Quantum Hall Effect

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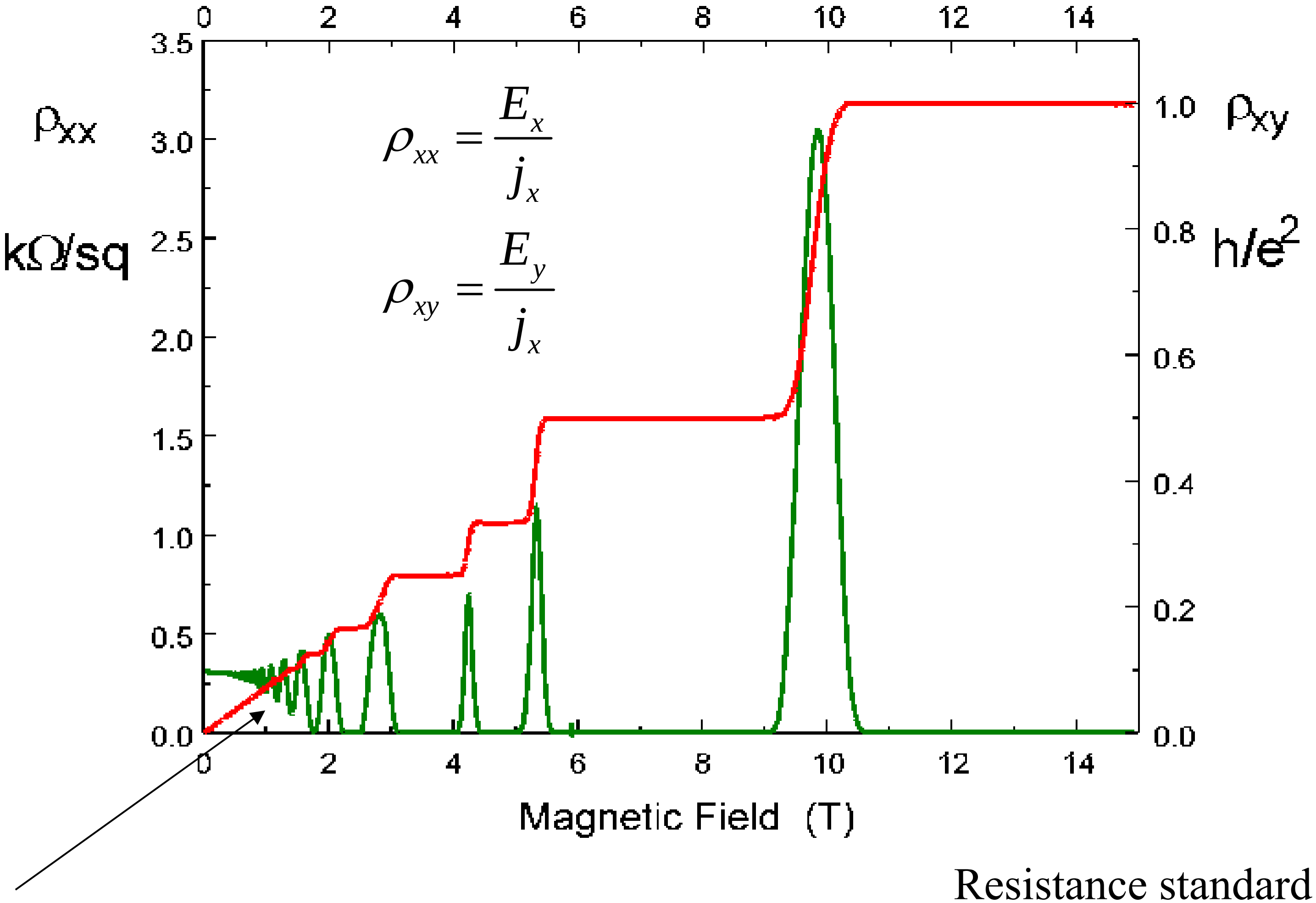


