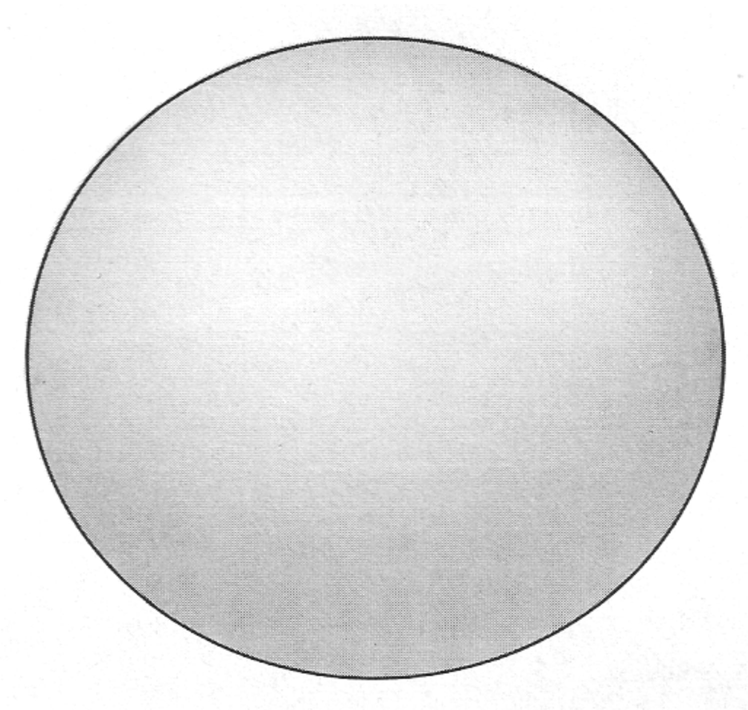
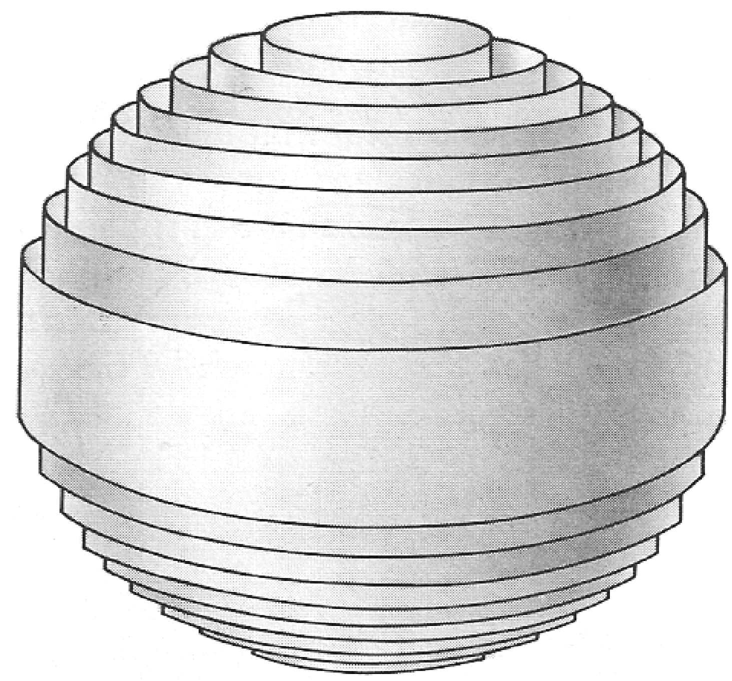
Fermi surfaces

