Magnetism

Paramagnetism

Paramagnetic materials have atoms with magnetic moments which can be aligned with an applied field so that the induced field will add to the applied field. This results in a larger B -field inside the material than outside and a positive magnetic susceptibility. Paramagnetism is similar to diamagnetism in the sense that the materials will only show magnetization when a magnetic field is applied. Once the applied field is switched off, the moments will randomize again and the magnetization decay exponentially to zero.

The magnetization of a paramagnet is the sum of the aligned magnetic moments per unit volume. The magnetic moment of an atom depends on the total angular momentum quantum number J. Since the magnetic quantum numbers are restricted to the values $m_J = (-J, -J+1, \cdots, J-1, J)$, the z-component of the magnetic moment can can take the values,

$$\mu_{m_J} = m_J g_J \mu_B,$$

where g_J is the Landé g factor, μ_B is the Bohr magneton. For a magnetic field applied in the z-direction, the energies of the magnetic states will be,

$$E_{m_J} = -\mu_{m_j} B_z = -m_J g_J \mu_B B_z.$$

The occupation probabilty of state m_J is given by a Boltzmann distribution,

$$p_{m_J} = \frac{\exp\left(\frac{-E_{m_J}}{k_B T}\right)}{\sum_{m_J=-J}^{m_J=J} \exp\left(\frac{-E_{m_J}}{k_B T}\right)}.$$

The $J = \frac{1}{2}$ case

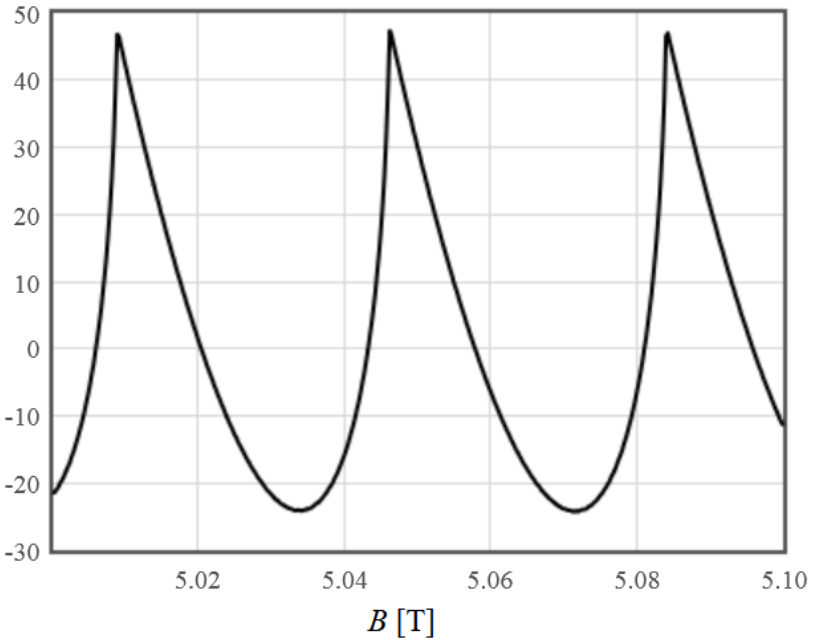
Magnetic Type of the elements

Text lists sorted by: Value | Atomic Number | Alphabetical
Plots: Shaded | Ball | Crossed Line | Scatter | Sorted Scatter
Log scale plots: Shaded | Ball | Crossed Line | Scatter | Sorted Scatter
Good for this property: Atomic Number

Hydrogen	Diamagnetic	Niobium	Paramagnetic	Thallium	Diamagnetic
Helium	Diamagnetic	Molybdenum	Paramagnetic	Lead	Diamagnetic
Lithium	Paramagnetic	Technetium	Paramagnetic	Bismuth	Diamagnetic
Beryllium	Diamagnetic	Ruthenium	Paramagnetic	Polonium	N/A
Boron	Diamagnetic	Rhodium	Paramagnetic	Astatine	N/A
Carbon	Diamagnetic	Palladium	Paramagnetic	Radon	N/A
Nitrogen	Diamagnetic	Silver	Diamagnetic	Francium	N/A
Oxygen	Paramagnetic	Cadmium	Diamagnetic	Radium	N/A
Fluorine	N/A	Indium	Diamagnetic	Actinium	N/A
Neon	Diamagnetic	Tin	Diamagnetic	Thorium	Paramagnetic
Sodium	Paramagnetic	Antimony	Diamagnetic	Protactinium	Paramagnetic
Magnesium	Paramagnetic	Tellurium	Diamagnetic	Uranium	Paramagnetic
Aluminum	Paramagnetic	Iodine	Diamagnetic	Neptunium	N/A
Silicon	Diamagnetic	Xenon	Diamagnetic	Plutonium	Paramagnetic
Phosphorus	Diamagnetic	Cesium	Paramagnetic	Americium	Paramagnetic
Sulfur	Diamagnetic	Barium	Paramagnetic	Curium	N/A

de Haas - van Alphen effect

$$\hbar\omega_c \ll k_B T$$



Magnetic Type of the elements

Text lists sorted by: Value | Atomic Number | Alphabetical
Plots: Shaded | Ball | Crossed Line | Scatter | Sorted Scatter
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Good for this property: Atomic Number

Hydrogen Diamagnetic	Niobium Paramagnetic	Thallium Diamagnetic
Helium Diamagnetic	Molybdenum Paramagnetic	Lead Diamagnetic
Lithium Paramagnetic	Technetium Paramagnetic	Bismuth Diamagnetic
Beryllium Diamagnetic	Ruthenium Paramagnetic	Polonium N/A
Boron Diamagnetic	Rhodium Paramagnetic	Astatine N/A
Carbon Diamagnetic	Palladium Paramagnetic	Radon N/A
Nitrogen Diamagnetic	Silver Diamagnetic	Francium N/A
Oxygen Paramagnetic	Cadmium Diamagnetic	Radium N/A
Fluorine N/A	Indium Diamagnetic	Actinium N/A
Neon Diamagnetic	Tin Diamagnetic	Thorium Paramagnetic
Sodium Paramagnetic	Antimony Diamagnetic	Protactinium Paramagnetic
Magnesium Paramagnetic	Tellurium Diamagnetic	Uranium Paramagnetic
Aluminum Paramagnetic	Iodine Diamagnetic	Neptunium N/A
Silicon Diamagnetic	Xenon Diamagnetic	Plutonium Paramagnetic
Phosphorus Diamagnetic	Cesium Paramagnetic	Americium Paramagnetic
Sulfur Diamagnetic	Barium Paramagnetic	Curium N/A

$$\chi_M = \frac{dM}{dH}$$

Pauli paramagnetism

Paramagnetic contribution due to free electrons.

Electrons have an intrinsic magnetic moment μ_B.

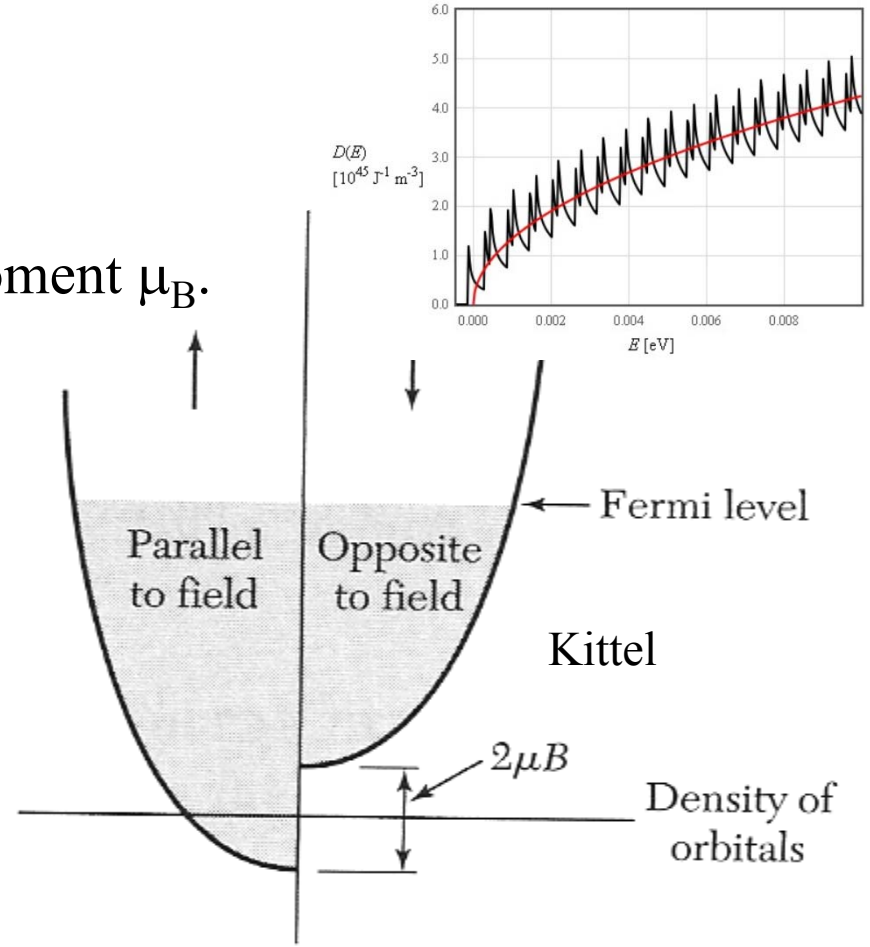
$$n_+ \approx \frac{1}{2}n + \frac{1}{2}\mu_B B D(E_F)$$