$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$

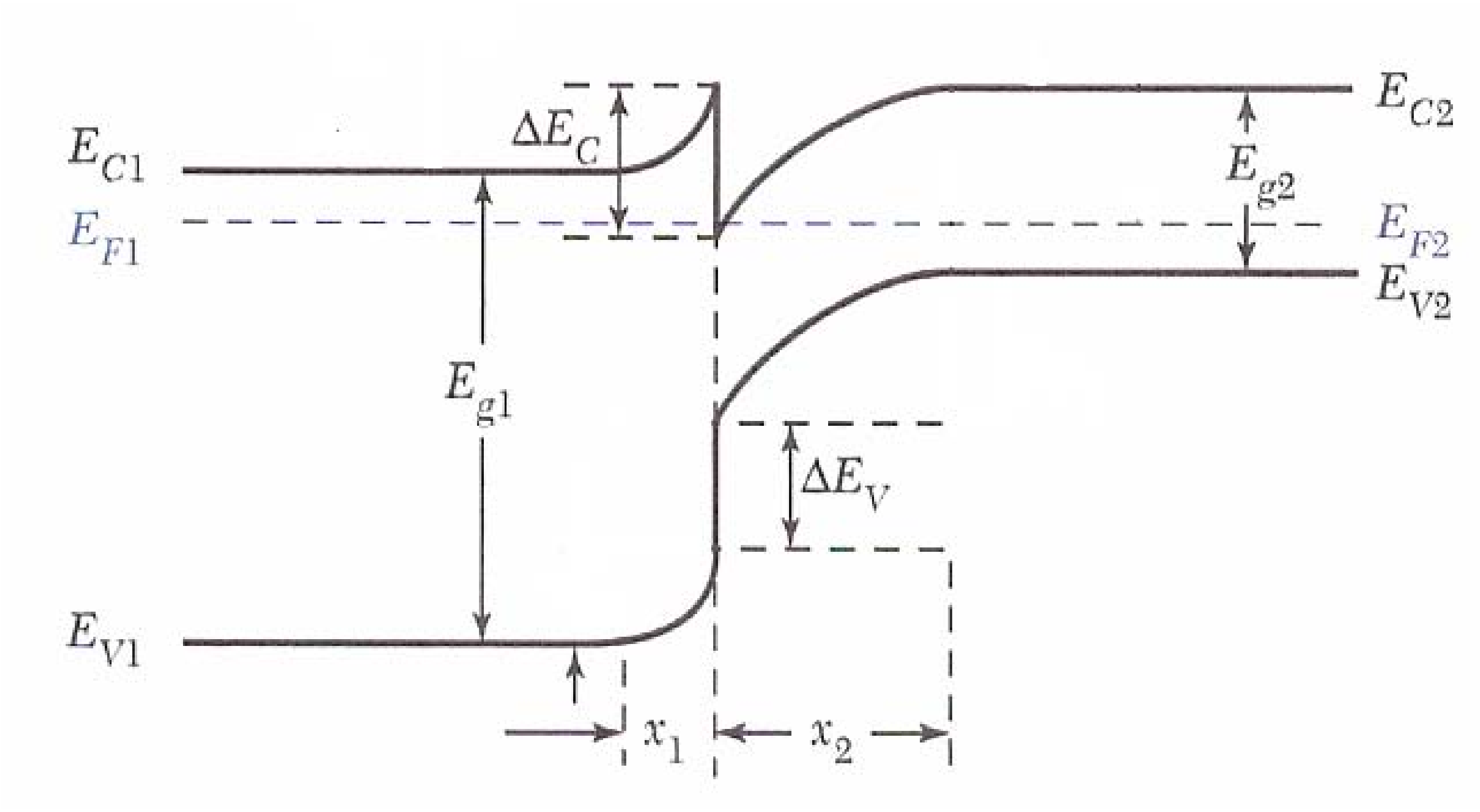
Klaus von
Klitzing

Shubnikov-De Haas oscillations

Resistance standard
25812.807557(18) Ω

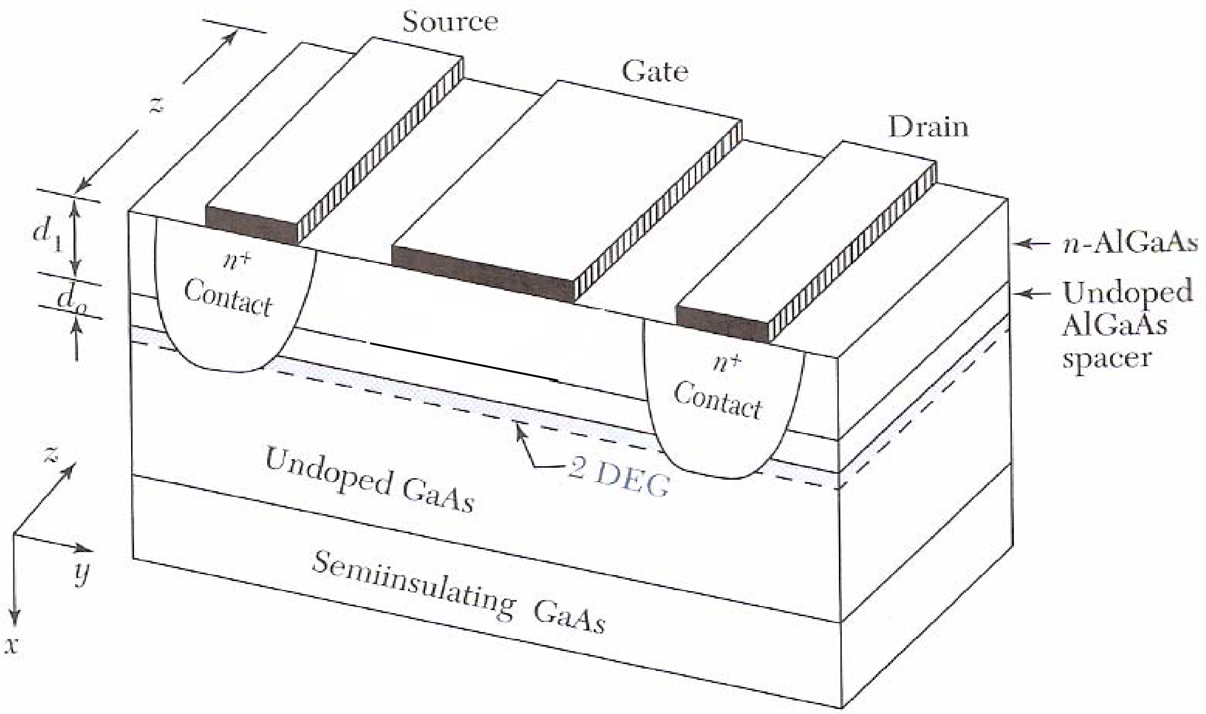
Heterostructure

pn junction formed from two semiconductors with different band gaps



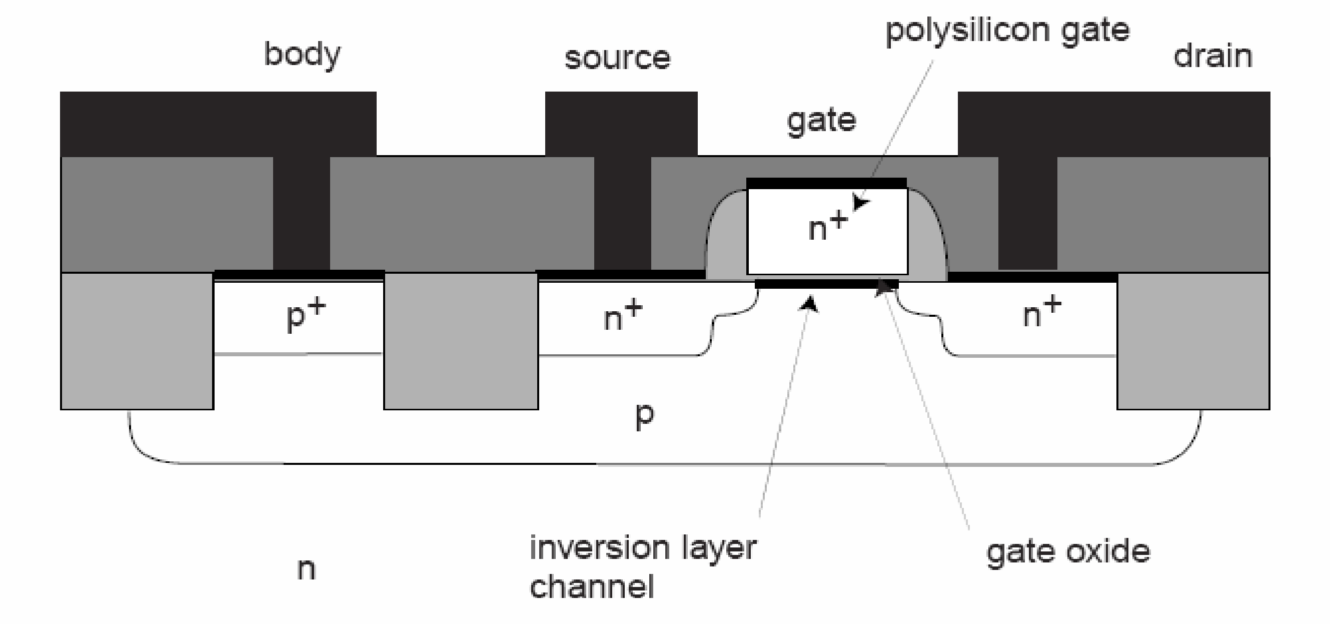
MODFET (HEMT)

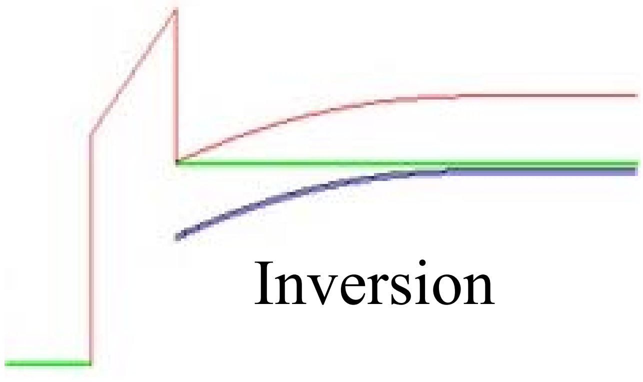
Modulation doped field effect transistor (MODFET)
High electron mobility transistor (HEMT)



The magnetic field can be at an angle to the 2DEG. The Landau splitting experiences the component perpendicular to the plane. The Zeeman splitting experiences the full field.

