Advanced Solid State Physics

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Oct 4, 2021

Advanced Solid State Physics

Solid state physics is the study of how atoms arrange themselves into solids and what properties these solids have.

Calculate the macroscopic properties from the microscopic structure.

Solid state physics

Every property of every solid can be calculated using multi-particle quantum mechanics.

$$H_{\mathrm{mp}} = -\sum_i \frac{\hbar^2}{2m_e}\nabla_i^2 - \sum_a \frac{\hbar^2}{2m_a}\nabla_a^2 - \sum_{a,i}\frac{Z_a e^2}{4\pi\epsilon_0|\vec{r}_i - \vec{r}_a|} + \sum_{i<j}\frac{e^2}{4\pi\epsilon_0|\vec{r}_i - \vec{r}_j|} + \sum_{a<b}\frac{Z_a Z_b e^2}{4\pi\epsilon_0|\vec{r}_a - \vec{r}_b|}$$

http://en.wikipedia.org/wiki/File:Erwin_Schr%C3%B6dinger.jpg

Introductory Solid State Physics

Crystals, Bravais lattices, Miller indices
Diffraction, Fourier transforms, Brillouin zones
Phonon dispersion and density of states
Free electrons dispersion and density of states
Calculation of thermodynamic properties from the DOS
Band structure calculations, empty lattice approximation, tight binding, plane wave method
Bloch waves, translation operator

TU Graz

Advanced Solid State Physics

Solid-state physics, the largest branch of condensed matter physics, is the study of rigid matter, or solids. The bulk of solid-state physics theory and research is focused on crystals, largely because the periodicity of atoms in a crystal, its defining characteristic, facilitates mathematical modeling, and also because crystalline materials often have electrical, magnetic, optical, or mechanical properties that can be exploited for engineering purposes. The framework of most solid-state physics theory is the Schrödinger (wave) formulation of non-relativistic quantum mechanics.

- Solid state physics in Wikipedia

The most remarkable thing is the great variety of *qualitatively different* solutions to Schrödinger's equation that can arise. We have insulators, semiconductors, metals, superconductors—all obeying different macroscopic laws: an electric field causes an electric dipole moment in an insulator, a steady current in a metal or semiconductor and a steadily accelerated current in a superconductor. Solids may be transparent or opaque, hard or soft, brittle or ductile, magnetic or non-magnetic.

From *Solid State Physics* by H. E. Hall

To a large extent, our success in understanding solids is a consequence of nature's kindness in organizing them for us... By the term solid we shall really always mean crystalline solid, and, moreover, infinite perfect crystalline solid at that.

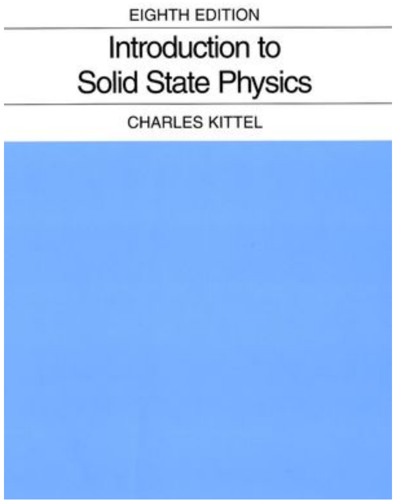
From *States of Matter* by David L. Goodstein

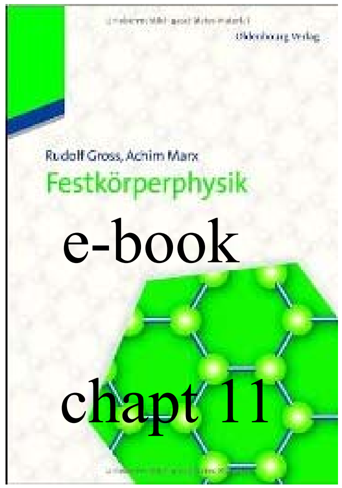
http://lampx.tugraz.at/~hadley/ss2/

TUG -> Institute of Solid State Physics -> Courses

Advanced Solid State Physics

Review: Fourier series and reciprocal space
Review: Free electrons (noninteracting fermions), electrons in crystals
Review: Phonons (noninteracting bosons)
Semiconductors
Electrons in a magnetic field
Fermi surfaces
Quantum Hall effect
Magnetism
Superconductivity
Linear response theory
Dielectric function / optical properties
Transport properties
Ferroelectricity, pyroelectricity, piezoelectricity
Landau theory of phase transitions
Quasiparticles (phonons, magnons, plasmons, exitons, polaritons)
Mott transition, Fermi Liquid Theory