$$n_- \approx \frac{1}{2}n - \frac{1}{2}\mu_B B D(E_F)$$

$$M = \mu_B(n_+ - n_-)$$

$$M = \mu_B^2 D(E_F)B = \mu_0 \mu_B^2 D(E_F)H$$

$$\chi = \frac{dM}{dH} = \mu_0 \mu_B^2 D(E_F)$$



Pauli paramagnetism is much smaller than the paramagnetism due to atomic moments and almost temperature independent because $D(E_F)$ doesn't change very much with temperature.

Ferromagnetism

Below a critical temperature (called the Curie temperature) a magnetization spontaneously appears in a ferromagnet even in the absence of a magnetic field.

Iron, nickel, and cobalt are ferromagnetic.

Ferromagnetism overcomes the magnetic dipole-dipole interactions. It arises from the Coulomb interactions of the electrons. The energy that is gained when the spins align is called the exchange energy.

Schrödinger equation for two particles

$$-\frac{\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2\right)\psi+V_1(\vec{r}_1)\psi+V_2(\vec{r}_2)\psi+V_{1,2}(\vec{r}_1,\vec{r}_2)\psi=E\psi$$

$\psi(\vec{r}_1,\vec{r}_2)=\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)$ is a solution to the noninteracting Hamiltonian, $V_{1,2}=0$

$$\psi_A\left(\vec{r}_1,\vec{r}_2\right)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)\begin{pmatrix}\uparrow\uparrow\\ \frac{1}{\sqrt{2}}\left(\uparrow\downarrow+\downarrow\uparrow\right)\\ \downarrow\downarrow\end{pmatrix}$$

$$\psi_S\left(\vec{r}_1,\vec{r}_2\right)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)\frac{1}{\sqrt{2}}\left(\uparrow(\vec{r}_1)\downarrow(\vec{r}_2)-\downarrow(\vec{r}_1)\uparrow(\vec{r}_2)\right)$$

Exchange (Austauschwechselwirking)

$$\psi_A(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)$$

$$\langle\psi_A|H|\psi_A\rangle=\frac{1}{2}[\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle-\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle$$
$$-\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle+\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle]$$

$$\psi_S(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\left(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right)$$

$$\langle\psi_S|H|\psi_S\rangle=\frac{1}{2}[\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle+\langle\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle$$
$$+\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\rangle+\langle\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)|H|\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\rangle]$$

The difference in energy between the ψ_A and ψ_S is twice the **exchange energy**.

H2 ↑↓ Helium ↑↑

Exchange

The exchange energy can only be defined when you speak of multi-electron wave functions. There is a contribution from each pair of electrons. It is the difference in energy between the symmetric solution and the antisymmetric solution. There is only a difference when the electron-electron term is included. Coulomb repulsion determines the exchange energy.

In ferromagnets, the antisymmetric state has a lower energy. Thus the state with parallel spins has lower energy.

In antiferromagnets, the symmetric state has a lower energy. Neighboring spins are antiparallel.

Ordered states have a lower entropy than free electrons.

Mean field theory (Molekularfeldtheorie)

Heisenberg Hamiltonian $H = -\sum_{i,j} J_{i,j} \vec{S}_i \cdot \vec{S}_j - g\mu_B \vec{B} \cdot \sum_i \vec{S}_i$

Exchange energy

Mean field approximation

$$H_{MF} = \sum_i \vec{S}_i \cdot \left(\sum_\delta J_{i,\delta} \left\langle \vec{S} \right\rangle + g\mu_B \vec{B} \right)$$

δ sums over the neighbors of spin *i*

Looks like a magnetic field B_{MF}

$$\vec{B}_{MF} = \frac{1}{g\mu_B} \sum_\delta J_{i,\delta} \left\langle \vec{S} \right\rangle$$

magnetization → $\vec{M} = g\mu_B \frac{N}{V} \left\langle \vec{S} \right\rangle$

eliminate <*S*>

Mean field theory

$$\vec{B}_{MF} = \frac{V}{Ng^2\mu_B^2} zJ\vec{M}$$

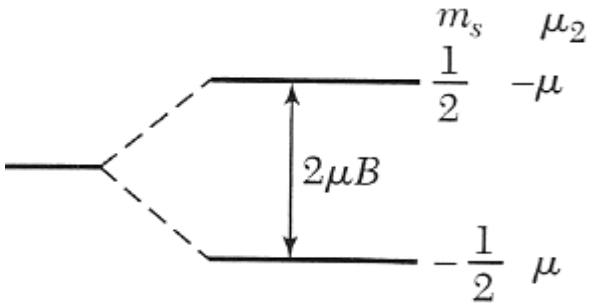
z is the number of nearest neighbors

In mean field, the energy of the spins is

$$E = \pm\frac{1}{2} g\mu_B (B_{MF} + B_a)$$

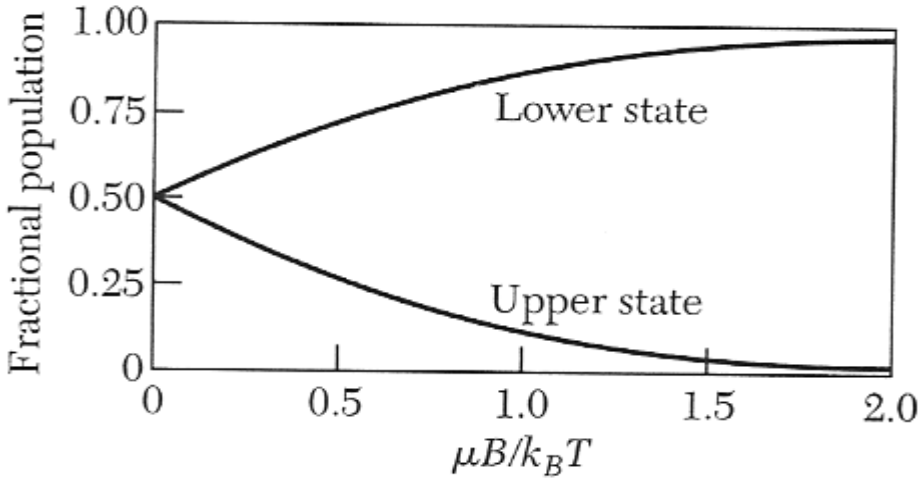
We calculated the populations of the spins in the paramagnetism section

Spin populations



$$\frac{N_1}{N} = \frac{\exp(\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$



$$M = (N_1 - N_2)\mu$$

$$= N\mu \frac{\exp(\mu B / k_B T) - \exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$= N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$$

Mean field theory

$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh\left(\frac{g \mu_B \left(B_{MF} + B_a \right)}{2 k_B T} \right)$$

For zero applied field

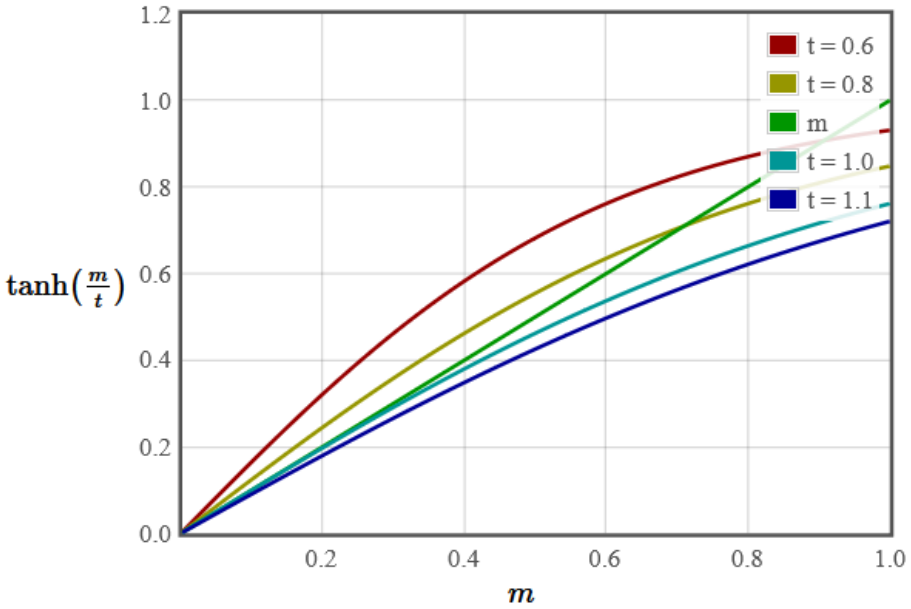
$$M = M_S \tanh\left(\frac{T_c}{T} \frac{M}{M_s} \right)$$