Fermi sphere in a magnetic field



$B = 0$

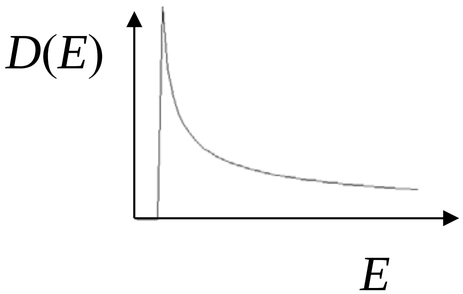


$B \neq 0$

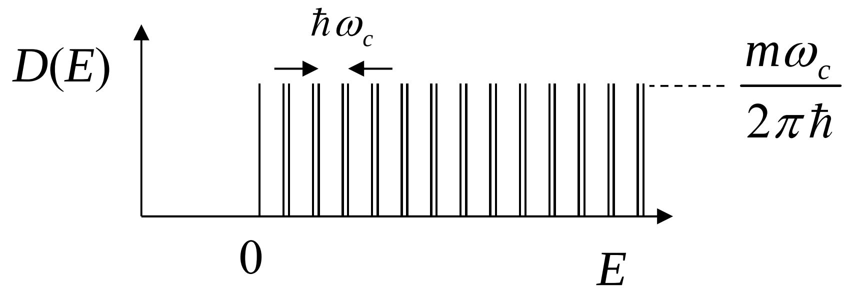
Landau cylinders

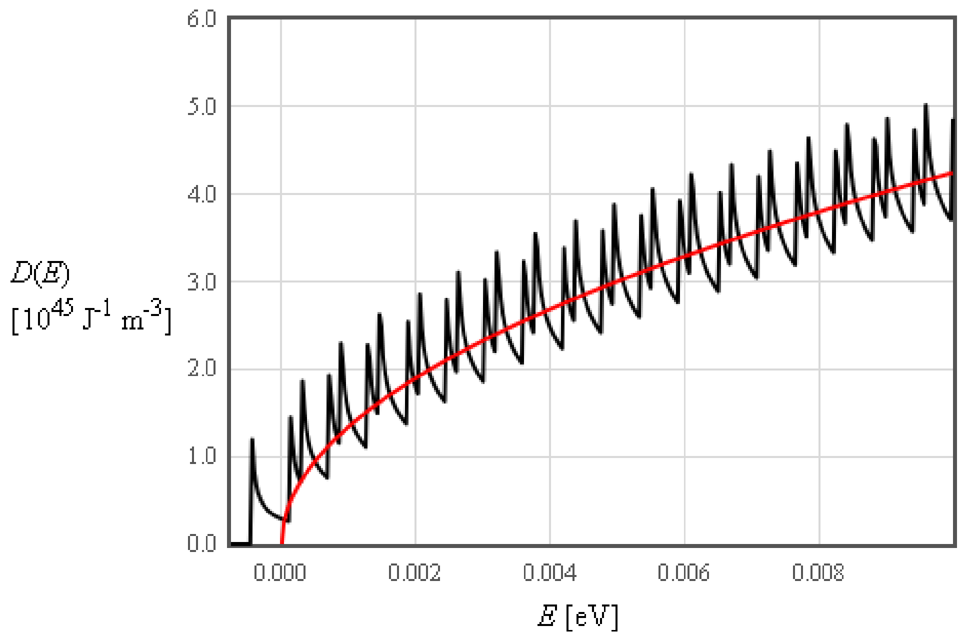
Density of states 3d

convolution of



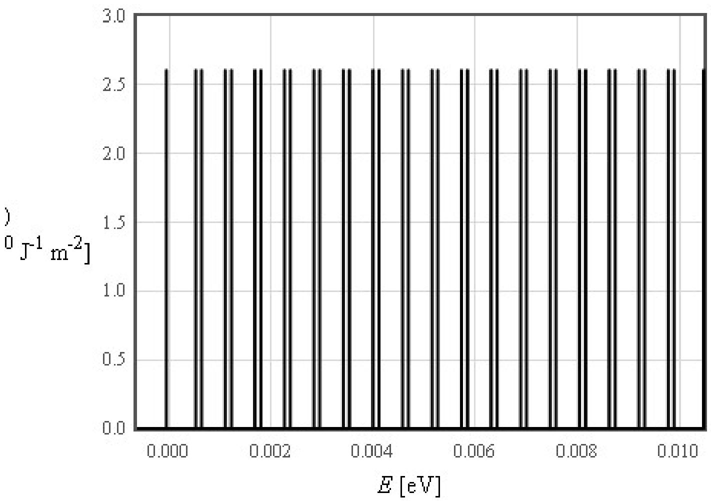
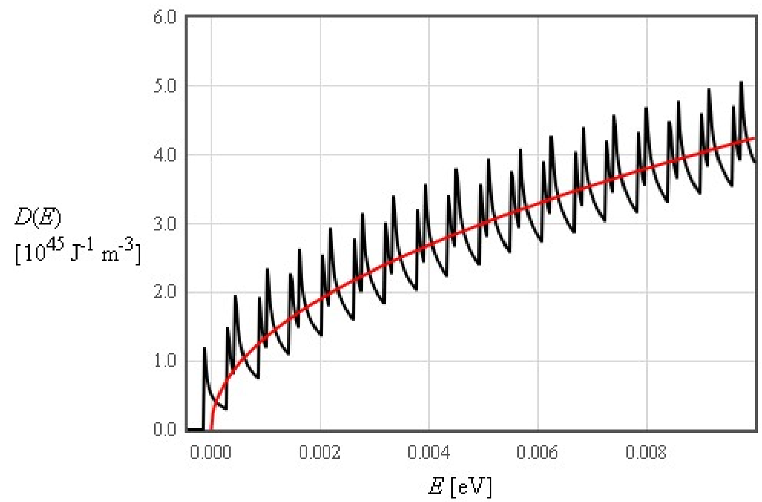
and





$$D(E) = \frac{(2m)^{3/2}\omega_c}{8\pi^2\hbar^2}\left(\sum_{\nu=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)}}+\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)}}\right) \quad \mathrm{J^{-1}m^{-3}}$$

quation for free electrons a magnetic field in 2 and 3 dimensions.

2-D Schrödinger equation	3-D Schrödinger equation
$i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$	$i\hbar\frac{d\psi}{dt}=\frac{1}{2m}\left(-i\hbar\nabla-\lvert e\rvert\vec{A}\right)^2\psi$
$\psi=g_\nu(x)\exp\left(ik_y y\right)$ $g_\nu(x)$ is a harmonic oscillator wavefunction	$\psi=g_\nu(x)\exp\left(ik_y y\right)\exp\left(ik_z z\right)$ $g_\nu(x)$ is a harmonic oscillator wavefunction
