Superconductivity: BCS theory, flux quantization

BCS theory (1957)

Electrons form Cooper pairs

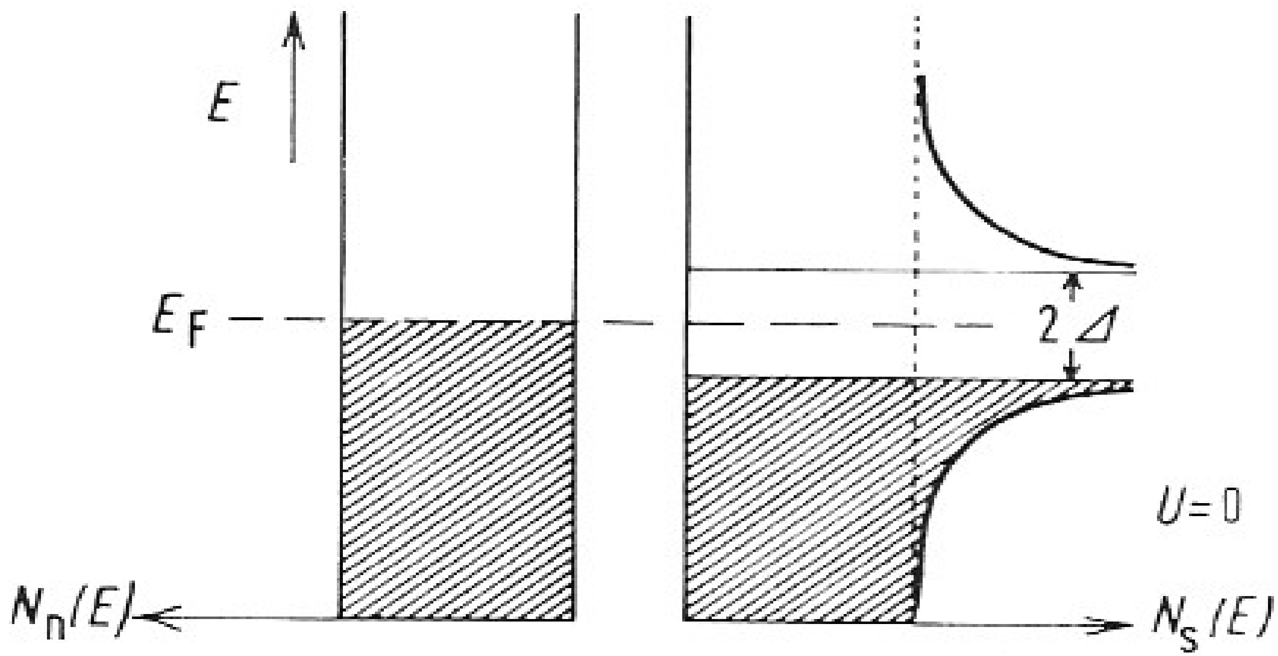
Electrons condense into a coherent state. Similar to:

- Superfluidity
- Bose-Einstein condensates
- Lasers

Pauli exclusion: the sign of the wavefunction changes when two electrons are exchanged.