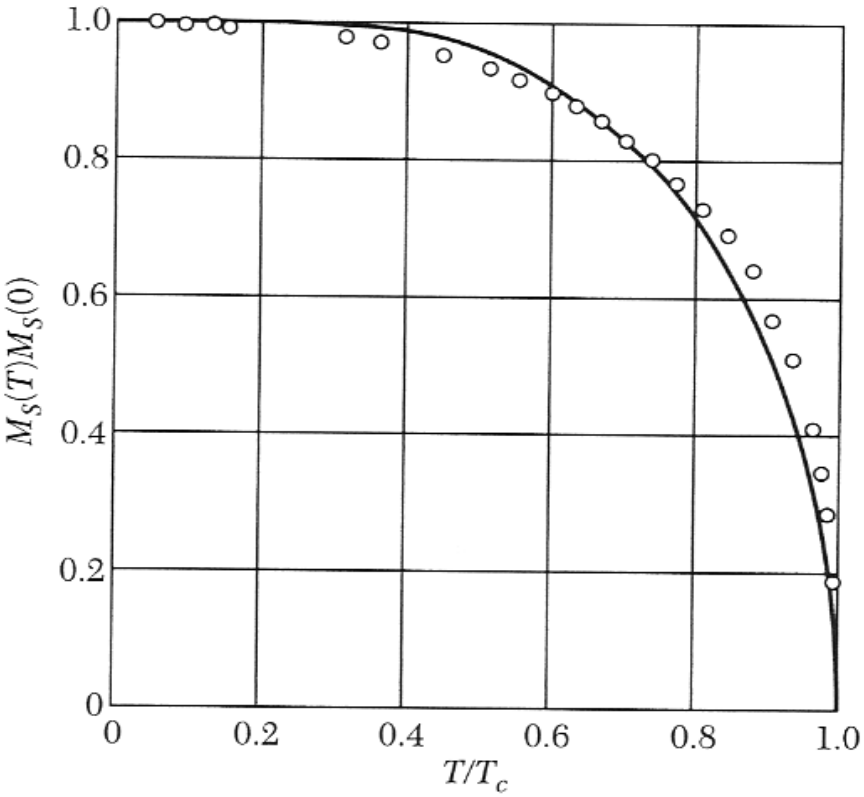
$$M_S = \frac{N}{2V} g \mu_B \qquad \text{and} \qquad T_c = \frac{z}{4 k_B} J$$

M_s = saturation magnetization $\qquad$ T_c = Curie temperature

Mean field theory

$$M = M_S \tanh\left(\frac{T_c}{T}\frac{M}{M_s}\right)$$





$$m = \tanh\left(\frac{m}{t}\right)$$

Experimental points for Ni.

$$M_S = \frac{N}{2V} g\mu_B \quad \text{and} \quad T_c = \frac{z}{4k_B} J$$

Source: Kittel

Ferromagnetism

Material	Curie temp. (K)	
Co	1388	
Fe	1043	
$FeOFe_2O_3$	858	
$NiOFe_2O_3$	858	
$CuOFe_2O_3$	728	
$MgOFe_2O_3$	713	
MnBi	630	
Ni	627	
MnSb	587	
$MnOFe_2O_3$	573	
$Y_3Fe_5O_{12}$	560	
CrO_2	386	
MnAs	318	
Gd	292	
Dy	88	
EuO	69	Electrical insulator
$Nd_2Fe_{14}B$	353	$M_s = 10\ M_s(\mathrm{Fe})$
Sm_2Co_{17}	700	rare earth magnets

$$M_S = \frac{N}{2V} g \mu_B$$

$$T_c = \frac{z}{4k_B} J$$

Curie - Weiss law

$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh\left(\frac{g \mu_B \left(B_{MF} + B_a \right)}{2 k_B T} \right) \qquad \vec{B}_{MF} = \frac{V}{N g^2 \mu_B^2} z J \vec{M}$$

Above T_c we can expand the hyperbolic tangent $\tanh(x) \approx x$ for $x << 1$

$$M \approx \frac{1}{4} g^2 \mu_B^2 \frac{N}{V k_B T} \left(\frac{V}{N g^2 \mu_B^2} z J M + B_a \right)$$

Solve for M

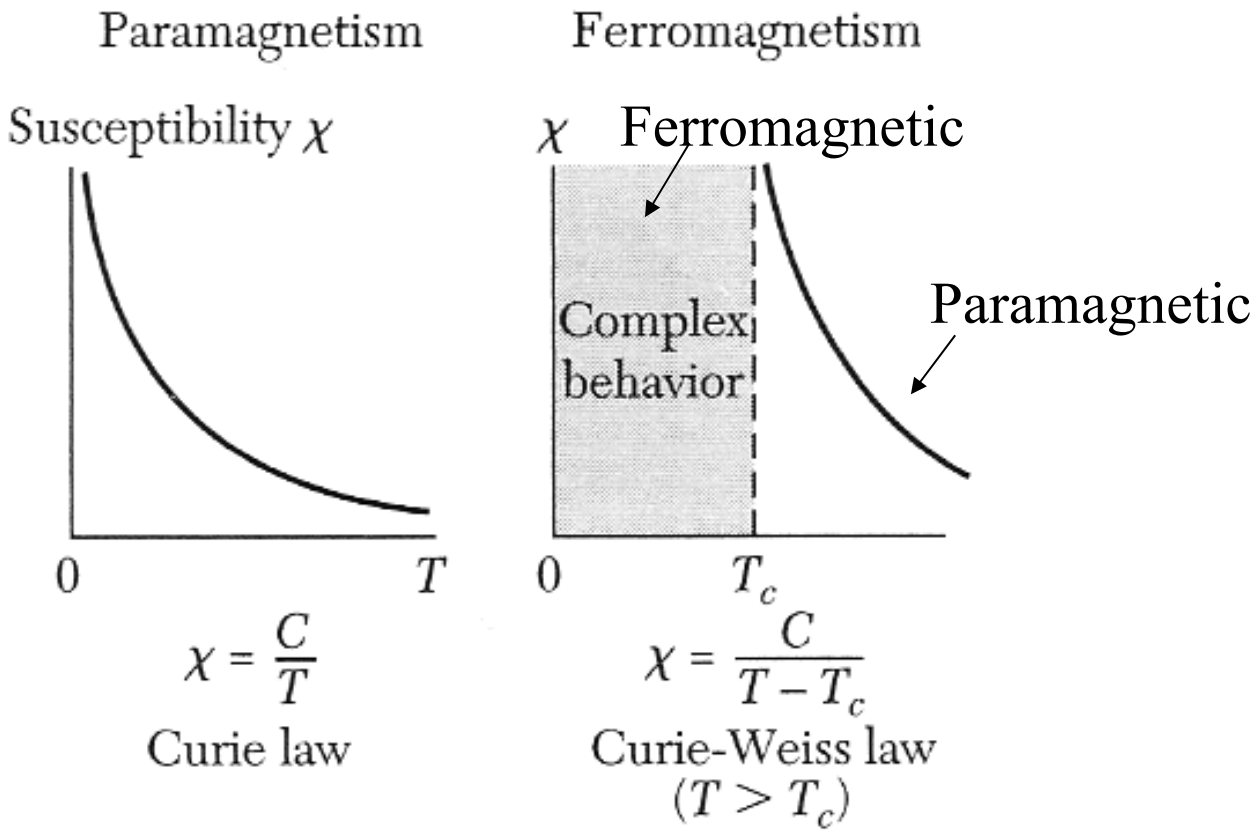
$$M \approx \frac{g^2 \mu_B^2 N}{4 V k_B} \frac{B_a}{T - T_c} \qquad T_c = \frac{z}{4 k_B} J$$

Curie Weiss Law $\chi = \frac{dM}{dH} \approx \frac{C}{T - T_c}$

Critical fluctuations near T_c

Ferromagnets are paramagnetic above T_c

Source: Kittel



Critical fluctuations near T_c.