$E=\hbar\omega_c\left(\nu+\frac{1}{2}\right)$ J $\nu=0,1,2,\cdots \qquad \omega_c=\frac{\lvert eB_z\rvert}{m}$	$E=\frac{\hbar^2k_z^2}{2m}+\hbar\omega_c\left(\nu+\frac{1}{2}\right)$ J $\nu=0,1,2,\cdots \qquad \omega_c=\frac{\lvert eB_z\rvert}{m}$
$\sum_{\nu=0}^{\infty}\delta\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}\right)-\frac{g\mu_B}{2}B\right)+\delta\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}\right)+\frac{g\mu_B}{2}B\right)$ $\text{J}^{-1}\text{m}^{-2}$  Calculate DoS	$D(E)=\frac{(2m)^{3/2}\omega_c}{8\pi^2\hbar^2}\left(\sum_{\nu=0}^{\infty}\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}-g/4\right)}}+\frac{H\left(E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)\right)}{\sqrt{E-\hbar\omega_c\left(\nu+\frac{1}{2}+g/4\right)}}\right)$ $\text{J}^{-1}\text{m}^{-}$  Calculate DoS
$E_F=\hbar\omega\left(\text{Int}\left(\frac{\pi\hbar n}{\ }\right)+\frac{1}{\ }\right)$	