1972

Density of states



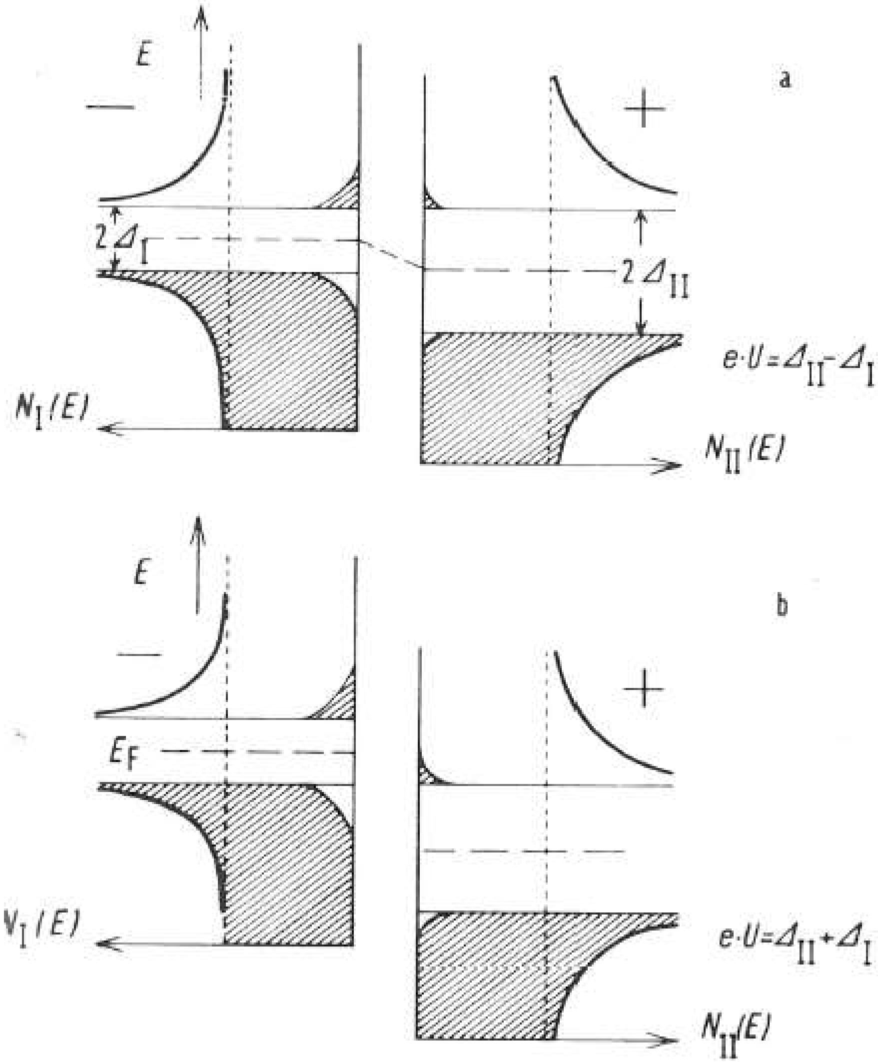
Condensate at E_F

Build wave packets out of states near E_F - Cooper pairs

exchange electrons $\Psi \rightarrow -\Psi$ exchange CP $\Psi \rightarrow \Psi$

no states within Δ of E_F

Tunneling spectroscopy



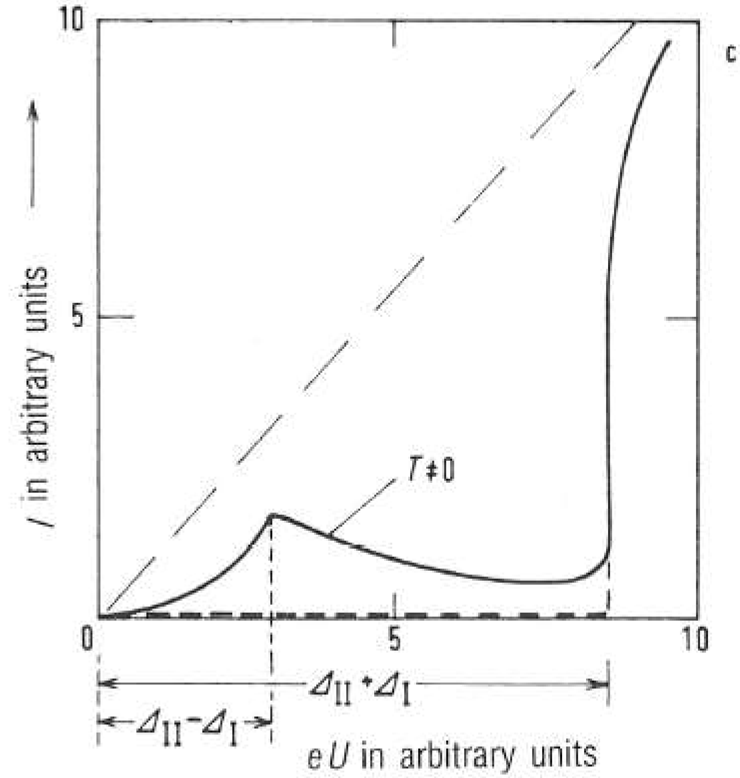


Fig. 31. Tunneling current between two superconductors $0 < T < T_c$.