MOSFETs





Inversion

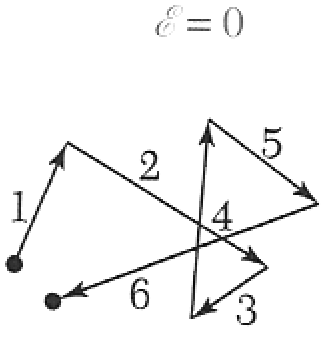
Drift

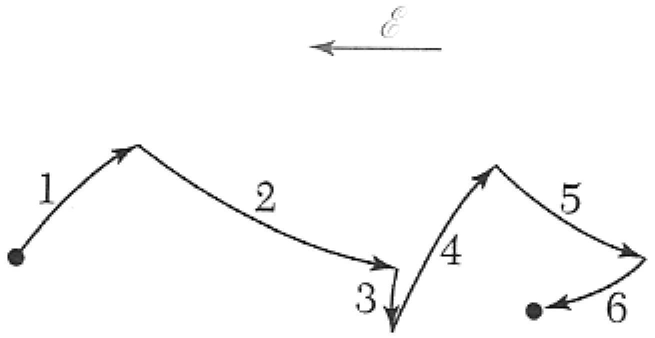
The electrons scatter and change direction after a time τ_{sc}.

Classical equipartition: $\frac{1}{2} m v_{th}^2 = \frac{3}{2} k_B T$

At 300 K, $v_{th} \sim 10^7$ cm/s.

mean free path: $\ell = v_{th} \tau_{sc} \sim 10$ nm ~ 200 atoms

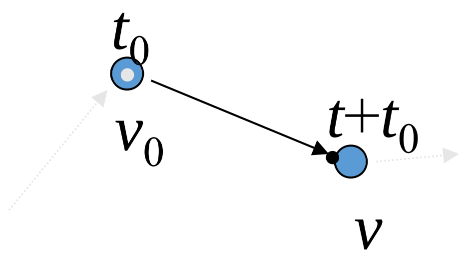




Drift (diffusive transport)

$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

time between two collisions



$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

$<v_0> = 0$ $<t - t_0> = \tau_{sc}$

$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v}$$

drift velocity: $\overrightarrow{v_{d,n}} = -\mu_n\overrightarrow{E}$ $\overrightarrow{v_{d,p}} = \mu_p\overrightarrow{E}$

Review of the Hall effect

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{\mathrm{sc}}} \longleftarrow$$ diffusive regime

$$\vec{F} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right) = m\frac{\vec{v}_d}{\tau_{\mathrm{sc}}}$$

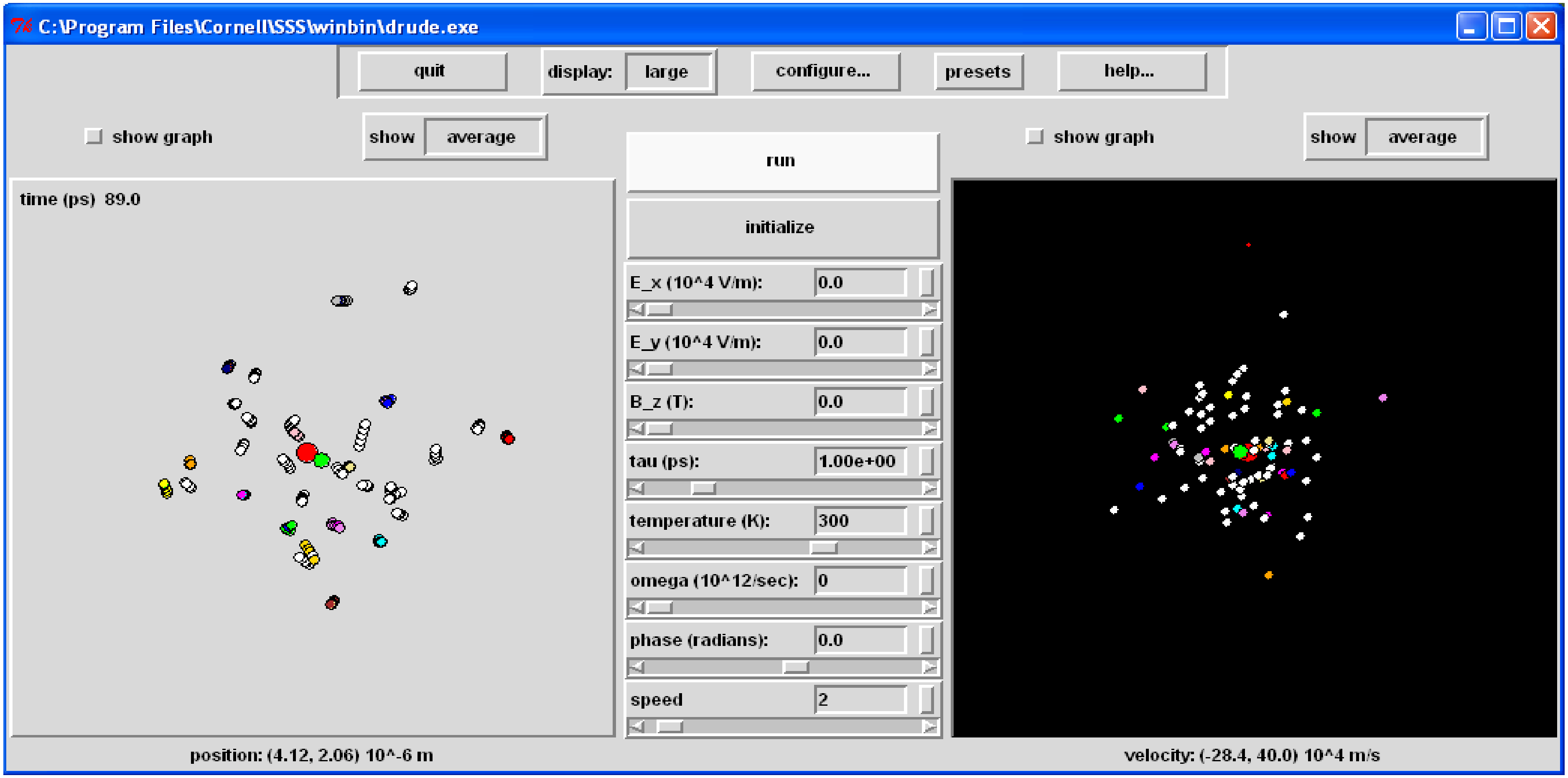
If *B* is in the *z*-direction, and *E* is in the *x*- direction, the three components of the force are

