# Ferromagnetism

# Ferromagnetism

Below a critical temperature (called the Curie temperature) a magnetization spontaneously appears in a ferromagnet even in the absence of a magnetic field.

Iron, nickel, and cobalt are ferromagnetic.

Ferromagnetism overcomes the magnetic dipole-dipole interactions. It arises from the Coulomb interactions of the electrons. The energy that is gained when the spins align is called the exchange energy.

# Schrödinger equation for two particles

$$-\frac{\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2\right)\psi+V_1(\vec{r}_1)\psi+V_2(\vec{r}_2)\psi+V_{1,2}(\vec{r}_1,\vec{r}_2)\psi=E\psi$$

$\psi(\vec{r}_1,\vec{r}_2)=\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)$ is a solution to the noninteracting Hamiltonian, $V_{1,2}=0$

$$\psi_A\left(\vec{r}_1,\vec{r}_2\right)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)\begin{pmatrix}\uparrow\uparrow\\ \frac{1}{\sqrt{2}}\left(\uparrow\downarrow+\downarrow\uparrow\right)\\ \downarrow\downarrow\end{pmatrix}$$

$$\psi_S\left(\vec{r}_1,\vec{r}_2\right)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)\frac{1}{\sqrt{2}}\left(\uparrow(\vec{r}_1)\downarrow(\vec{r}_2)-\downarrow(\vec{r}_1)\uparrow(\vec{r}_2)\right)$$

# Exchange (Austauschwechselwirking)

$$\psi_A(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)$$

$$\langle\psi_A|H|\psi_A\rangle=\frac{1}{2}[\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle-\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle$$
$$-\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle+\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle]$$

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$$\psi_S(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)$$

$$\langle\psi_S|H|\psi_S\rangle=\frac{1}{2}[\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle+\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle$$
$$+\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle+\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle]$$

The difference in energy between the $\psi_A$ and $\psi_S$ is twice the **exchange energy**.

# Exchange

The exchange energy can only be defined when you speak of multi-electron wavefunctions. It is the difference in energy between the symmetric solution and the antisymmetric solution. There is only a difference when the electron-electron term is included. Coulomb repulsion determines the exchange energy.

In ferromagnets, the antisymmetric state has a lower energy. Thus the state with parallel spins has lower energy.

In antiferromagnets, the symmetric state has a lower energy. Neighboring spins are antiparallel.

Ordered states have a lower entropy than free electrons.

# Mean field theory (Molekularfeldtheorie)

Heisenberg Hamiltonian $H = -\sum_{i,j} J_{i,j} \vec{S}_i \cdot \vec{S}_j - g\mu_B \vec{B} \cdot \sum_i \vec{S}_i$

Exchange energy

Mean field approximation

$$H_{MF} = \sum_i \vec{S}_i \cdot \left( \sum_\delta J_{i,\delta} \left\langle \vec{S} \right\rangle + g\mu_B \vec{B} \right)$$

Looks like a magnetic field $B_{MF}$

δ sums over the neighbors of spin *i*

$$\vec{B}_{MF} = \frac{1}{g\mu_B} \sum_\delta J_{i,\delta} \left\langle \vec{S} \right\rangle$$

