Optical Properties of Metals

Diffusive metal

The current is related to the electric field

$$j_n = \sigma_{nm} E_m \qquad v_n = -\mu_{nm} E_m$$ ← Steady state solution

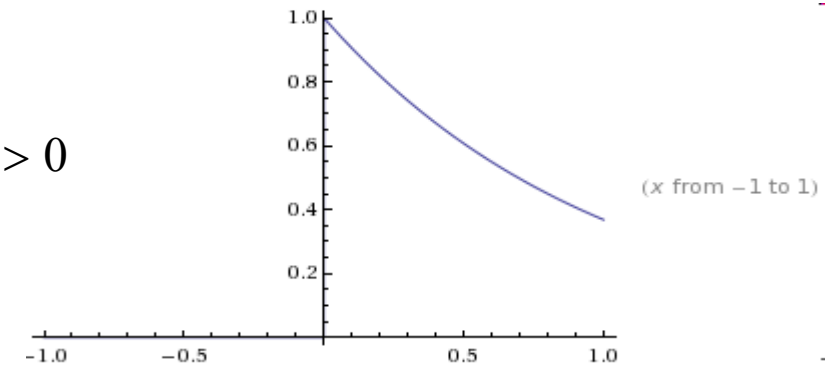
The differential equation that describes how the velocity changes in time is:

$$m\frac{dv(t)}{dt} + \frac{ev(t)}{\mu} = -eE(t)$$

Inertial term

The impulse response function :

$$g(t) = \frac{1}{m}\exp\left(\frac{-et}{\mu m}\right) \qquad t > 0$$



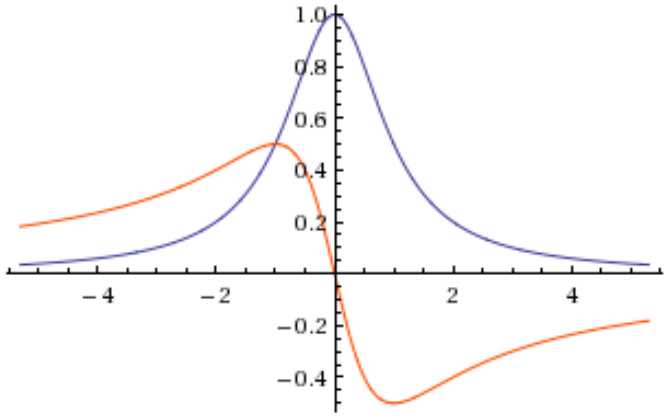
Diffusive metal

The differential equation is:

$$m\frac{dv(t)}{dt}+\frac{ev(t)}{\mu}=-eE(t)$$

Assume a harmonic solution $E(\omega)e^{i\omega t}$, $v(\omega)e^{i\omega t}$

$$\left(-\frac{i\omega m}{e}-\frac{1}{\mu}\right)v(\omega)=E(\omega)$$



$$\frac{v(\omega)}{E(\omega)}=\left(-\frac{i\omega m}{e}-\frac{1}{\mu}\right)^{-1}=-\mu\left(1+i\omega\tau\right)^{-1}=\frac{-\mu\left(1-i\omega\tau\right)}{1+\omega^2\tau^2}$$

$$\sigma(\omega)=\frac{j(\omega)}{E(\omega)}=-ne\frac{v(\omega)}{E(\omega)}=ne\mu\left(\frac{1-i\omega\tau}{1+\omega^2\tau^2}\right)$$