Buckel - Superconductivity

BCS results

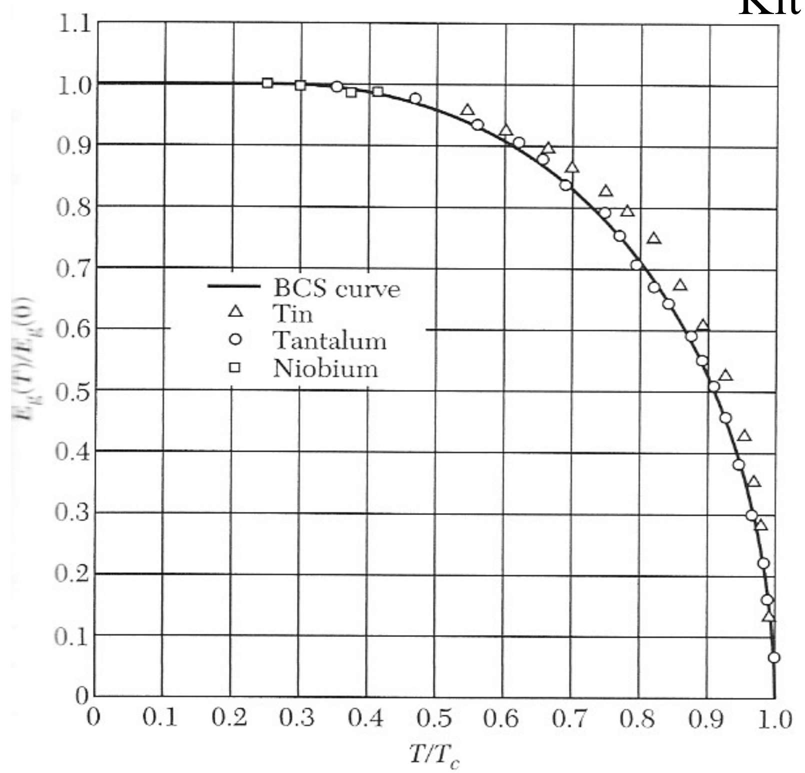
$$\frac{\Delta(0)}{k_B T} = 1.76$$

Al	1.7
Cd	1.6
In	1.8
Hg	2.3
Nb	1.9
Pb	2.1
Sn	1.7

$$\left.\frac{c_s - c_n}{c_n}\right|_{T=T_c} = 1.43$$

Al	1.4
Cd	1.4
In	1.7
Hg	2.4
Nb	1.9
Pb	2.7
Sn	1.6

Kittel



Superconductivity

Perfect diamagnetism

Jump in the specific heat like a 2nd order phase transition, not a structural transition

Superconductors are good electrical conductors but poor thermal conductors, electrons no longer conduct heat

There is a dramatic decrease of acoustic attenuation at the phase transition, no electron-phonon scattering

Dissipationless currents - quantum effect

Electrons condense into a single quantum state - low entropy.

Electron decrease their energy by Δ but loose their entropy.

London equations

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}(-i\hbar\nabla - qA)^2 \psi + V\psi$$

+ cooper pairs condense into the same state

First London equation:

$$\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e}\vec{E}$$

Second London equation:

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e}\vec{B}$$

Meissner effect

Combine second London equation with Ampere's law

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B} \qquad \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla \times \nabla \times \vec{B} = \frac{-n_s e^2 \mu_0}{m_e} \vec{B}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \left(\nabla \cdot \vec{B} \right) - \nabla^2 \vec{B}$$

Helmholtz equation: $\lambda^2 \nabla^2 \vec{B} = \vec{B}$

London penetration depth: $\lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$