magnetization →

$$\vec{M} = g\mu_B \frac{N}{V} \left\langle \vec{S} \right\rangle$$

eliminate $<S>$

# Mean field theory

$$\vec{B}_{MF} = \frac{V}{Ng^2\mu_B^2} zJ\vec{M}$$

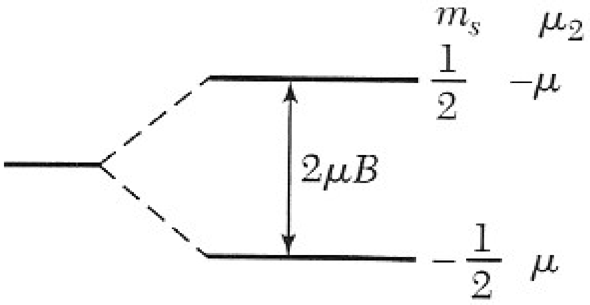
$z$ is the number of nearest neighbors

In mean field, the energy of the spins is

$$E = \pm\frac{1}{2} g\mu_B (B_{MF} + B_a)$$

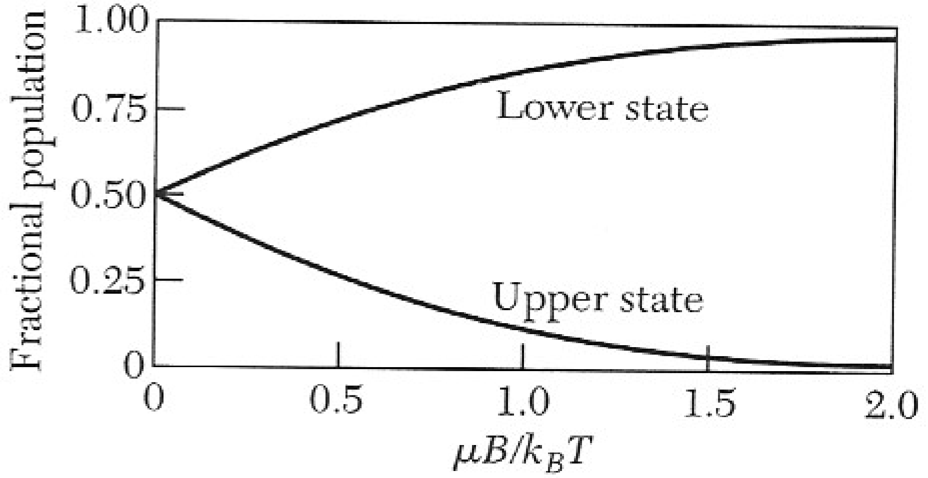
We calculated the populations of the spins in the paramagnetism section

# Spin populations



$$\frac{N_1}{N} = \frac{\exp(\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$



$$\begin{aligned} M &= (N_1 - N_2)\mu \\ &= N\mu \frac{\exp(\mu B / k_B T) - \exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)} \\ &= N\mu \tanh\left(\frac{\mu B}{k_B T}\right) \end{aligned}$$

# Mean field theory

$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh\left( \frac{g \mu_B \left( B_{MF} + B_a \right)}{2 k_B T} \right)$$

For zero applied field

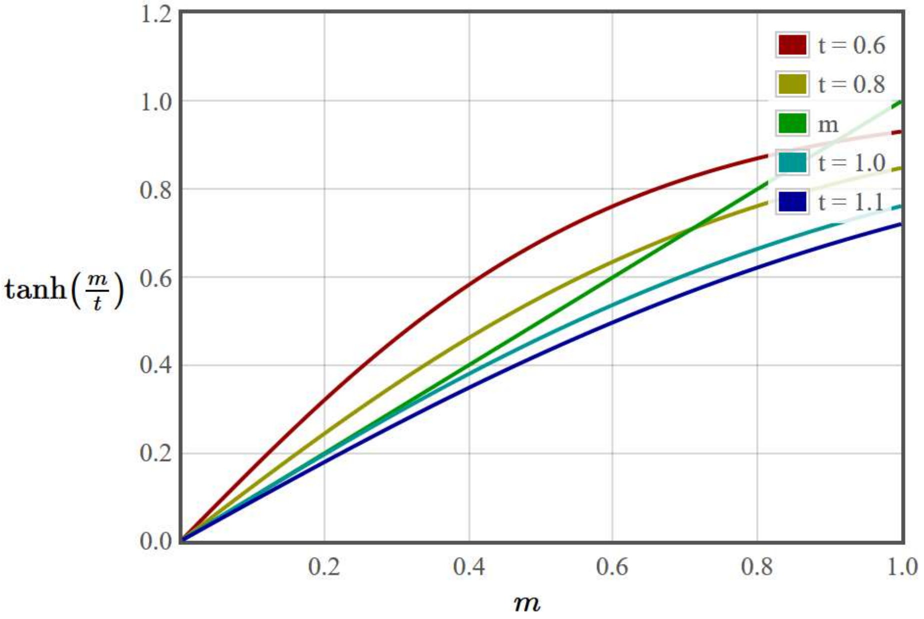
$$M = M_S \tanh\left( \frac{T_c}{T} \frac{M}{M_s} \right)$$