$$-e\left(E_x + v_{dy}B_z\right) = m\frac{v_{dx}}{\tau_{\mathrm{sc}}}$$

$$ev_{dx}B_z = m\frac{v_{dy}}{\tau_{\mathrm{sc}}} \quad \Longrightarrow \quad \tan\theta_H = -\frac{eB_z}{m}\tau_{sc}$$

$$0 = m\frac{v_{dz}}{\tau_{\mathrm{sc}}}$$

Hall angle



C:\Program Files\Cornell\SSS\winbin\drude.exe

time (ps) 89.0

E_x (10^4 V/m): 0.0

E_y (10^4 V/m): 0.0

B_z (T): 0.0

tau (ps): 1.00e+00

temperature (K): 300

omega (10^12/sec): 0

phase (radians): 0.0

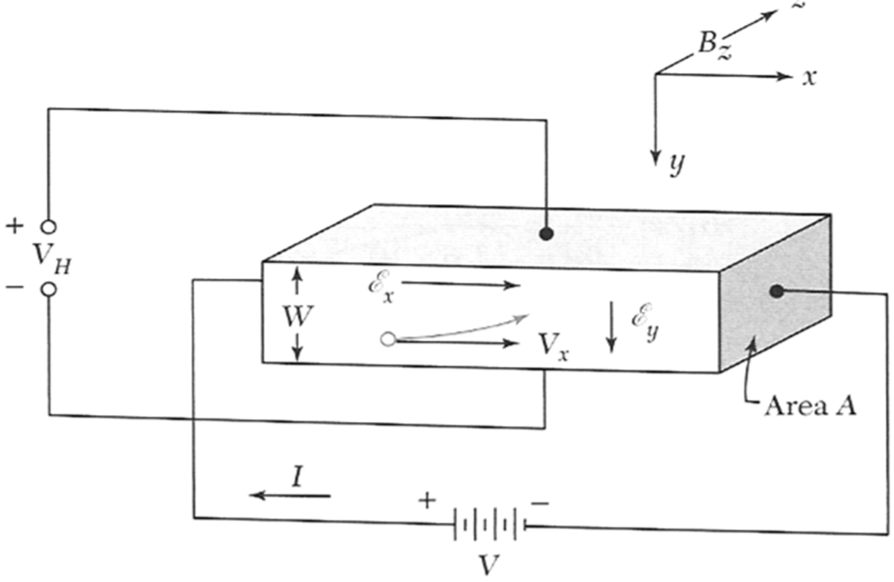
speed 2

position: (4.12, 2.06) 10^-6 m

velocity: (-28.4, 40.0) 10^4 m/s

If no forces are applied, the electrons diffuse.
The average velocity moves against an electric field.
In just a magnetic field, the average velocity is zero.
In an electric and magnetic field, the electrons move in a straight line at the Hall angle.
The drift velocity decreases as the B field increases.

The Hall Effect (diffusive regime)