Meissner effect

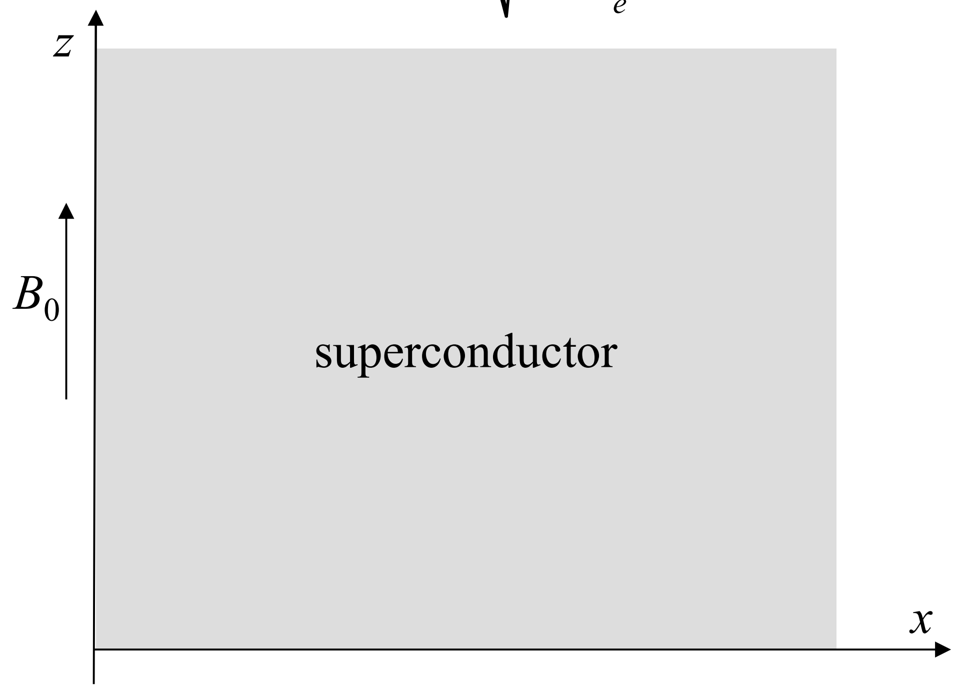
$$\lambda^2 \nabla^2 \vec{B} = \vec{B}$$

$$\lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$$

solution to Helmholtz equation:

$$\vec{B} = \vec{B}_0 \exp\left(\frac{-x}{\lambda}\right)\hat{z}$$

Al	λ = 50 nm
In	λ = 65 nm
Sn	λ = 50 nm
Pb	λ = 40 nm
Nb	λ = 85 nm



$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

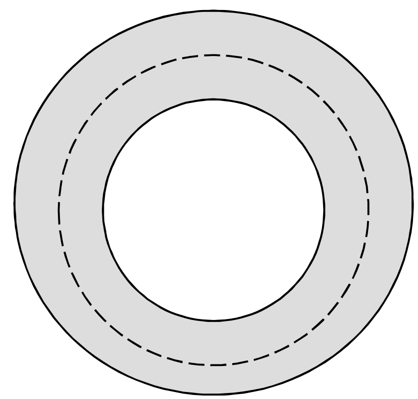
$$\vec{j} = \frac{\vec{B}_0}{\mu_0 \lambda} \exp\left(\frac{-x}{\lambda}\right)\hat{y}$$

Flux quantization

$$\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$

For a ring much thicker than the penetration depth, $j = 0$ along the dotted path.

$$0 = \left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)$$



Integrate once along the dotted path.

$$\oint \nabla\theta \cdot d\vec{l} = -\frac{2e}{\hbar}\oint \vec{A}\cdot d\vec{l} = -\frac{2e}{\hbar}\int_S \nabla\times\vec{A}\cdot d\vec{s} = -\frac{2e}{\hbar}\int_S \vec{B}\cdot d\vec{s} = -\frac{2e}{\hbar}\Phi$$