Mean-field Theory

In contrast to dia- and paramagnetism, ferromagnetic materials (e.g. Fe, Ni, Co) have a non-vanishing magnetization $\vec{M} \neq 0$ also at a vanishing external magnetic field $\vec{B}_a = 0$ below a critical temperature T_c. In this section the magnetization of the isotropic Heisenberg model in the mean field approximation (german *Molekularfeldnäherung* or *mittlere Feldnäherung*) is derived. The Heisenberg model is a very good model for describing the exchange interaction between spins and therefore ideal for modelling ferromagnetism. The Hamiltonian of the isotropic Heisenberg model is given by

$$H = -J' \sum_{i\ n.n.\ j} \vec{S}_i \cdot \vec{S}_j - \mu B_a \sum_i S_i^z.$$

Here, the first sum goes only over the nearest neighbor sites j of lattice site i ($i\ n.n.\ j$). This is because it is (reasonably) assumed that the spin on site i only interacts directly with the neighboring spins but the interaction is vanishing for all lattice sites further away. The exchange coupling constant between the spins on sites i and j is given by J' (in this simple case independent of the sites). In ferromagnetism, parallel spins decrease the energy and $J' < 0$. In the case of antiferromagnetism, antiparallel spins decrease the energy and $J' > 0$. For paramagnetism $J' = 0$. In case of a system of spin $\frac{1}{2}$ particles (for example electrons) the projection of the magnetic moment onto the magnetic field can take two values , $\vec{\mu} \cdot \vec{B} = \pm \frac{g\mu_B}{2} B$ with g being the Landé g factor (for $J = 1/2$) and μ_B the Bohr magneton.

Now mean field approximation is applied: The main idea is that the interaction of two neighboring spins $\vec{S}_i \cdot \vec{S}_j$ can be replaced by the interaction of a single spin of site i with the *mean spin value* of its surrounding:

$$\vec{S}_i \cdot \vec{S}_j \to \vec{S}_i \cdot \langle \vec{S} \rangle.$$

The *mean field Hamiltonian* can then be written as:

$$H_{MF} = -\sum_i \vec{S}_i \cdot \left(zJ' \langle \vec{S} \rangle + \mu \vec{B}_a \right).$$

In this equation z is the so-called coordination number (number of nearest neighbor sites of spin i) and comes from the sum over the nearest neighbor sites. The term in the brackets doesn't depend on any lattice site and acts like a **total magnetic field** consisting of the external magnetic field, B_a and the *mean magnetic field* B_{MF}:

lampx.tugraz.at/~hadley/ss2/magnetism/mft_2.php

Magnetization of a Magnetite Single Crystal Near the Curie Point*

D. O. SMITH†
Laboratory for Insulation Research, Massachusetts, Institute of Technology, Cambridge, Massachusetts
(Received January 20, 1956)

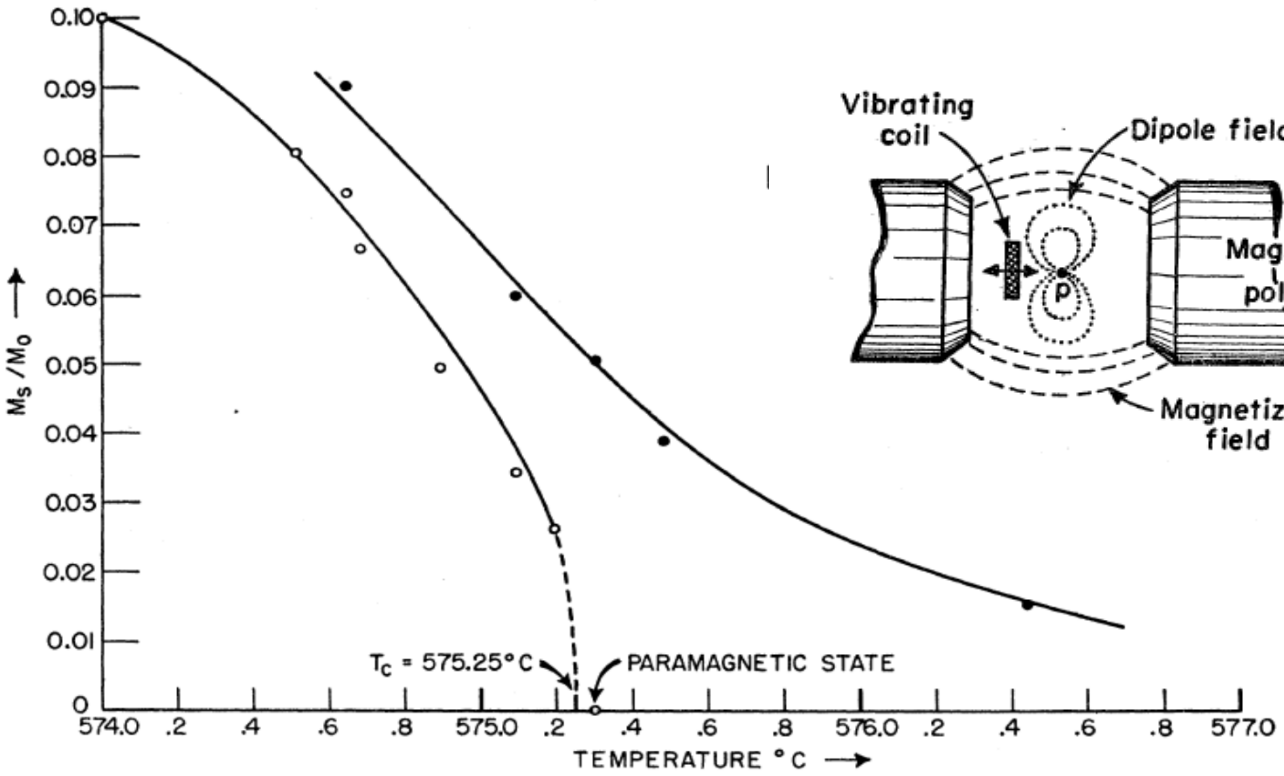


FIG. 9. M_s/M_0 *vs* T in the [111] direction near the Curie point for single-crystal magnetite.

FIG. 2. Principle of the vibrating-coil magnetometer.

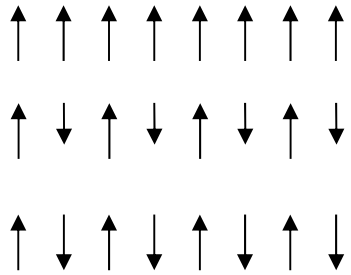
Magnetic ordering

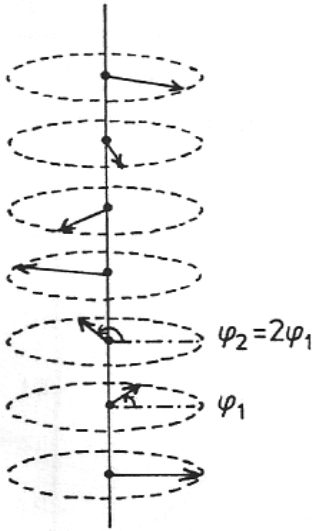
Ferromagnetism

Ferrimagnetism

Antiferromagnetism

Helimagnetism





All ordered magnetic states have excitations called magnons

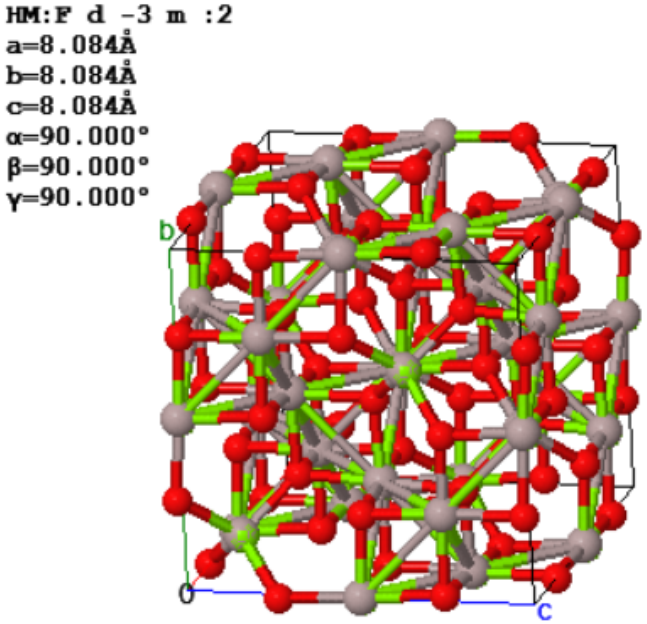
Ferrimagnets

Magnetite Fe_3O_4
(Magneteisen)

Ferrites $MO \cdot Fe_2O_3$

M = Fe, Zn, Cd, Ni, Cu,
Co, Mg

Two sublattices A and B.