$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $v_{d,y} = 0$,

$E_y = v_{d,x}B_z = V_H/W = R_H j_x B_z$ V_H = Hall voltage, R_H = Hall Constant

$v_{d,x} = - j_x/ne$

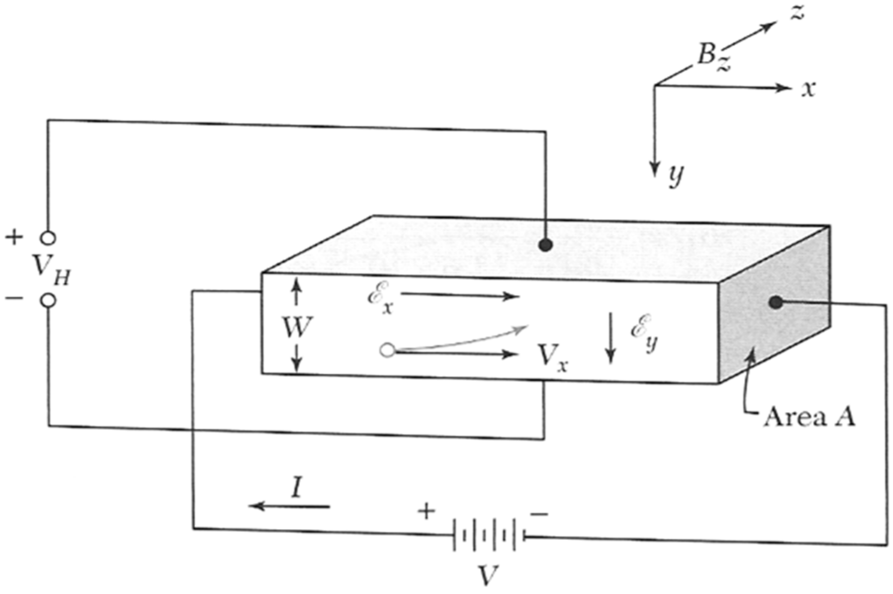
$$\boxed{R_H = E_y/j_xB_z = - 1/ne}$$

The Hall Effect (diffusive regime)

$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{xy} = \frac{E_y}{j_x}$$

$R_H = E_y/j_x B_z = -1/ne$



multiply both sides by B_z

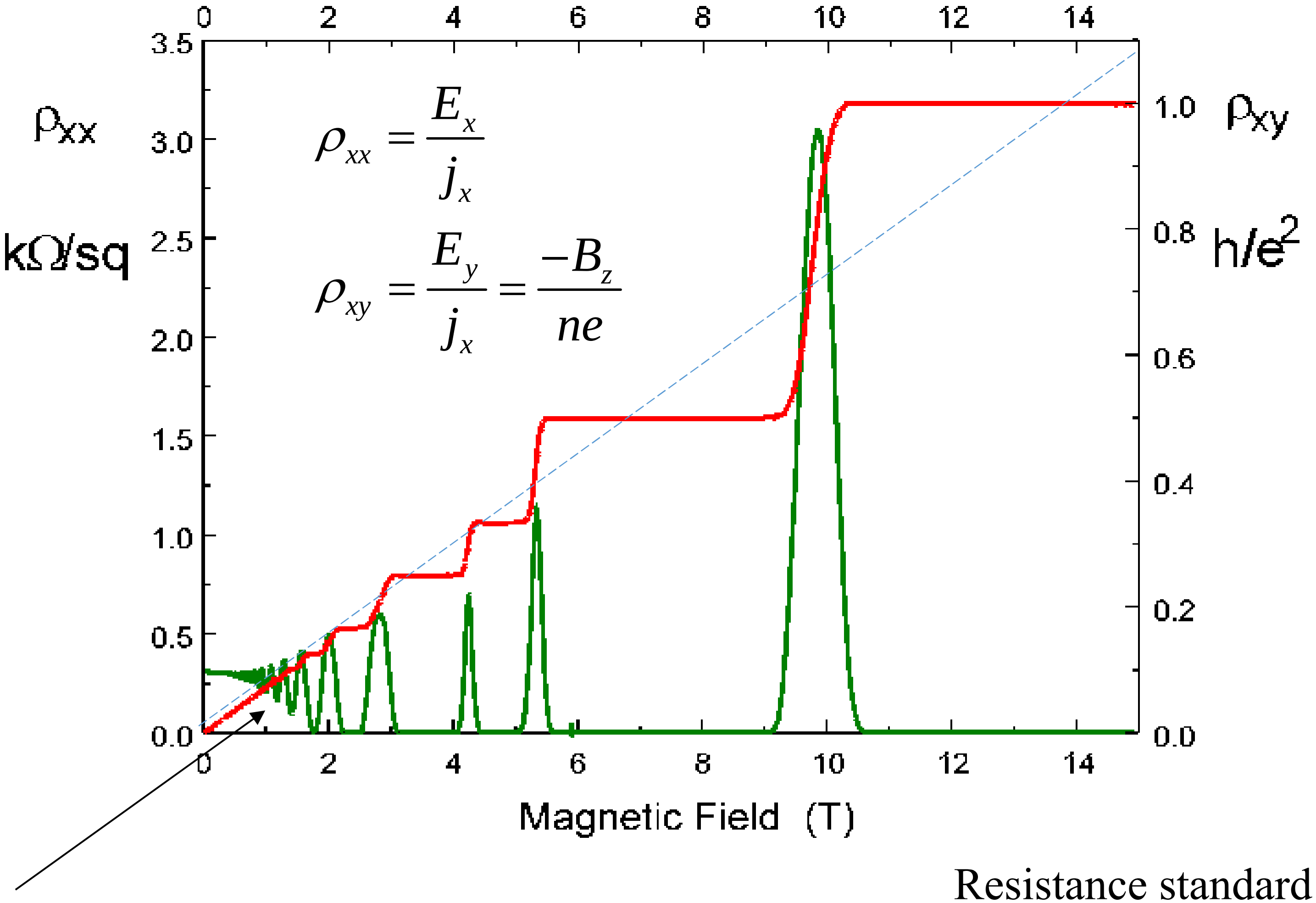
$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

In 2D, j has units of A/m and n has units of $1/\mathrm{m}^2$.

