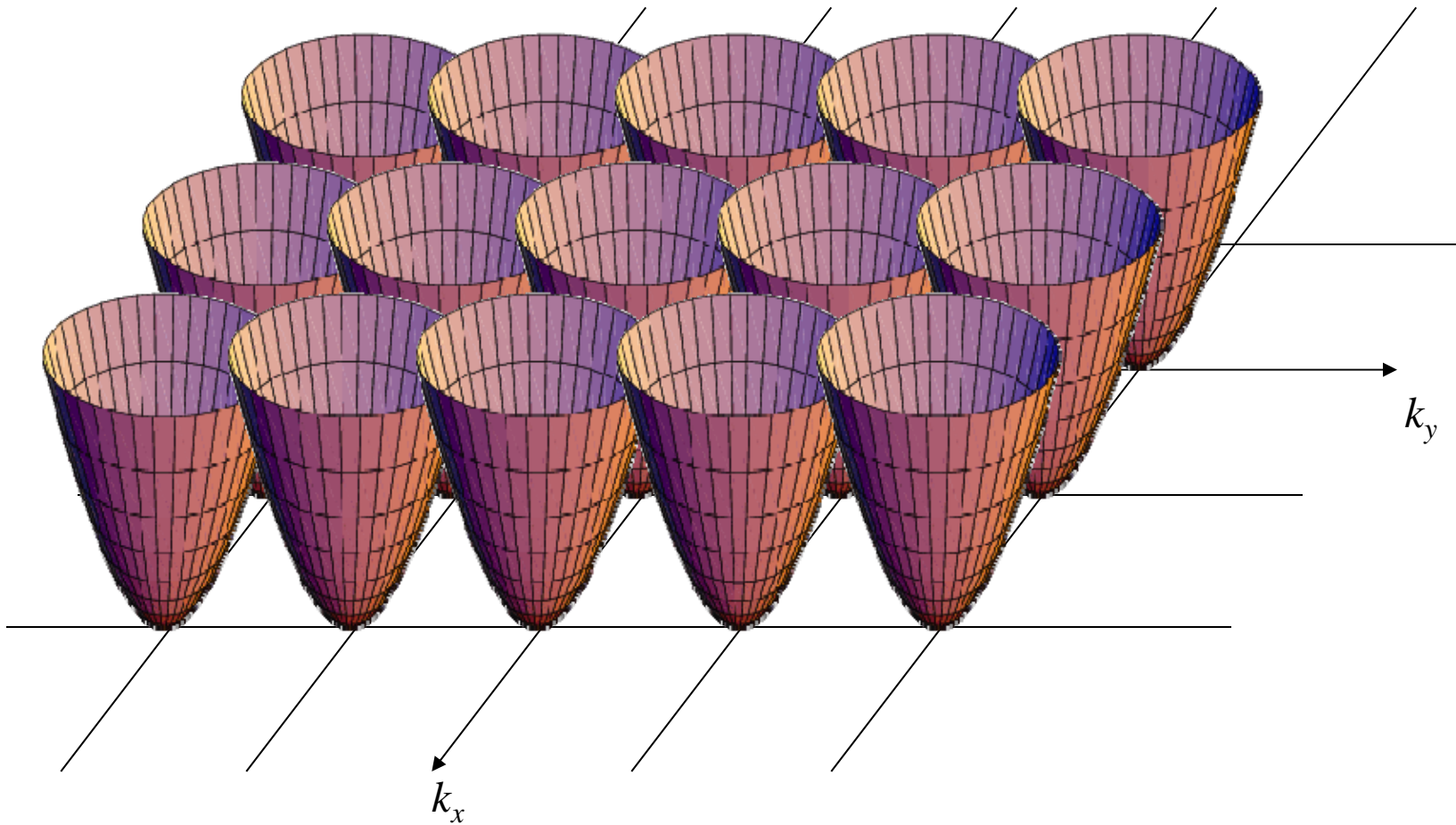


## Hexagonal



# Empty lattice approximation

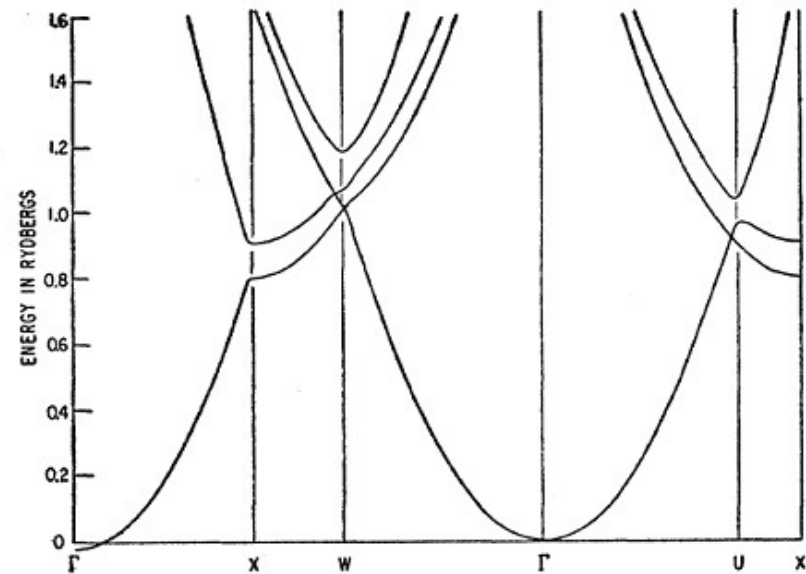
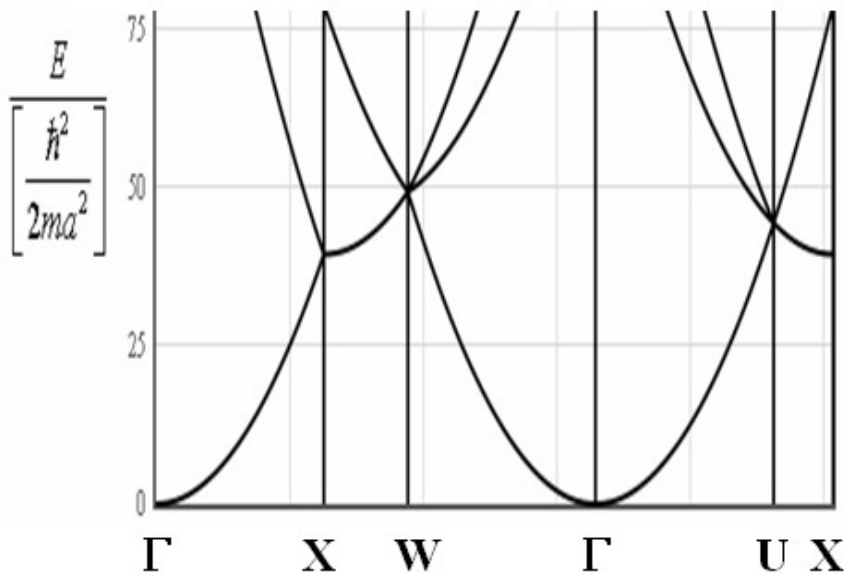
---



$$e^{i\vec{k}\cdot\vec{r}} = e^{i\vec{k}'\cdot\vec{r}} e^{i\vec{G}\cdot\vec{r}}$$

# Empty lattice approximation

---



W. Harrison, Phys. Rev. 118 p. 1182 (1960)

aluminum