$$\tau=\frac{\mu m}{e}\longleftarrow\text{Scattering time}$$

$$\sigma(\text{low }\omega)=ne\mu$$

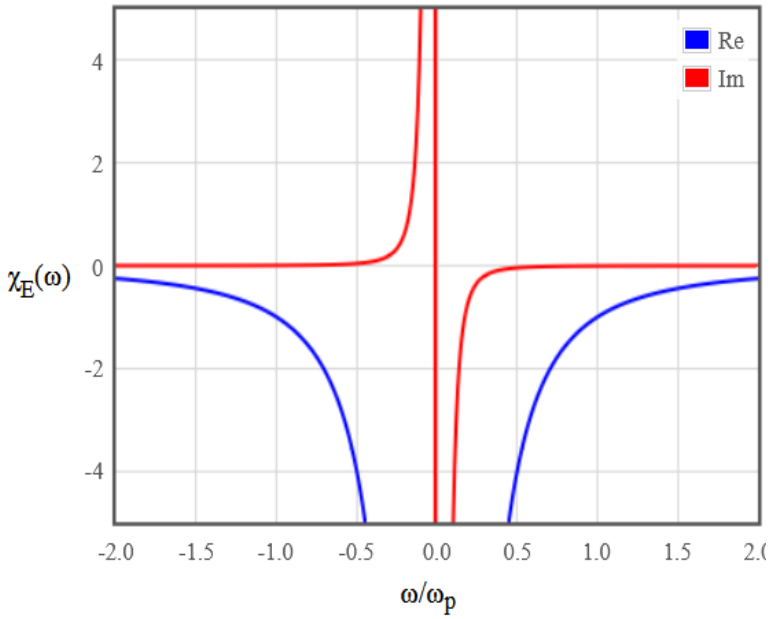
$$\sigma(\text{high }\omega)=\frac{-ine^2}{\omega m}$$

Diffusive metal

$$\chi(\omega) = \frac{\sigma(\omega)}{i\omega\varepsilon_0} = \frac{ne\mu}{i\omega\varepsilon_0}\left(\frac{1 - i\omega\tau}{1 + \omega^2\tau^2}\right)$$

$$\varepsilon(\omega) = 1 + \chi = 1 - \frac{ne\mu}{\omega\varepsilon_0}\left(\frac{\omega\tau + i}{1 + \omega^2\tau^2}\right)$$

$$\varepsilon(\omega) = 1 - \omega_p^2\left(\frac{\omega\tau^2 + i\tau}{\omega + \omega^3\tau^2}\right) \qquad \omega_p^2 = \frac{ne^2}{m\varepsilon_0}$$

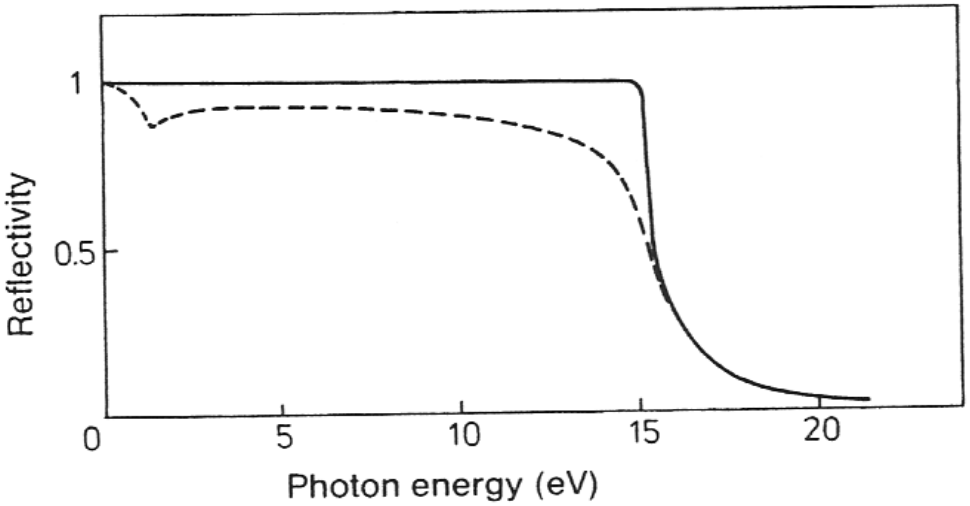


Take the limit as τ goes to infinity

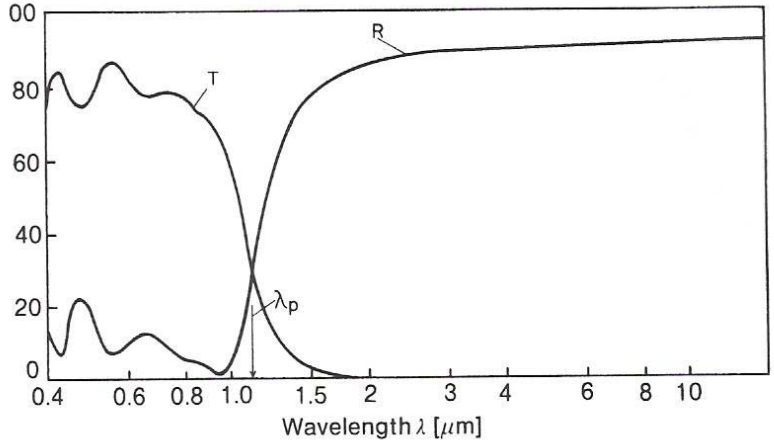
$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