$MgAl_2O_4$

Spinel crystal structure XY_2O_4

8 tetrahedral sites A (surrounded by 4 O) $5\mu_B \uparrow$

16 octahedral sites B (surrounded by 6 O) $9\mu_B \downarrow$

per unit cell

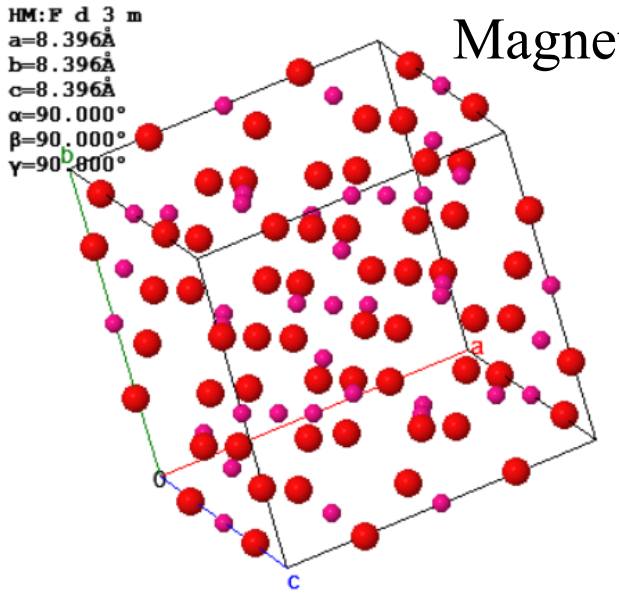
Ferrimagnets

Magnetite Fe_3O_4

Ferrites $MO{\cdot}Fe_2O_3$

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓



Magnetite

Exchange integrals J_{AA}, J_{AB}, and J_{BB} are all negative (antiparallel preferred)

$|J_{AB}| > |J_{AA}|,|J_{BB}|$

Ferrimagnetism

Ferrimagnetic materials (e.g. Magnetite) exhibit a crystalstructure with two sublattices A and B with different net magnetic moments. The exchange integrals J_{AA}, J_{BB} and J_{AB} are all negative with $|J_{AB}| > |J_{AA}|, |J_{BB}|$. Therefore, the spins of the A site align antiparallel with the spins of the B site. Due to the different net magnetic moments, ferrimagnetic materials have a non vanishing magnetization $M \neq 0$ even without an external field ($B_a = 0$). Similar to the derivation of ferromagnetism, the Hamiltonian of the isotropic Heisenberg model is used for the interacting spins

$$H = -\sum_{i,j} J_{i,j} \vec{S}_i \cdot \vec{S}_j - g\mu_b \overrightarrow{B_a} \sum_i \overrightarrow{S_i}.$$

with with g being the Landé g factor and μ_B the Bohr magneton. For simplification, it is assumed that only neighbouring spins interact with each other and the interaction vanishes for all lattice sites further away. Again, mean field approximation is applied for each of the two sublattices. The main idea is that the interaction of two neighboring spins $\vec{S}_i \cdot \vec{S}_j$ can be replaced by the interaction of a single spin on site i with the *mean spin value* of its surrounding:

$$\vec{S}_i \cdot \vec{S}_j \to \vec{S}_i \cdot \langle \vec{S} \rangle.$$

As there are two different sublattices A and B, the mean field approximation is performed for both lattices. One obtains:

$$H_{MF,A} = -\sum_i \vec{S}_i \cdot \left(\sum_{\delta_B} J_{i,AB} \langle \vec{S}_B \rangle + \sum_{\delta_A} J_{i,AA} \langle \vec{S}_A \rangle + g\mu_b \vec{B} \right).$$

↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ Ferrimagnetism

gauss = 10^{-4} T
oersted = $10^{-4}/4\pi \times 10^{-7}$ A/m

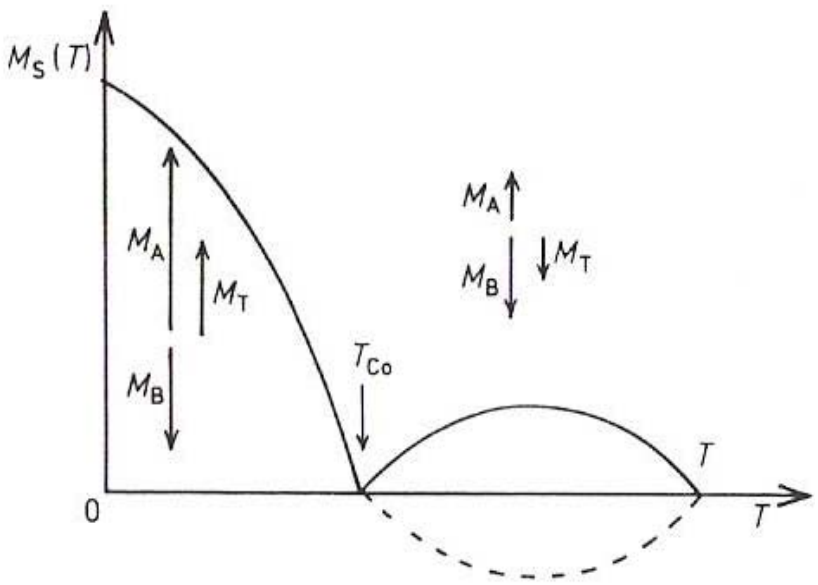


Table 33.3
SELECTED FERRIMAGNETS, WITH CRITICAL TEMPERATURES T_c AND SATURATION MAGNETIZATION M_0

MATERIAL	T_c (K)	M_0 (gauss)[a]
Fe_3O_4 (magnetite)	858	510
$CoFe_2O_4$	793	475
$NiFe_2O_4$	858	300
$CuFe_2O_4$	728	160
$MnFe_2O_4$	573	560
$Y_3Fe_5O_{12}$ (YIG)	560	195

[a] At $T = 0$(K).
Source: F. Keffer, *Handbuch der Physik*, vol. 18, pt. 2, Springer, New York, 1966.

Kittel

D. Gignoux, magnetic properties of Metallic systems

Antiferromagnetism

Negative exchange energy $J_{AB} < 0$.

At low temperatures, below the Neel temperature T_N, the spins are aligned antiparallel and the macroscopic magnetization is zero.

Spin ordering can be observed by neutron scattering.

At high temperature antiferromagnets become paramagnetic. The macroscopic magnetization is zero and the spins are disordered in zero field.

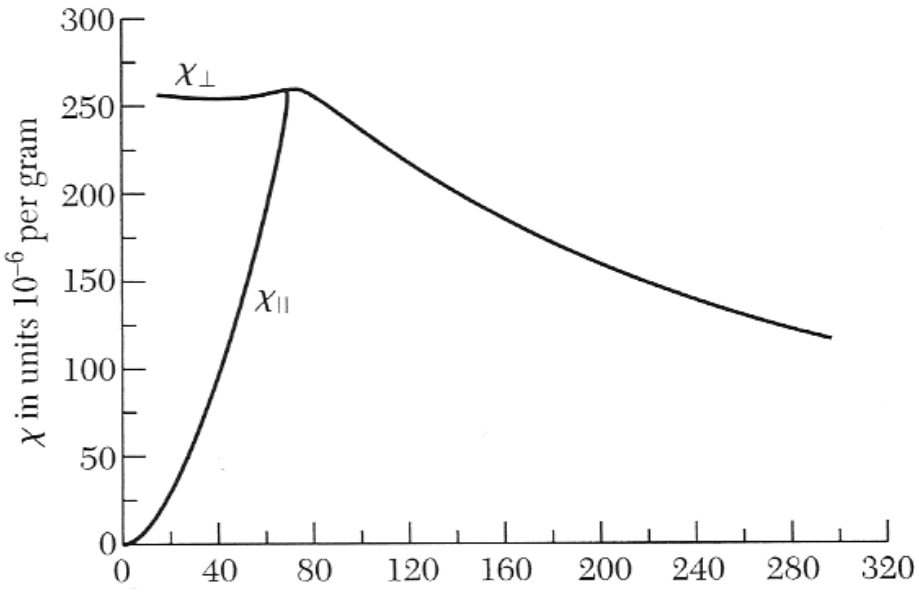
$$\chi = \mu_0 \frac{\vec{M}_A + \vec{M}_B}{\vec{B}_a} = \frac{C}{T + \Theta}$$

Curie-Weiss temperature

Antiferromagnetism ↑↓↑↓↑↓↑↓

Average spontaneous magnetization is zero at all temperatures.