In 3D, j has units of $\mathrm{A/m}^3$ and n has units of $1/\mathrm{m}^3$.

The Hall resistivity is proportional to the magnetic field.

Quantum Hall Effect



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Shubnikov-De Haas oscillations

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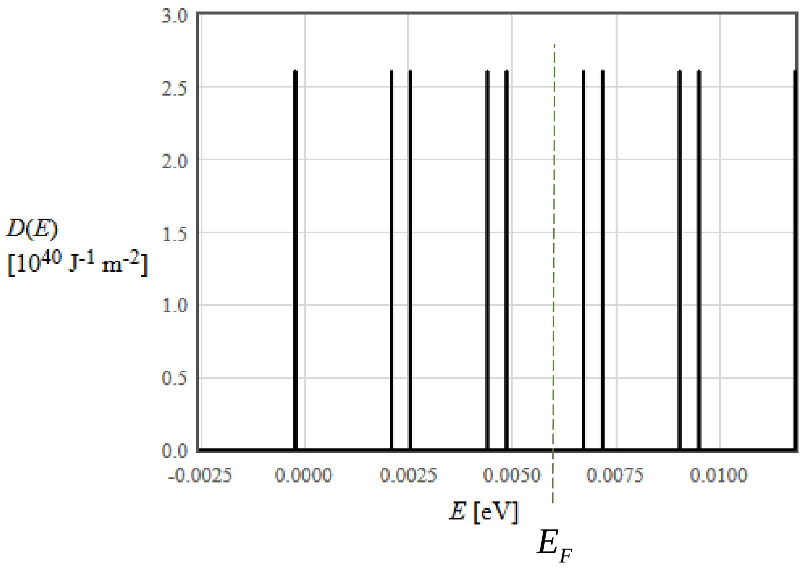
Quantum hall effect



If the Fermi energy is between Landau levels, the electron density n is an integer v times the degeneracy of the Landau level $n = D_0 v$

$$\rho_{xy} = \frac{E_y}{j_x} = \frac{-B_z}{ne}$$

Each Landau level can hold the same number of electrons.

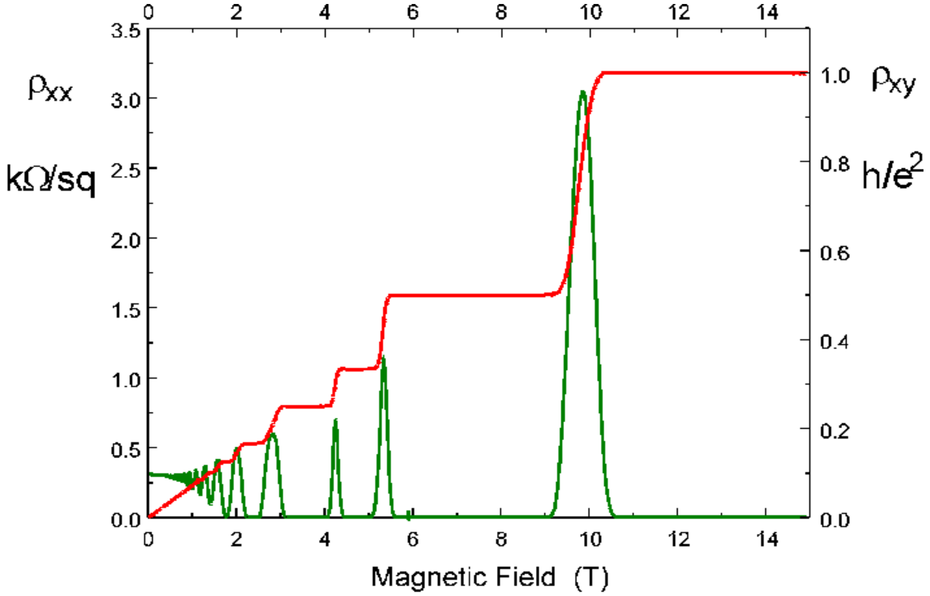
$$\rho_{xy} = \frac{-B_z}{ne} = \frac{-hD_0}{ve^2 D_0} = \frac{-h}{ve^2}$$