$$M_S = \frac{N}{2V} g \mu_B \quad \text{and} \quad T_c = \frac{z}{4 k_B} J$$

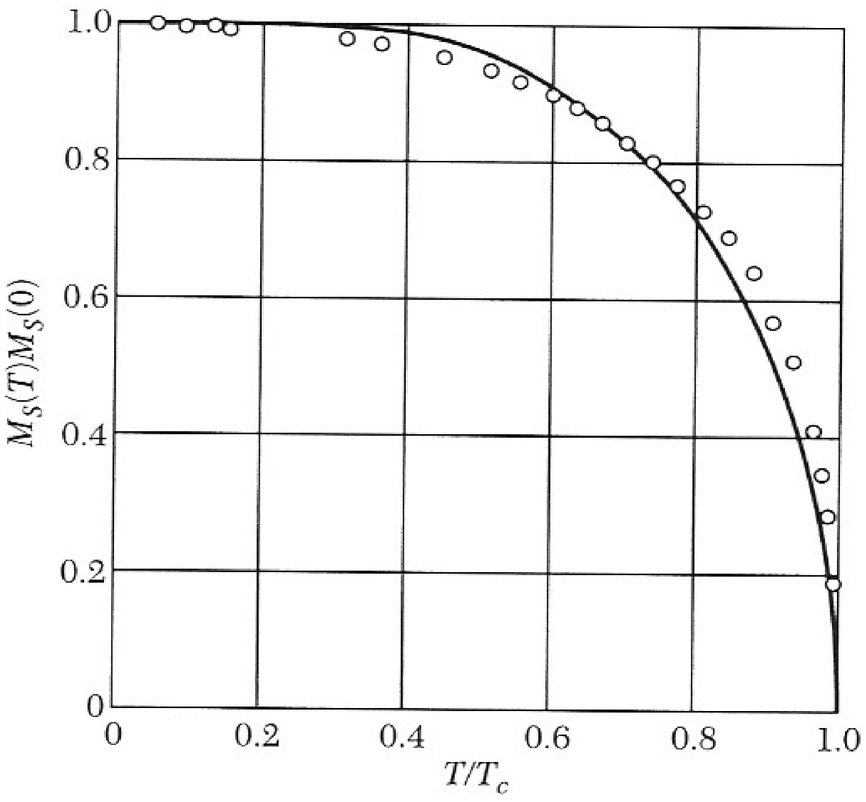
$M_s$ = saturation magnetization $T_c$ = Curie temperature

# Mean field theory

$$M = M_S \tanh\left(\frac{T_c}{T}\frac{M}{M_s}\right)$$



$$m = \tanh\left(\frac{m}{t}\right)$$



Experimental points for Ni.

$$M_S = \frac{N}{2V} g\mu_B \quad \text{and} \quad T_c = \frac{z}{4k_B}J$$

Source: Kittel

# Ferromagnetism

| **Material** | **Curie temp. (K)** | |
|---|---|---|
| Co | 1388 | |
| Fe | 1043 | |
| $FeOFe_2O_3$ | 858 | |
| $NiOFe_2O_3$ | 858 | |
| $CuOFe_2O_3$ | 728 | |
| $MgOFe_2O_3$ | 713 | |
| MnBi | 630 | |
| Ni | 627 | |
| MnSb | 587 | |
| $MnOFe_2O_3$ | 573 | |
| $Y_3Fe_5O_{12}$ | 560 | |
| $CrO_2$ | 386 | |
| MnAs | 318 | |
| Gd | 292 | |
| Dy | 88 | |
| EuO | 69 | Electrical insulator |
| $Nd_2Fe_{14}B$ | 353 | $M_s = 10\, M_s(\text{Fe})$ |
| $Sm_2Co_{17}$ | 700 | rare earth magnets |

$$M_S = \frac{N}{2V} g \mu_B$$

$$T_c = \frac{z}{4k_B} J$$

# Curie - Weiss law

$$M = \frac{1}{2} g\mu_B \frac{N}{V} \tanh\left( \frac{g\mu_B \left(B_{MF} + B_a\right)}{2k_B T} \right) \qquad \vec{B}_{MF} = \frac{V}{Ng^2\mu_B^2} zJ\vec{M}$$

Above $T_c$ we can expand the hyperbolic tangent $\tanh(x) \approx x$ for $x << 1$

$$M \approx \frac{1}{4} g^2\mu_B^2 \frac{N}{Vk_B T} \left( \frac{V}{Ng^2\mu_B^2} zJM + B_a \right)$$