low frequency metal / high frequency insulator

Ibach & Lueth



Aluminum



ITO

Conducting transparent contacts for LEDs and Solar cells

Windows that reflect infrared

Reflection of radio waves from ionosphere

$$\omega_p^2 \approx \frac{ne^2}{\varepsilon_0 m}$$

Optical properties of a diffusive metal

It is assumed that electrons in a diffusive metal scatter so often that we can average over the scatering events. The differential equation that describes the motion of the electrons is,

$$m\frac{d\vec{v}}{dt}+\frac{e\vec{v}}{\mu}=-e\vec{E}.$$

Here m is the mass of an electron, $\vec{v}$ is the velocity of the electron, $-e$ is the charge of an electron, and $\vec{E}$ is the electric field. When a constant electric field is applied, the solution is,

$$\vec{v}=-\mu\vec{E}.$$

Thus the (negatively charged) electrons move in the opposite direction as the electric field.

If the electric field is pulsed on, the reponse of the electrons is described by the impulse response function $g(t)$. The impulse response function satisfies the equation,

$$m\frac{dg}{dt}+\frac{eg}{\mu}=-e\delta(t).$$

When the electric field is pulsed on, the electrons suddenly start moving and then their velocity decays exponentially to zero in a time $\tau=m\mu/e$.

$$g(t)=-\frac{e}{m}\exp(-t/\tau).$$

The scattering time τ and the electron density n are the only two parameters that are needed to describe many of the optical properties of a diffusive metal. The form below can be used to input τ and n and then a script calculates and plots the impulse response function, the Fourier transform of the impulse response function, the mobility, the dc conductivity, the frequency dependent complex conductivity, the electric susceptibility, the dielectric function, the plasma frequency, the index of refraction, the extinction coefficient, and the reflectance.

$\tau =$ 1E-11 [s] $n =$ 1E28 [m^{-3}] submit

Mobility $\mu = 1.76\ m^2\ V^{-1}\ s^{-1}$
DC conductivity $\sigma_o = 2.82e{+}9\ \Omega^{-1}\ m^{-1}$
Plasma frequency $\omega_p = 5.64e{+}15$ rad/s, $\omega_p\tau = 5.64e{+}4$.

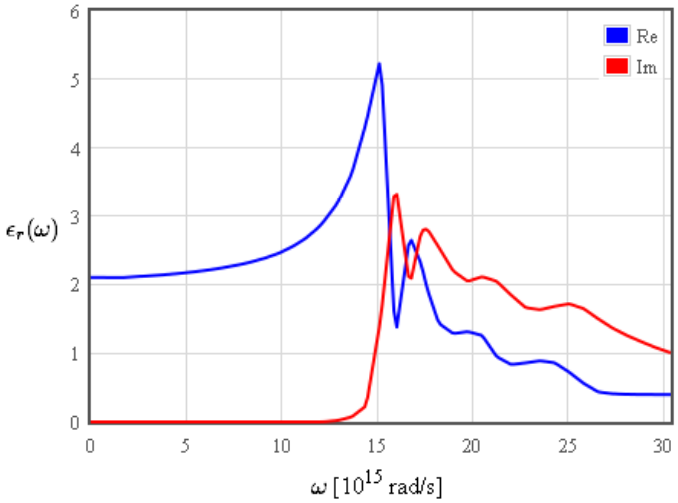
Impulse response function

The optical properties of SiO_2 (glass)

nanophotonics.csic.es

Dielectric function

The relative dielectric constant describes the relationship between the electric displacement $\vec{D}$ and the electric field $\vec{E}$, $\vec{D} = \epsilon_r \epsilon_0 \vec{E} = \vec{P} + \epsilon_0 \vec{E}$.