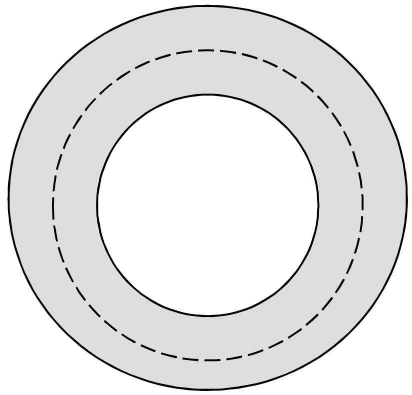
Stokes' theorem

magnetic flux

Flux quantization

$$\oint \nabla \theta \cdot d\vec{l} = 2\pi n = -\frac{2e}{\hbar}\Phi$$

$n = \ldots -2, -1, 0, 1, 2 \ldots$



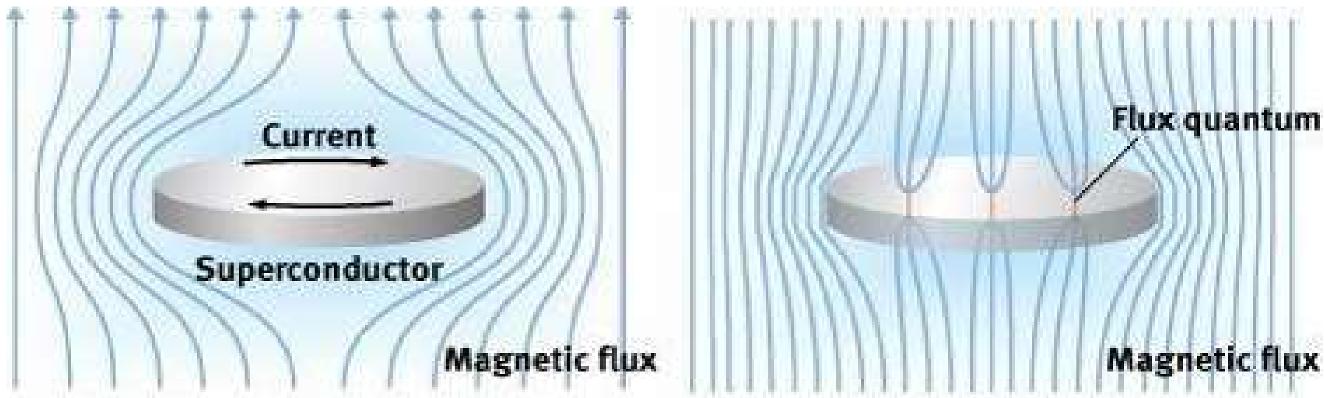
$$2\pi n = \frac{2e}{\hbar}\Phi = \frac{\Phi}{\Phi_0}$$

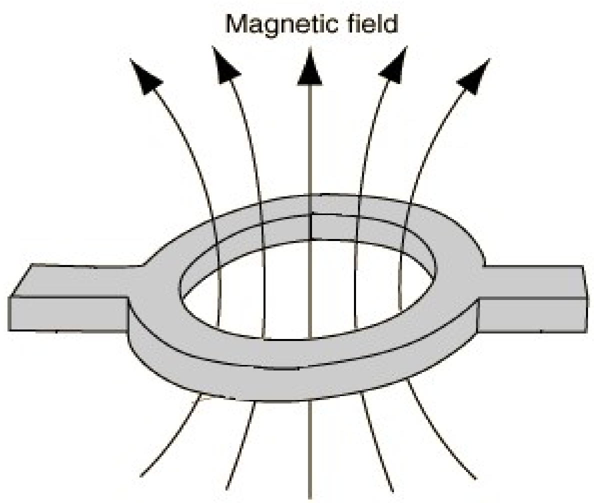
Flux quantization: $\boxed{\Phi = n\Phi_0}$