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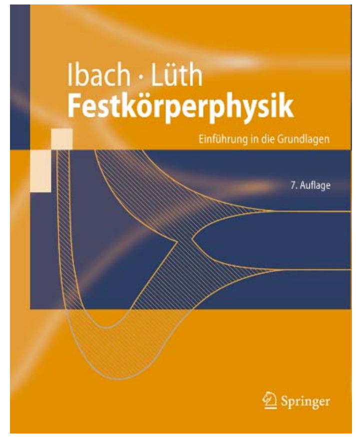

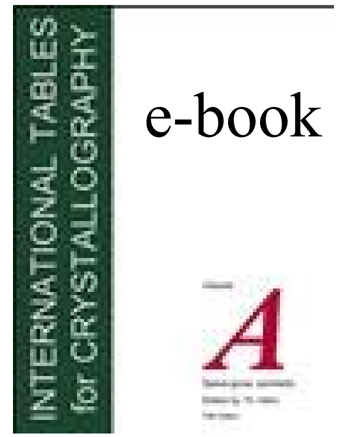

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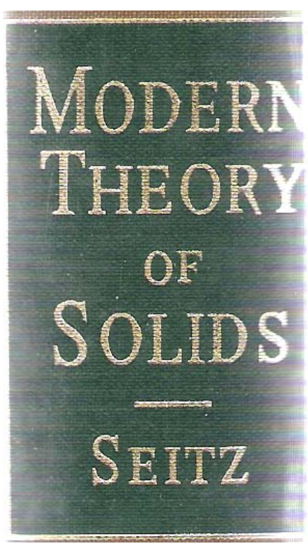
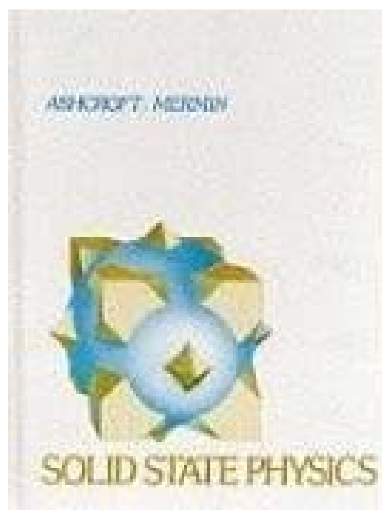
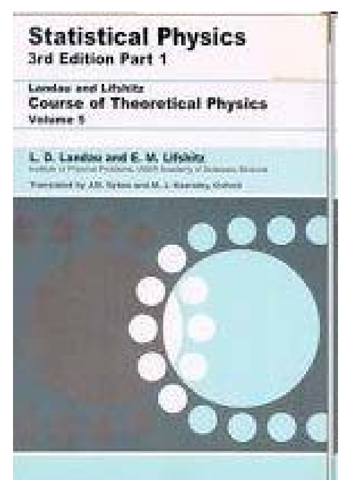






Student projects

Something that will help other students pass this course

2VO + 1UE

Derivation
Example calculations (phonon dispersion relation for GaAs)
Javascript calculations
Lecture videos

Examination

1 hour written exam

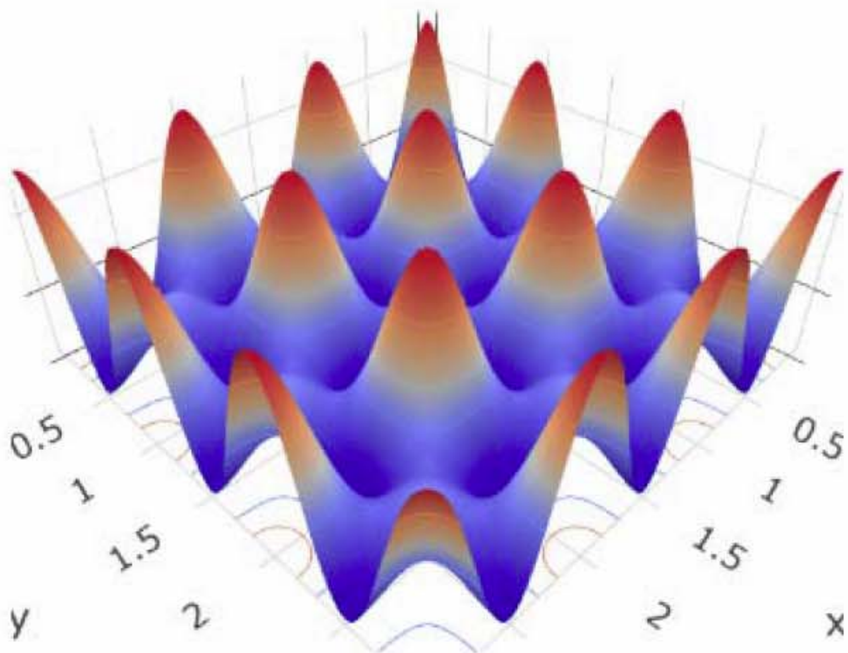
One page of handwritten notes

Oral exam

- Student project
- Mistakes on written exam
- General questions about the course