Source: Kittel



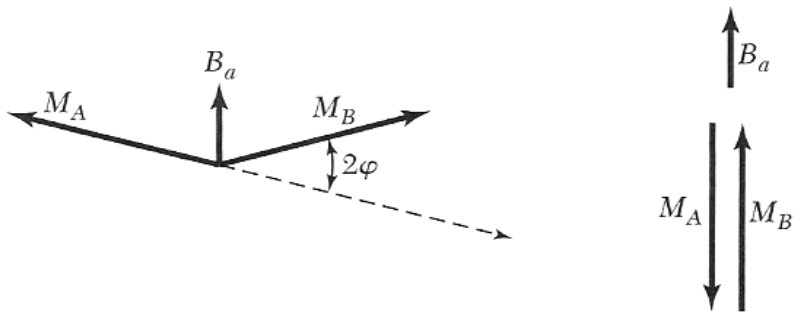
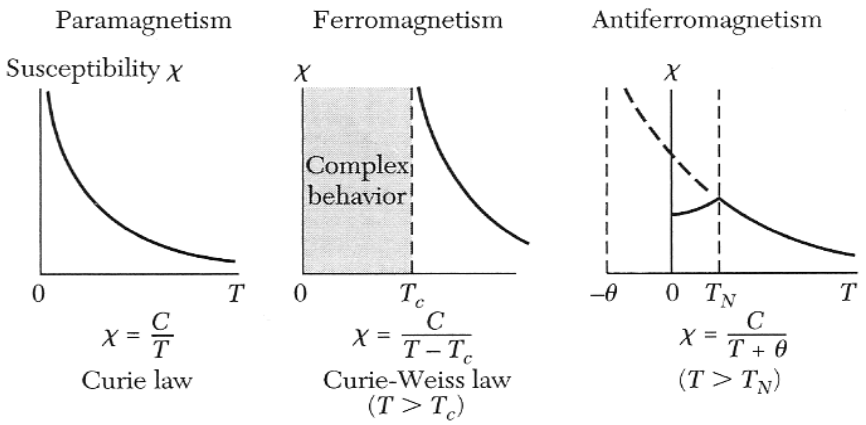


Table 2 Antiferromagnetic crystals

Substance	Paramagnetic ion lattice	Transition temperature, T_N, in K	Curie-Weiss θ, in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF_2	bc tetr.	67	82	1.24	0.76
FeF_2	bc tetr.	79	117	1.48	0.72
$FeCl_2$	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
$CoCl_2$	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
$NiCl_2$	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			



from Kittel

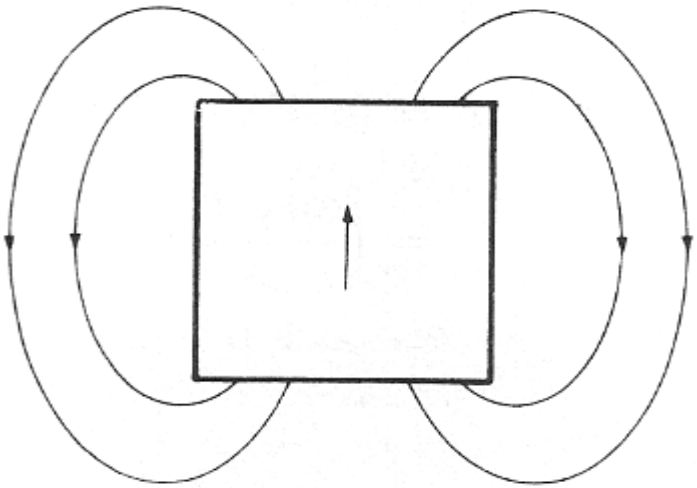
Applications of magnetism

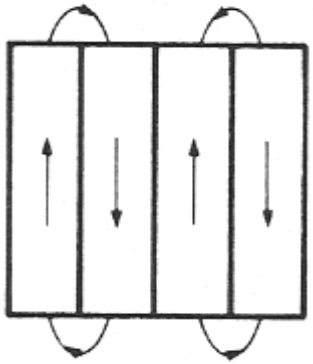
Hard magnets: permanent magnets, motors, generators, microphones

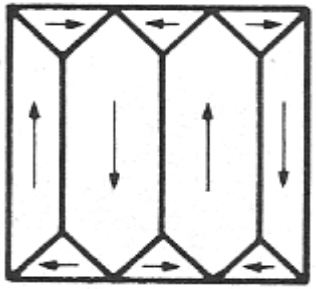
Soft magnets: transformers

Magnetic recording

Magnetic domains (weisssche Bezirke)



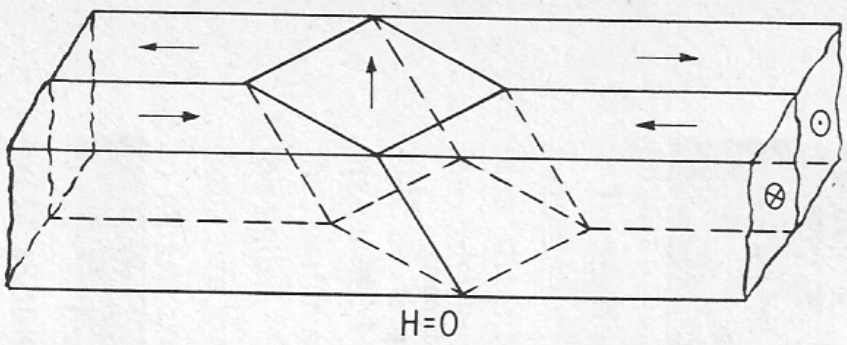


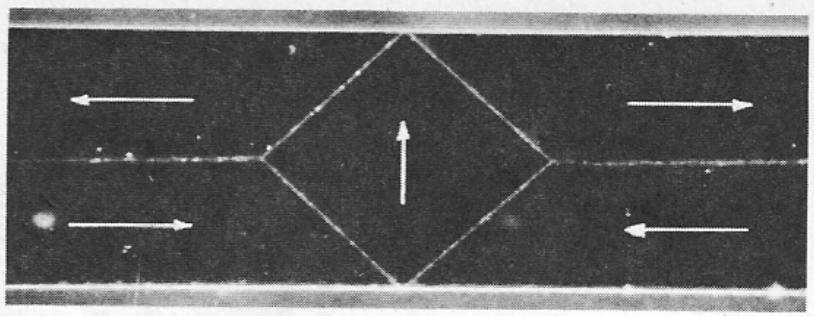


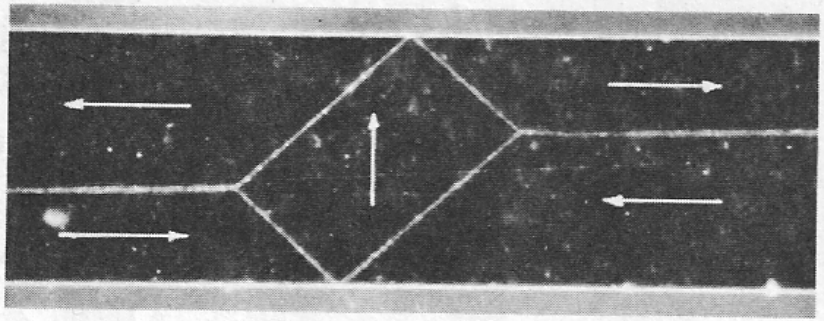
Magnetic energy density

$$\frac{B^2}{2\mu_0}$$

Costs energy to introduce domain walls where spin up regions are adjacent to spin down regions.


H=0


H = 0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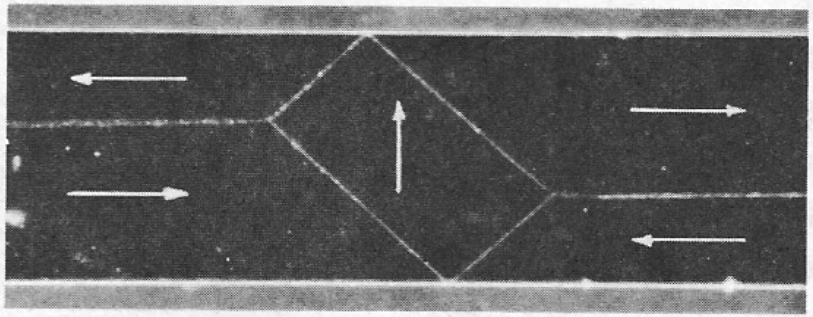
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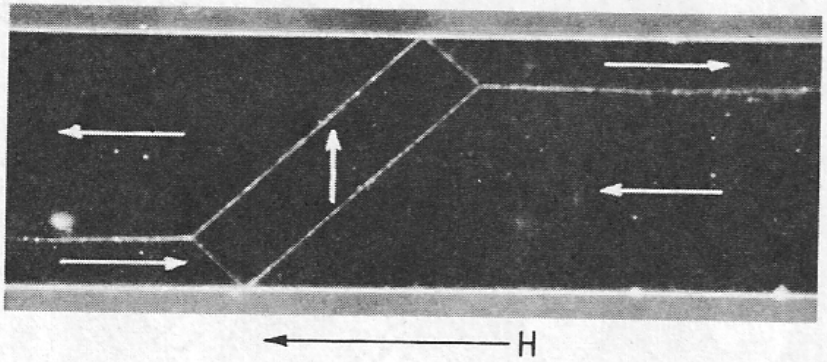
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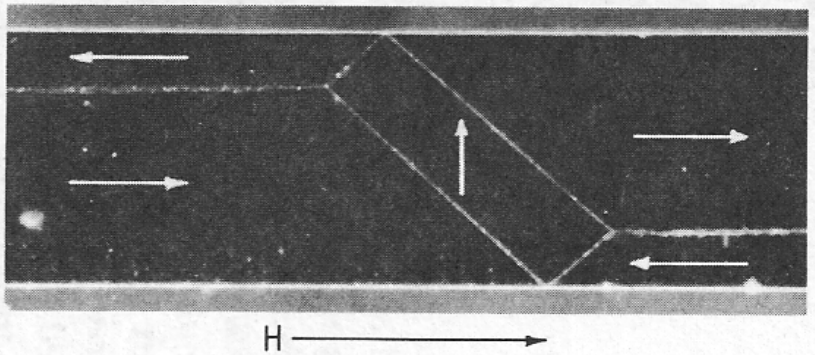