$$\Phi_0 = \frac{h}{2e} = 2.0679 \times 10^{-15} \ [\mathrm{W = T\ m^2}]$$

Superconducting flux quantum

Flux quantization

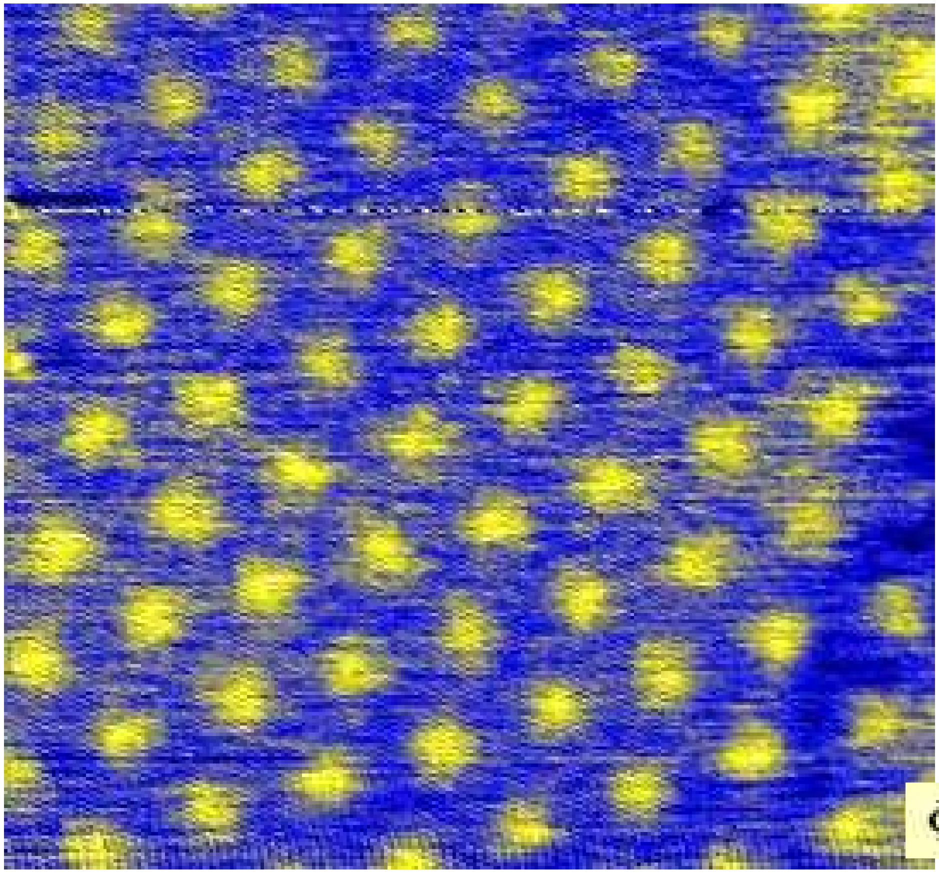


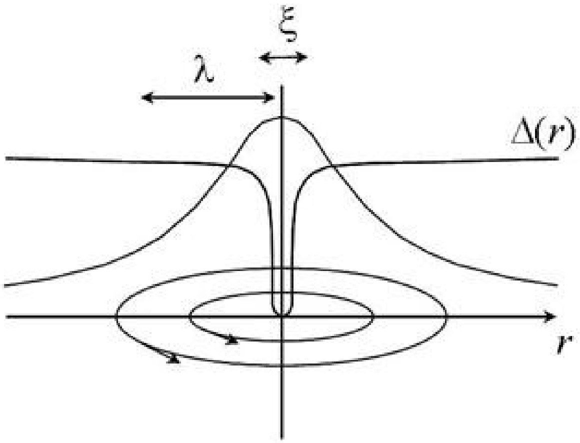


$$\Phi = n\Phi_0 = \frac{nh}{2e}$$

Flux is quantized through a superconducting ring.