Practically all properties are periodic in 1/*B*

Internal energy

$$u = \int_{-\infty}^{\infty} E D(E) f(E) dE$$

Specific heat

$$c_v = \left(\frac{\partial u}{\partial T}\right)_{V=const}$$

Entropy

$$s = \int \frac{c_v}{T} dT$$

Helmholtz free energy

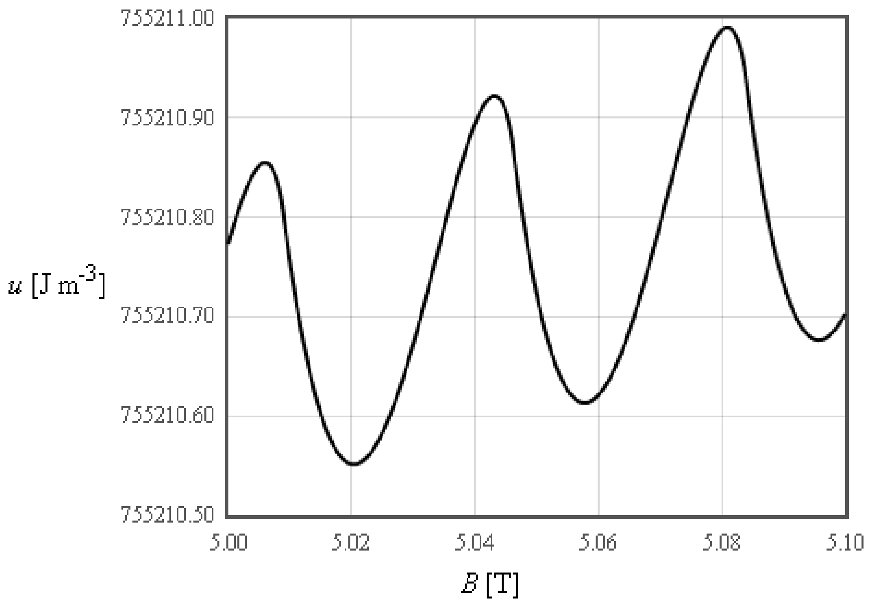
$$f = u - Ts$$

Pressure

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T=const}$$

Bulk modulus

$$B = -V \frac{\partial P}{\partial V}$$

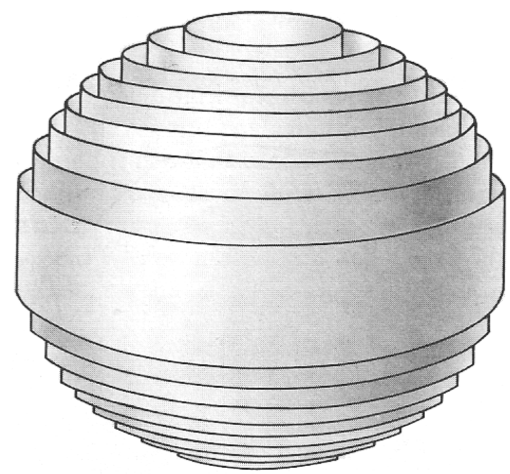


Magnetization

$$M = -\frac{dU}{dH}$$

Fermi sphere in a magnetic field