H

Ferromagnetic domains

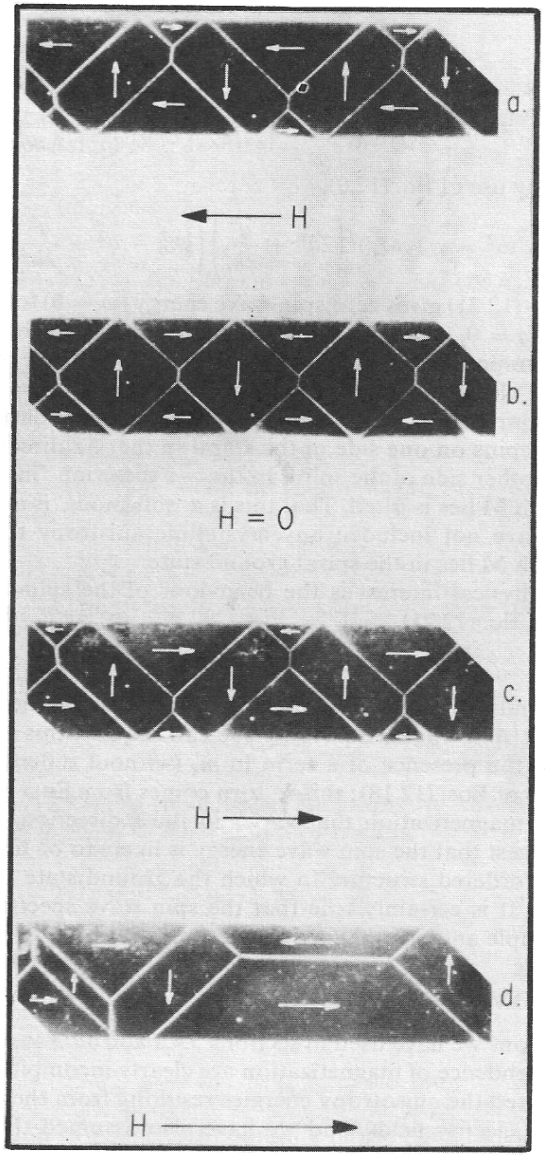
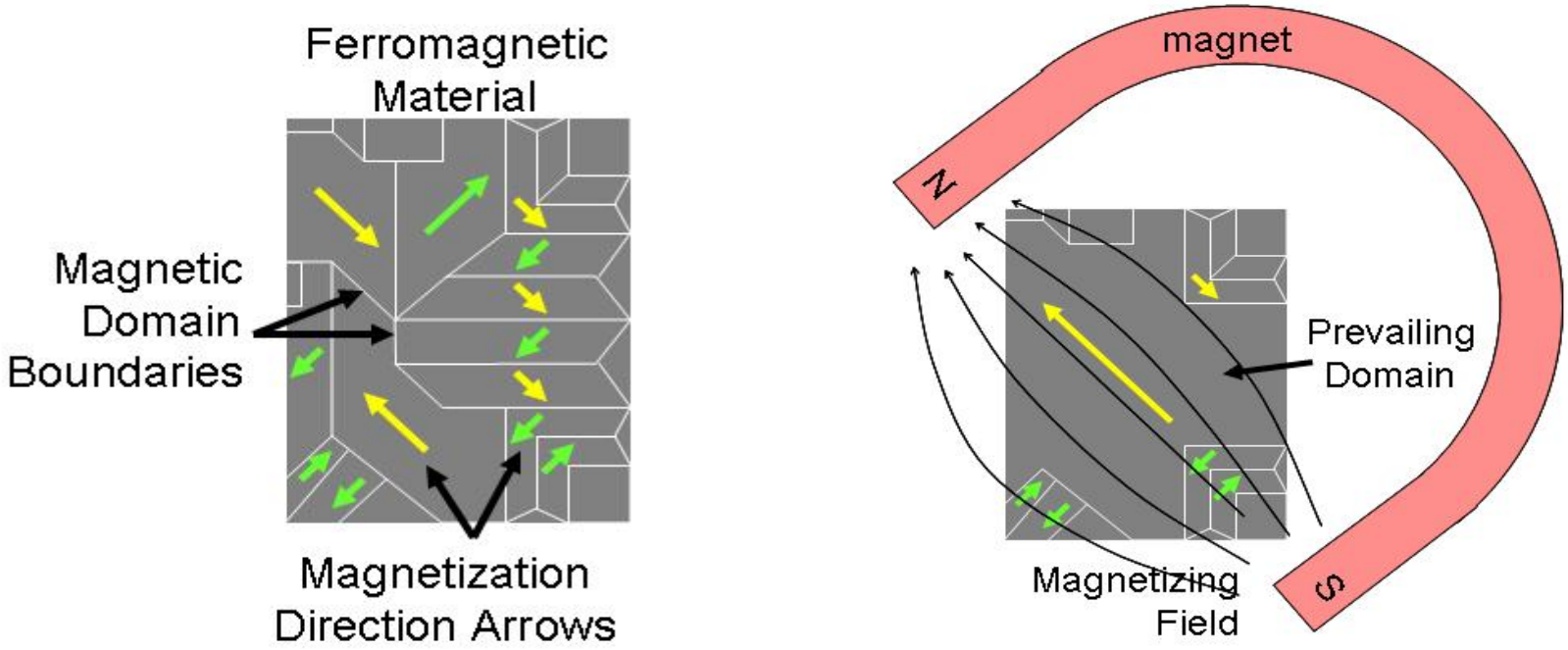


Fig. 12.5. Photographs showing reversible domain wall motion in a 50 μm whisker from (*a*) to (*b*) to (*c*), with an irreversible jump from (*c*) to (*d*).
{R. W. de Blois and C. D. Graham, *J. Appl. Phys.*, **29**, 931 (1958)}.

Weak fields: favorable domains expand
Strong fields: domains rotate to align with field

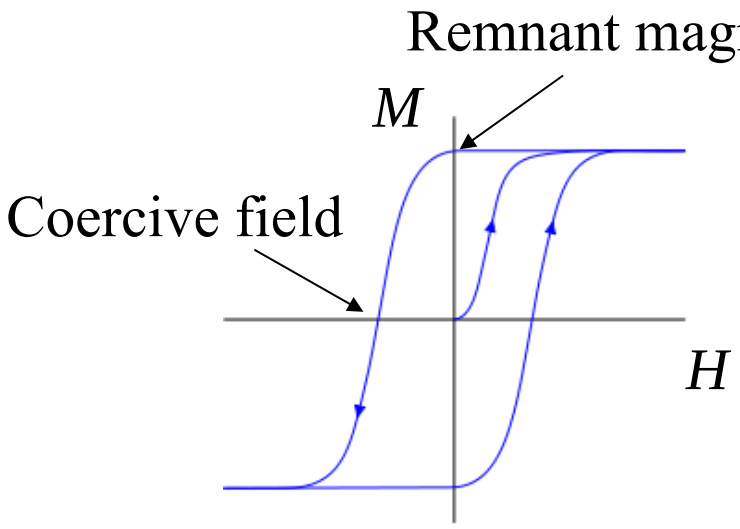
Irreproducible jump between c and d.