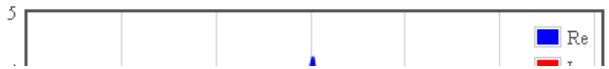


There are two conventions for dielectric function. Either it is assumed that the time dependence of $\vec{D}$, $\vec{P}$, and $\vec{E}$ is $\exp(-i\omega t)$ and the plot of the dielectric function looks as it is shown above, or it is assumed that the time dependence of $\vec{D}$, $\vec{P}$, and $\vec{E}$ is $\exp(i\omega t)$ and the imaginary part of the has the opposite sign as in the plot above. Here we will assume a time dependence of $\exp(-i\omega t)$.

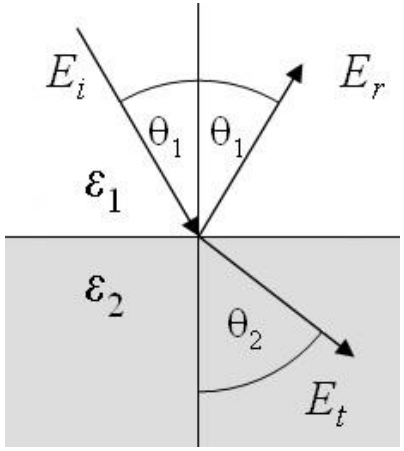
Electric susceptibility

The electric susceptibility χ_E describes the relationship between the polarization $\vec{P}$ and the electric field $\vec{E}$, $\vec{P} = \epsilon_0 \chi_E \vec{E}$.