Cross sectional area $S = \pi k_F^2$



$$\hbar\omega_c\left(v+\tfrac{1}{2}\right)=\frac{\hbar^2 k_F^2}{2m}$$

$$\hbar\frac{eB_v}{m}\left(v+\tfrac{1}{2}\right)=\frac{\hbar^2 k_F^2}{2m}$$

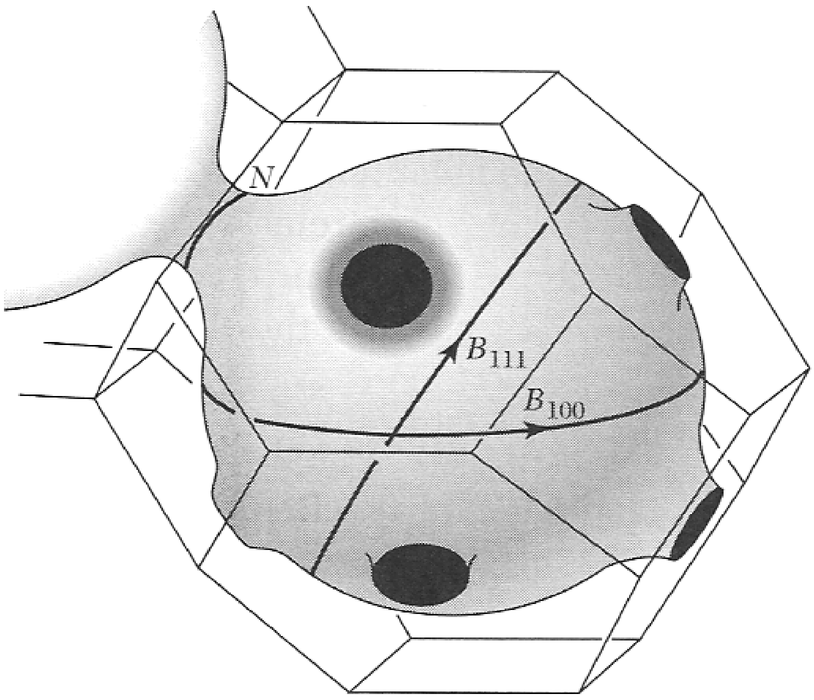
$$\frac{2\pi e}{\hbar}\left(v+1+\tfrac{1}{2}\right)=\frac{S}{B_{v+1}} \qquad \frac{2\pi e}{\hbar}\left(v+\tfrac{1}{2}\right)=\frac{S}{B_v}$$

Subtract right from left

$$S\left(\frac{1}{B_{v+1}}-\frac{1}{B_v}\right)=\frac{2\pi e}{\hbar}$$

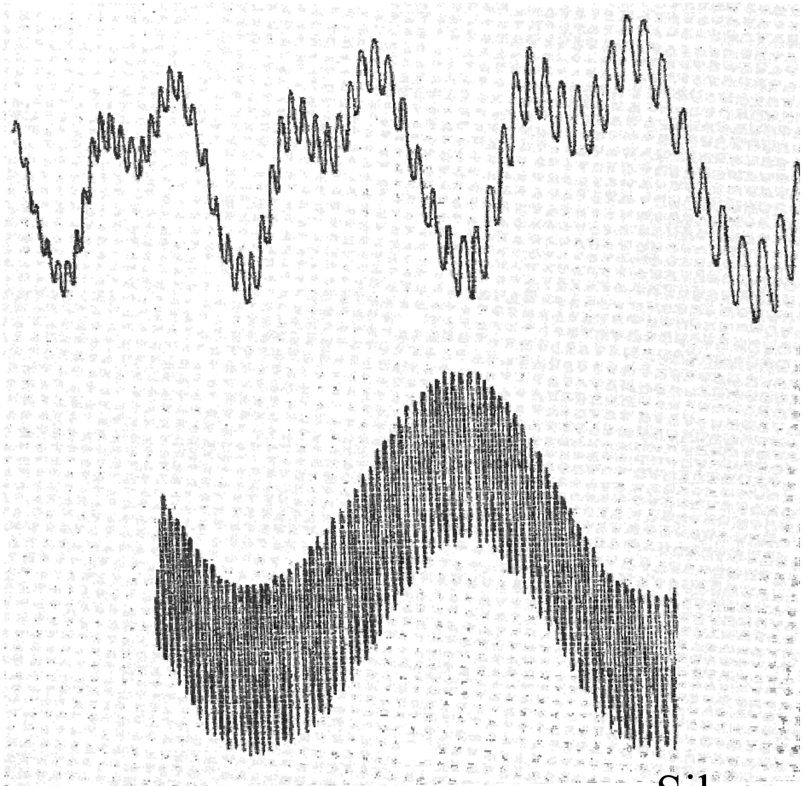
From the period of the oscillations, you can determine the cross sectional area *S*.