Magnetizing a magnet



Weak fields: favorable domains expand
Strong fields: domains rotate to align with field

Hysteresis



$$B = \mu_0 \left(H + M \right)$$

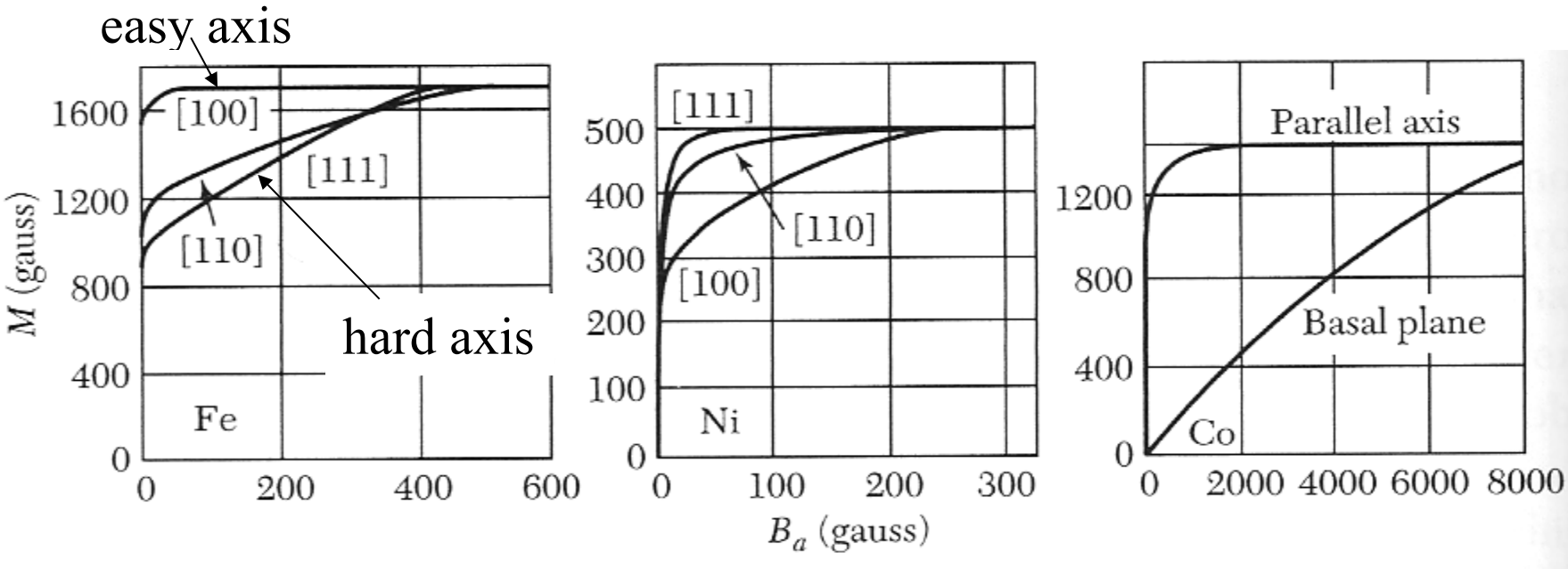
$$M = \chi H$$

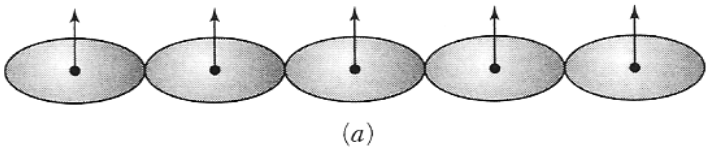
$$B = \mu_0 (1 + \chi) H = \mu_r \mu_0 H$$

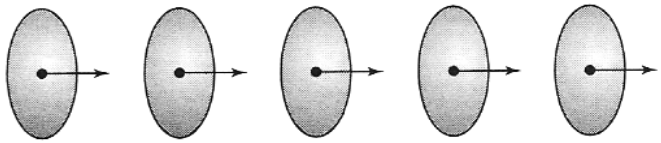
$$\mu_r = \left(1 + \chi\right)$$

Area of the loop is proportional to energy dissipated in traversing the loop.

Anisotropy energy







Spin-orbit coupling couples the shape of the wavefunction to the spin. The exchange energy depends on the overlap of the wavefunctions and thus on spin direction.

Bloch wall

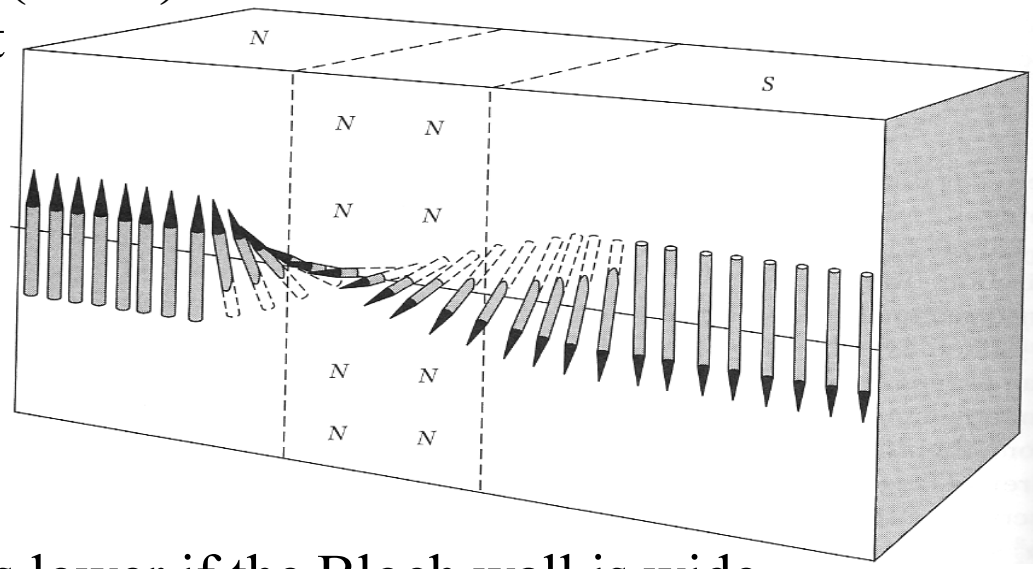
energy of two spins

$$w = -J\vec{S}_i \cdot \vec{S}_j = -JS^2 \cos\varphi \approx -JS^2\left(1 - \frac{\varphi^2}{2}\right)$$

neglecting the constant part

$$w \approx JS^2 \frac{\varphi^2}{2}$$

$$\varphi = \frac{\pi}{N}$$



energy of a line of spins

$$Nw \approx \frac{JS^2\pi^2}{2N}$$

energy is lower if the Bloch wall is wide

energy per unit area $\approx \frac{JS^2\pi^2}{2Na^2}$

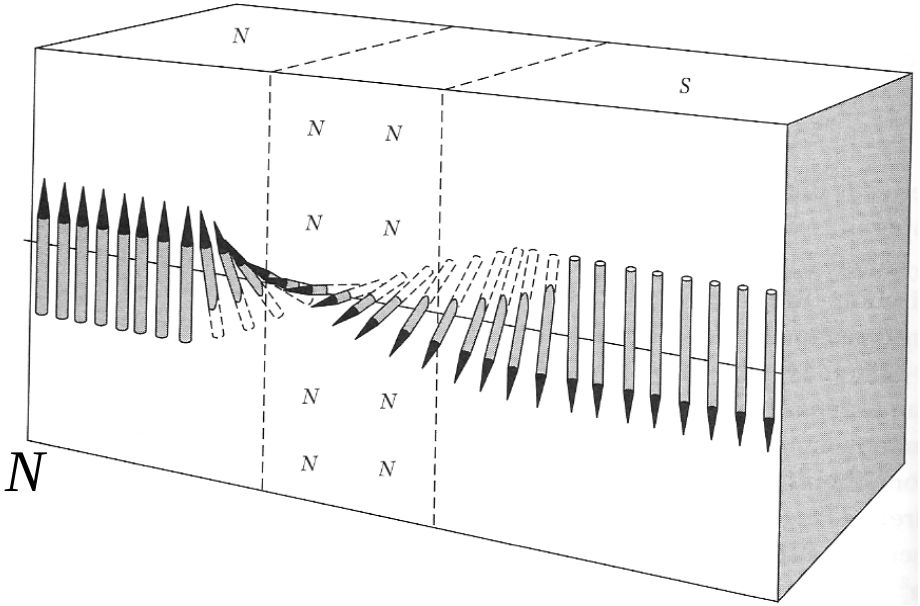
a is the lattice constant

Bloch wall

Anisotropy energy depends on the number of spins pointing in the hard direction

$\approx KNa$ ← Na = thickness of wall

anisotropy constant J/m^3



Total energy per unit area:

$$E = \frac{JS^2\pi^2}{2Na^2} + KNa \qquad [\mathrm{J/m^2}]$$

smaller for large N smaller for small N

$$\frac{dE}{dN} = 0 \Rightarrow -\frac{JS^2\pi^2}{2N^2a^2} + Ka = 0$$

$$N = \sqrt{\frac{JS^2\pi^2}{2Ka^3}}$$

$N \sim 300$ for iron