Solve for $M$

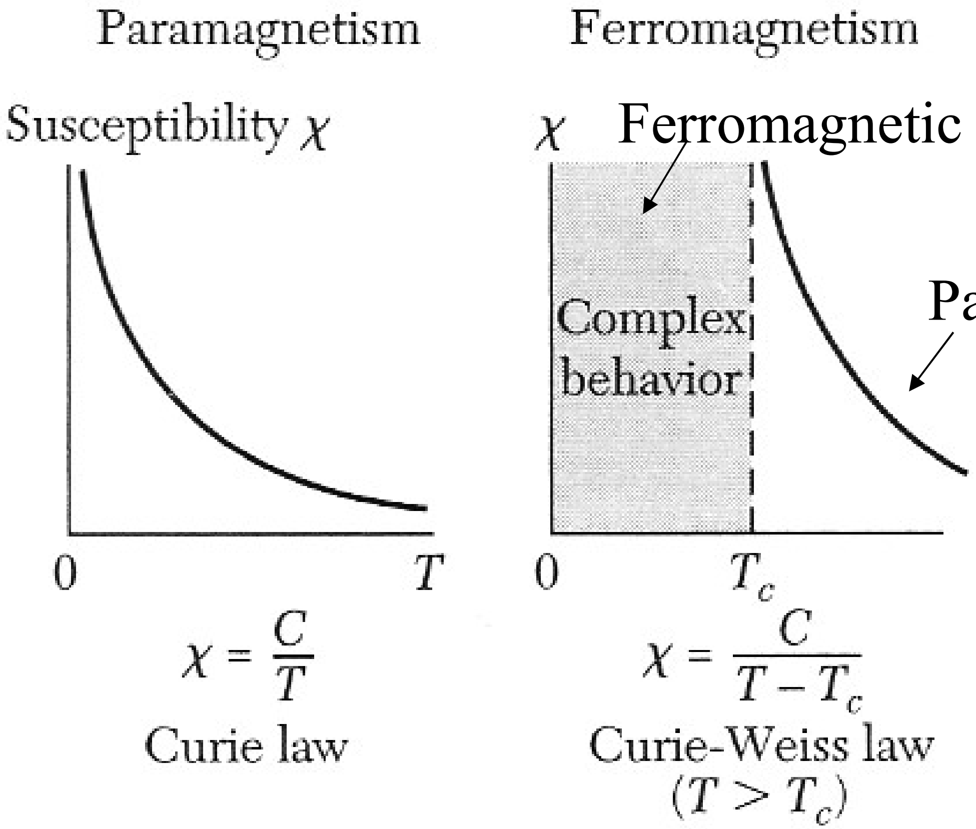
$$M \approx \frac{g^2\mu_B^2 N}{4Vk_B} \frac{B_a}{T - T_c} \qquad T_c = \frac{z}{4k_B} J$$

Curie Weiss Law $\chi = \frac{dM}{dH} \approx \frac{C}{T - T_c}$

Critical fluctuations near $T_c$

# Ferromagnets are paramagnetic above $T_c$

Source: Kittel



Ferromagnetic

Paramagnetic

Critical fluctuations near $T_c$.

PHYSICAL REVIEW VOLUME 102, NUMBER 4 MAY 15, 1956

# Magnetization of a Magnetite Single Crystal Near the Curie Point*

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(Received January 20, 1956)