$$\chi_E = \epsilon_r - 1$$



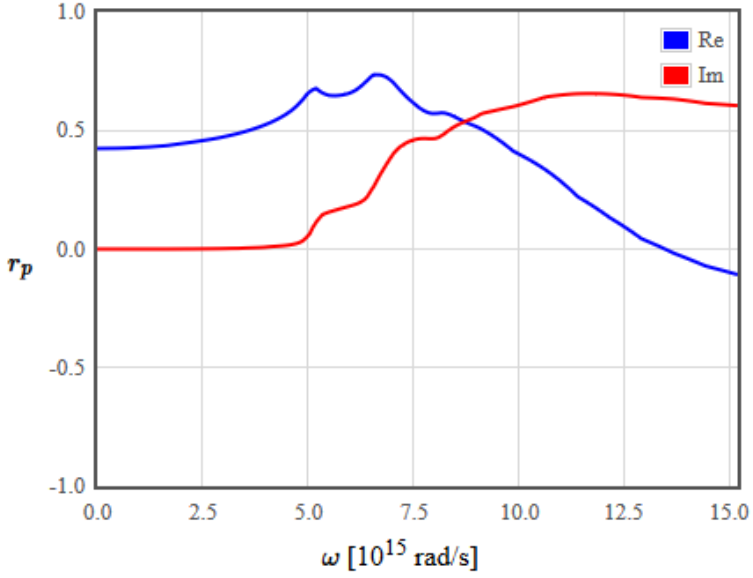
Ellipsometry

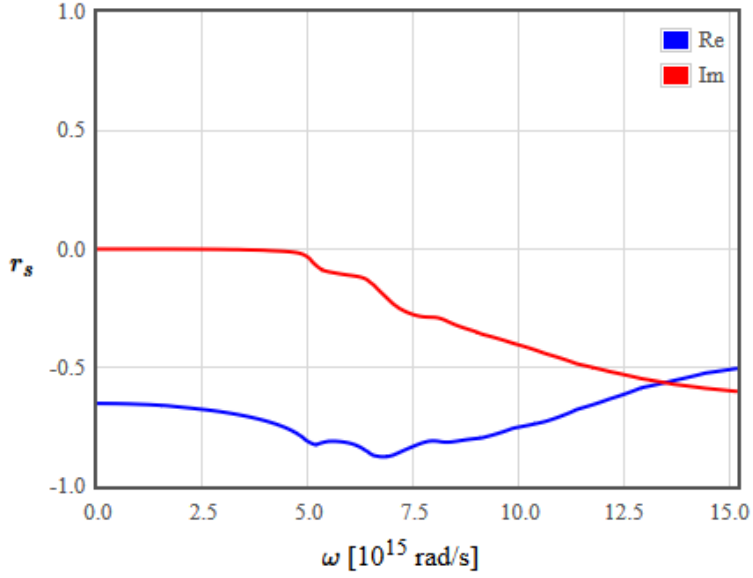


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

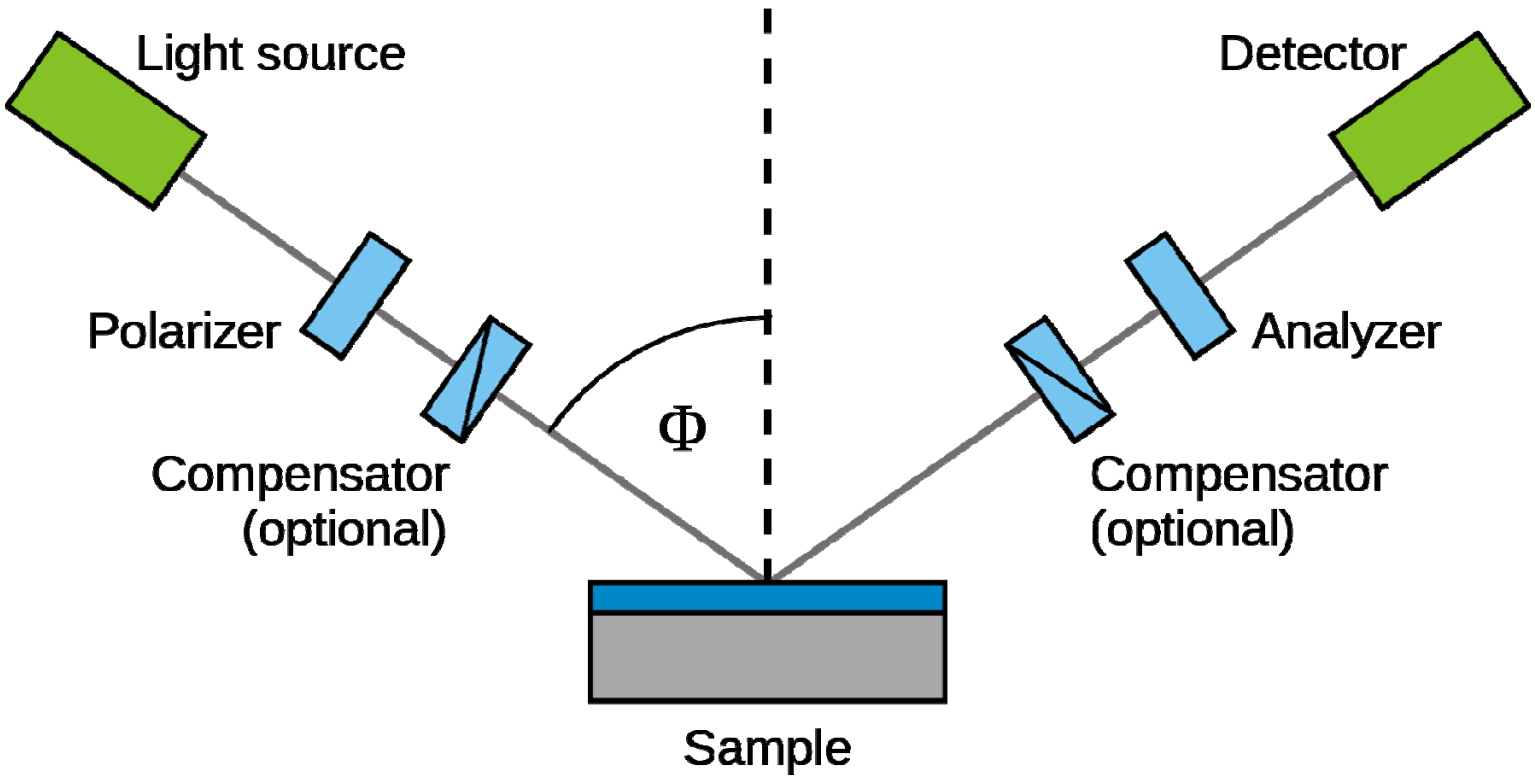
$$r_p = \frac{E_{rp}}{E_{ip}} = \frac{\sqrt{\epsilon_2}\cos\theta_1 - \sqrt{\epsilon_1}\cos\theta_2}{\sqrt{\epsilon_2}\cos\theta_1 + \sqrt{\epsilon_1}\cos\theta_2}$$