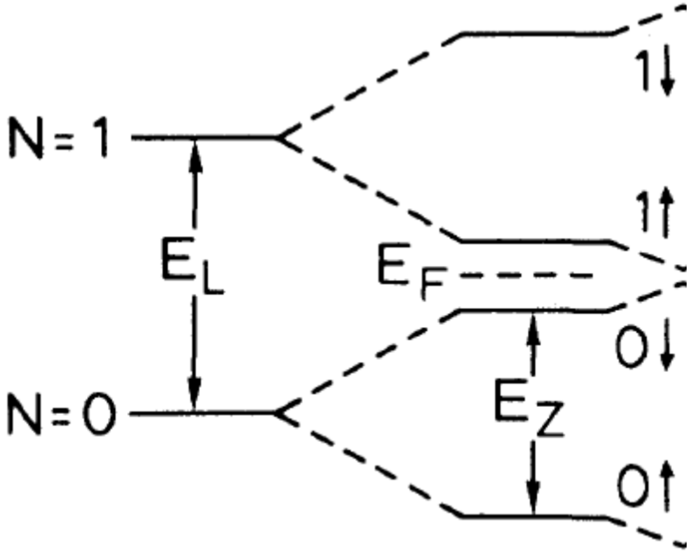
$$D_0 = \frac{m\omega_c}{2\pi\hbar} = \frac{eB_z}{h}$$

$$\omega_c = \frac{eB_z}{m} \qquad B_z = \frac{hD_0}{e}$$

Quantum hall effect

$$\boxed{\rho_{xy} = \frac{h}{\nu e^2}}$$



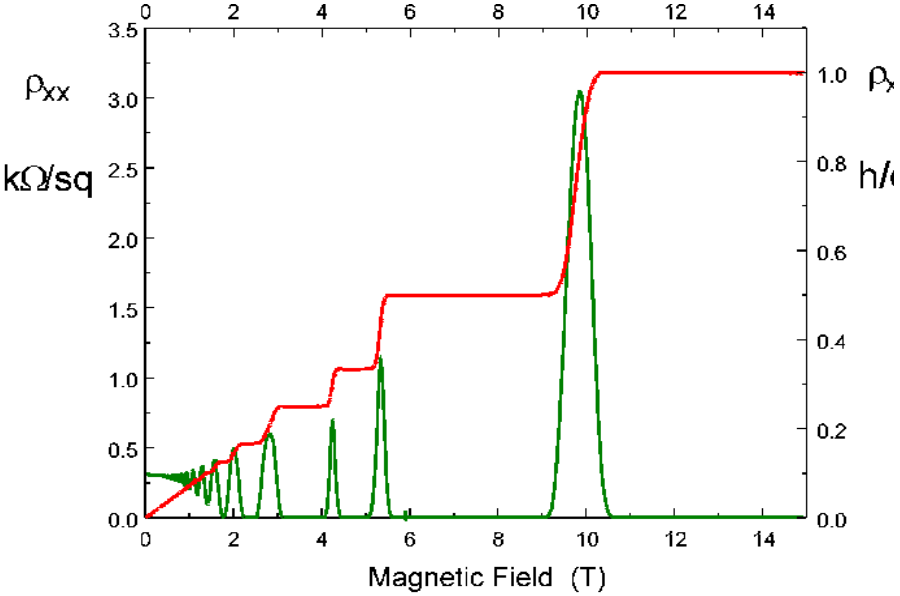


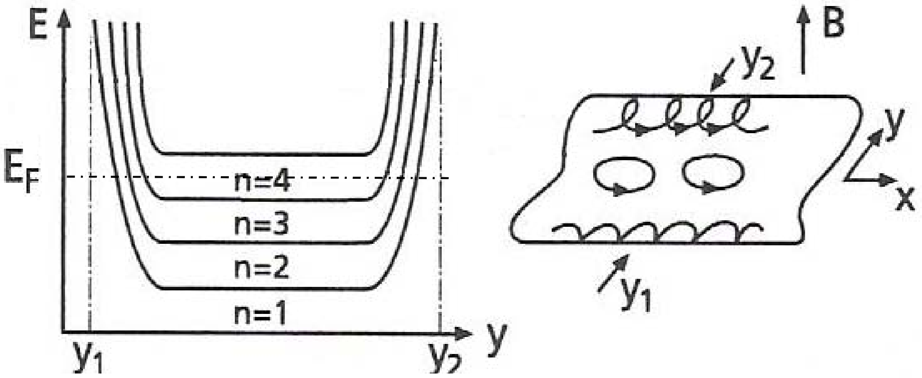
S. Koch, R. J. Haug, and K. v. Klitzing,
Phys. Rev. B 47, 4048–4051 (1993)

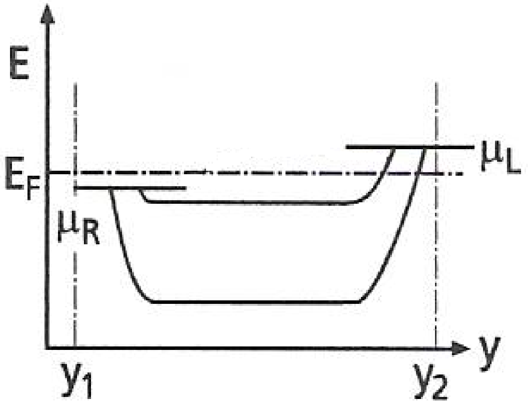
Quantum Hall effect

Edge states are responsible for the zero resistance in ρ_{xx}

On the plateaus, resistance goes to zero because there are no states to scatter into.







Ibach & Lueth (modified)