Vortices in Superconductors



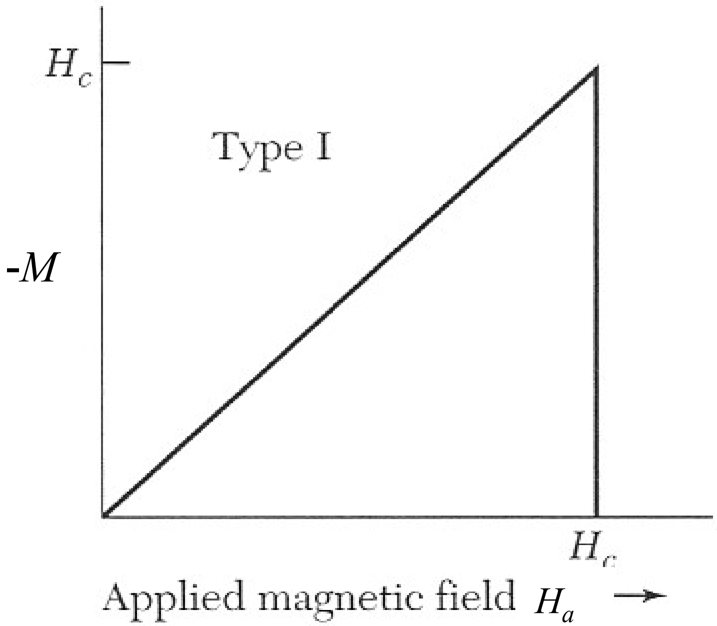


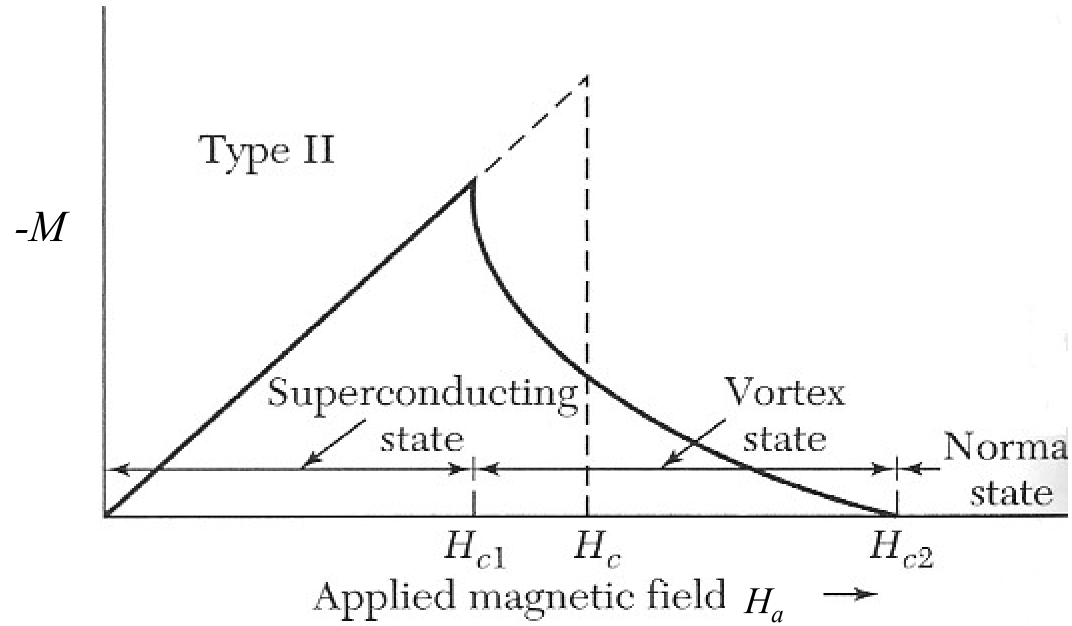
$$\Phi_0 = h/2e \simeq 2 \times 10^{-15} \text{ Tesla m}^2$$

STS image of the vortex lattice in $NbSe_2$.
(630 nm x 500 nm, B = .4 Tesla, T = 4 K)

http://www.insp.upmc.fr/axe1/Dispositifs%20quantiques/AxeI2_more/VORTICES/vortexHD.htm

Type I and Type II





$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right)$$

Superconductors are perfect diamagnets at low fields.
B=0 inside a bulk superconductor.

Type I and Type II