$$r_s = \frac{E_{sr}}{E_{si}} = \frac{\sqrt{\epsilon_2}\cos\theta_2 - \sqrt{\epsilon_1}\cos\theta_1}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2}\cos\theta_2}$$





Ellipsometry

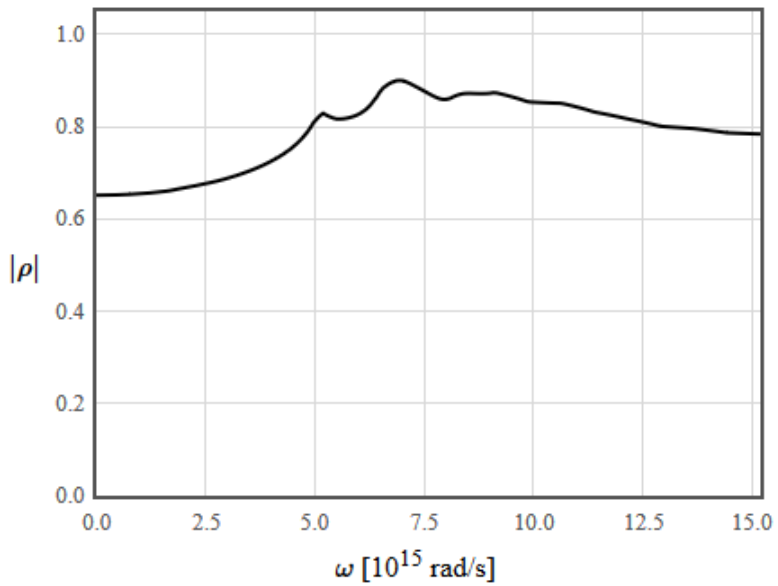


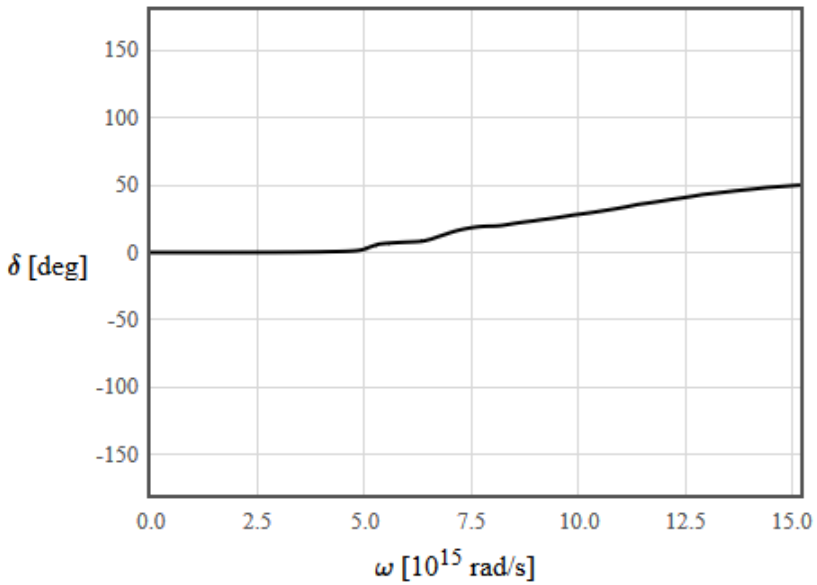
Ellipsometry measures the change of polarization upon reflection. The measured signal depends on the thickness and the dielectric constant.