Experimental determination of the Fermi surface



Kittel

Rhenium

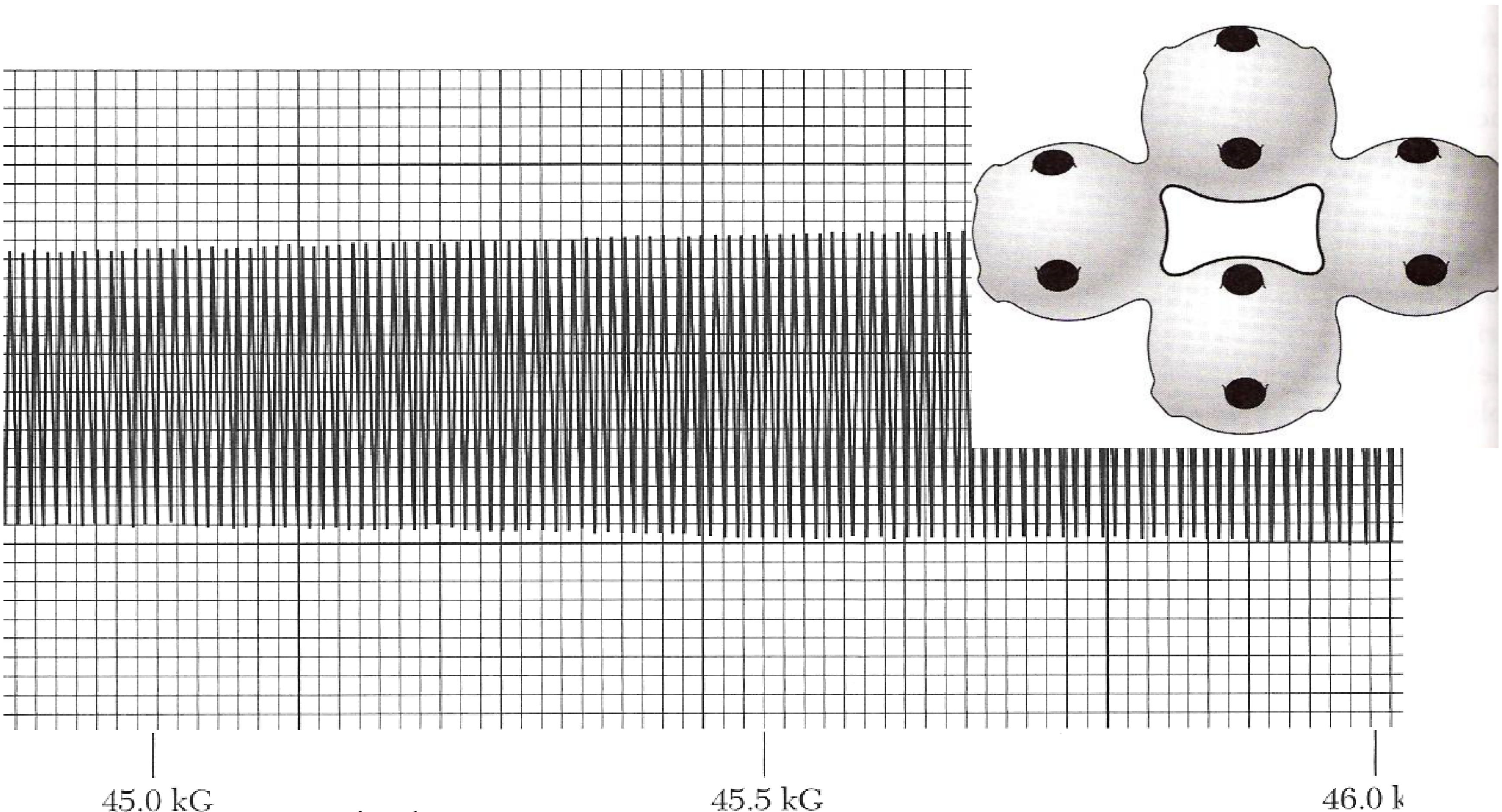


Silver

de Haas - van Alphen

De Haas - van Alphen effect

The magnetic moment of gold oscillates periodically with $1/B$



Kittel

http://www.phys.ufl.edu/fermisurface/