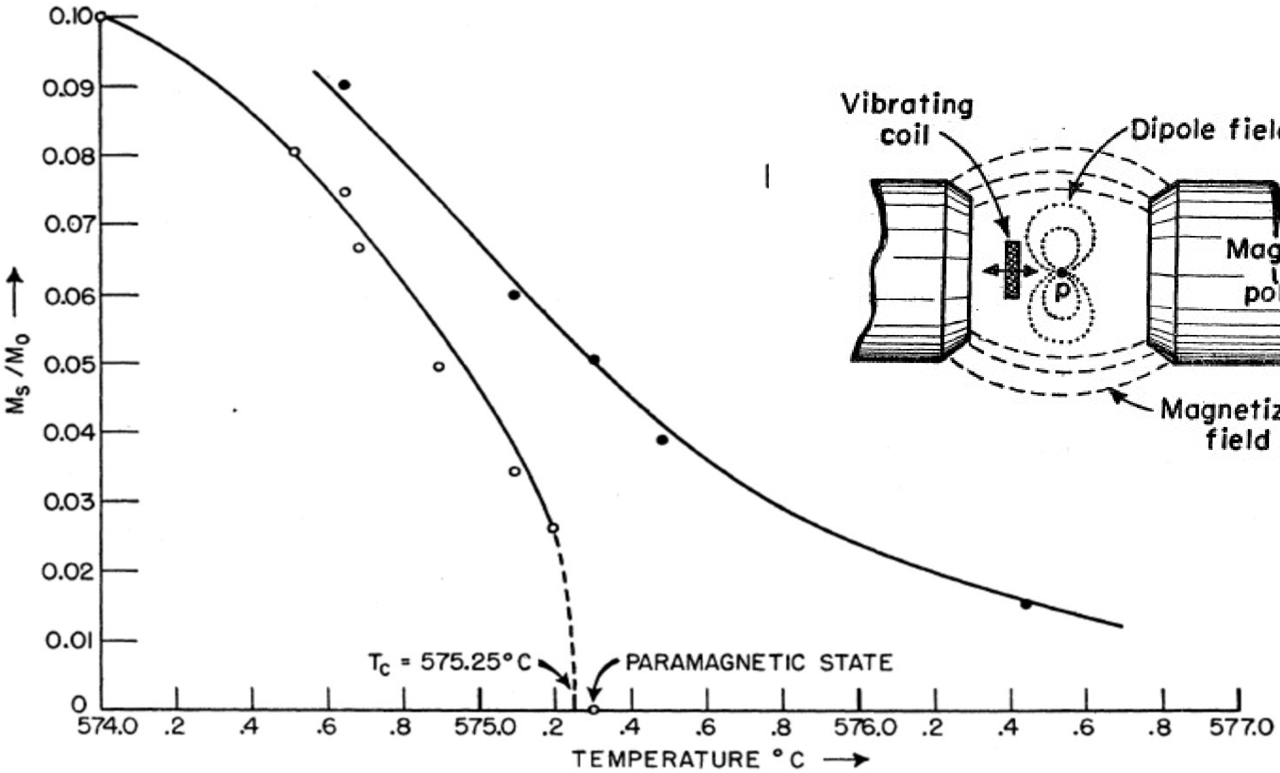


FIG. 9. $M_s/M_0$ *vs* $T$ in the [111] direction near the Curie point for single-crystal magnetite.

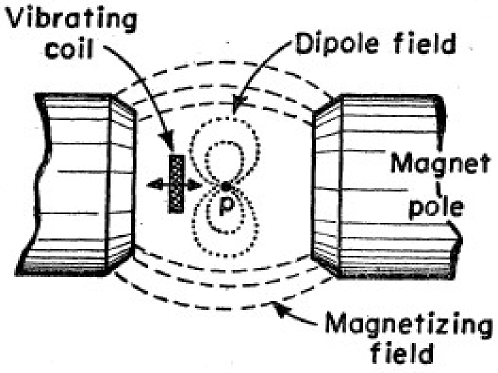


FIG. 2. Principle of the vibrating-coil magnetometer.

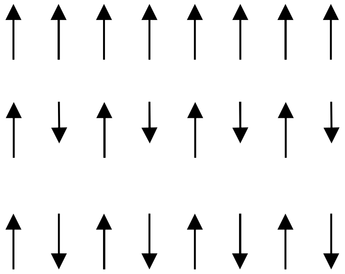
# Magnetic ordering

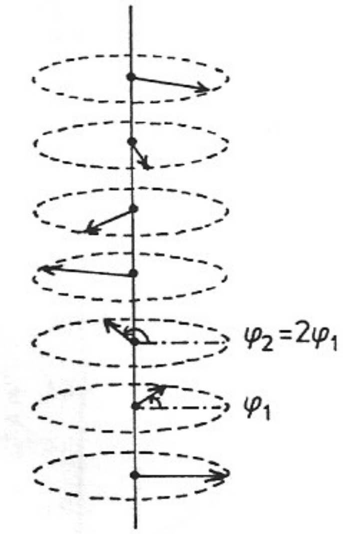
Ferromagnetism

Ferrimagnetism

Antiferromagnetism

Helimagnetism





All ordered magnetic states have excitations called magnons