http://en.wikipedia.org/wiki/Ellipsometry

Ellipsometry

$$\rho = \frac{r_p}{r_s} = |\rho| e^{i\delta}$$





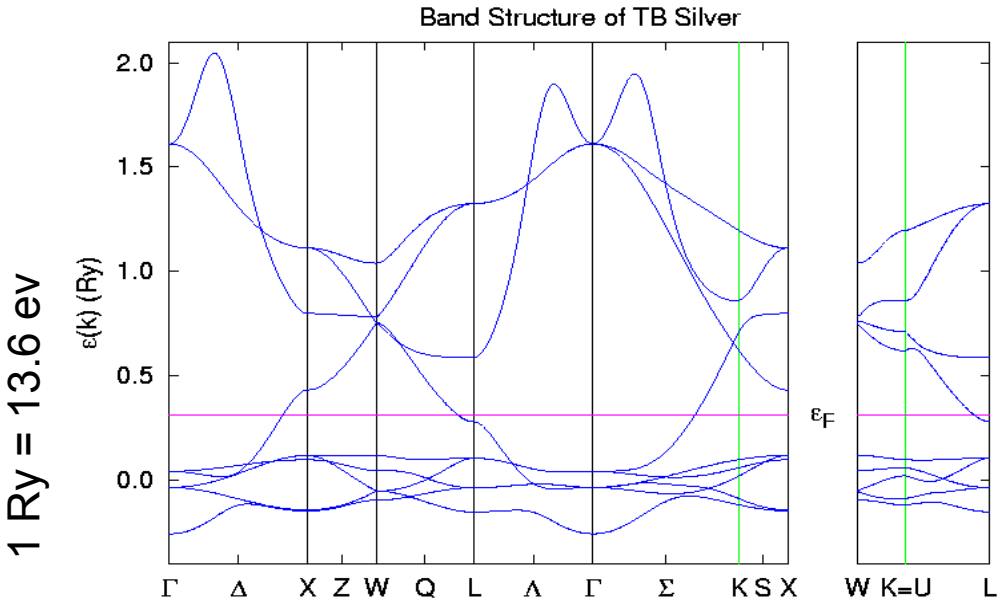
The ratio of the two reflected polarizations is insensitive to instabilities of light source or atmospheric absorption.

Intraband transitions

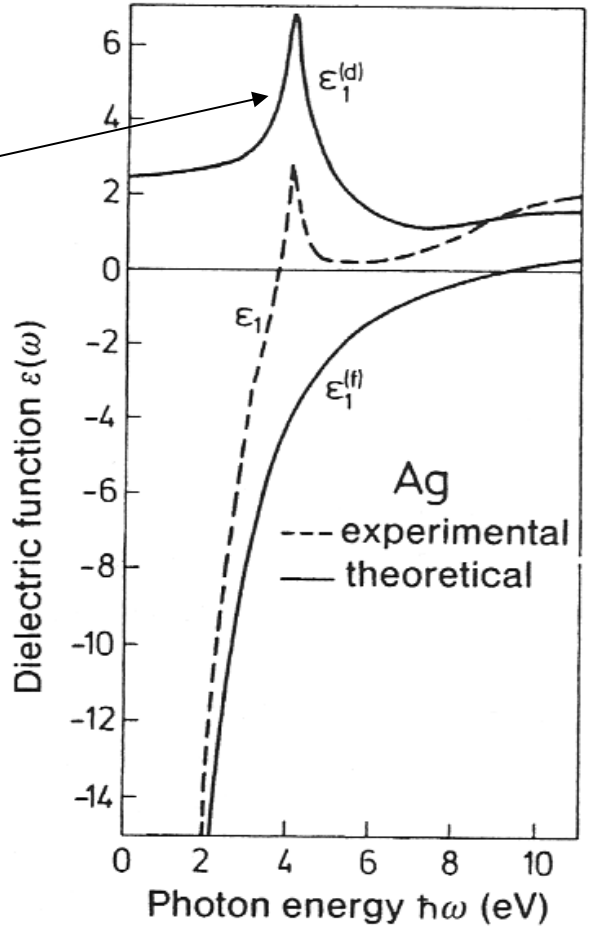
When the bands are parallel, there is a peak in the absorption (ε")

$$\hbar\omega = E_c(\vec{k}) - E_v(\vec{k})$$

Intraband (d-band) absorption



1 Ry = 13.6 ev



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