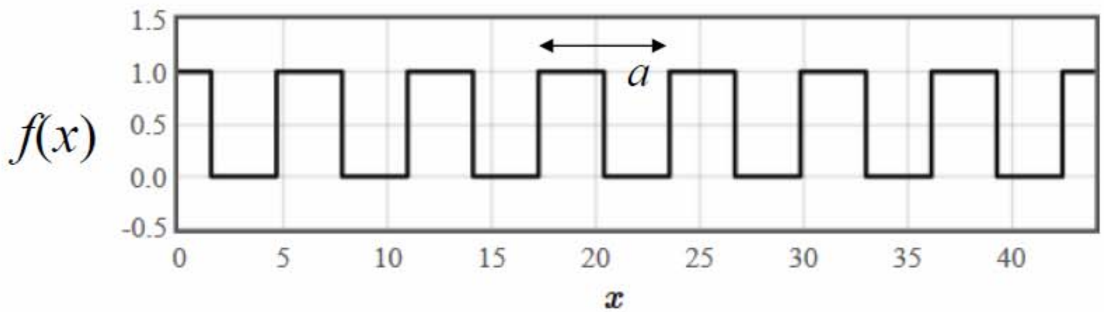
Fourier Series in one dimension

Periodic functions



Use a Fourier series to describe periodic functions

Expanding a 1-d function in a Fourier series



Any periodic function can be represented as a Fourier series.

$$f(x) = f_0 + \sum_{p=1}^{\infty} c_p \cos(2\pi px/a) + s_p \sin(2\pi px/a)$$

multiply by $\cos(2\pi p'x/a)$ and integrate over a period.

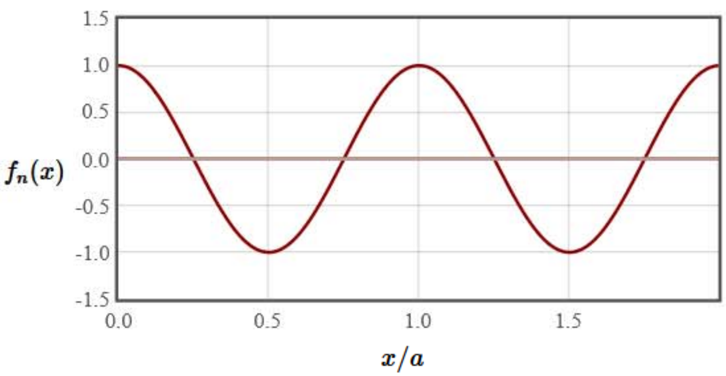
$$\int_0^a f(x)\cos\left(2\pi p'x/a\right)dx = c_p \int_0^a \cos\left(2\pi p'x/a\right)\cos\left(2\pi p'x/a\right)dx = \frac{ac_{p'}}{2}$$

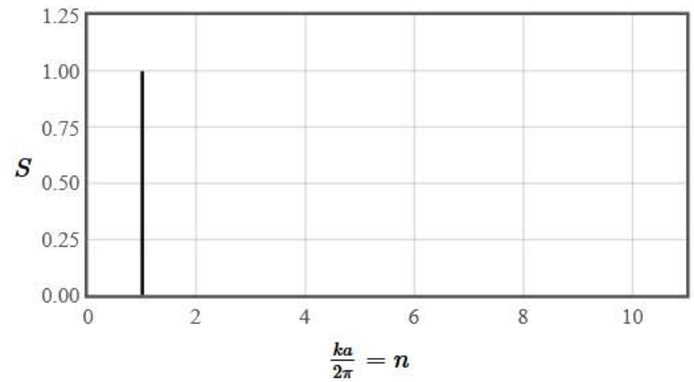
$$c_p = \frac{2}{a}\int_0^a f(x)\cos\left(2\pi px/a\right)dx$$

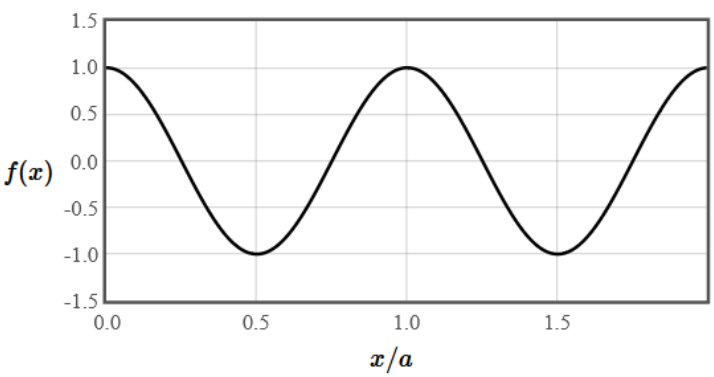
Fourier synthesis

A periodic function with period a can be written as a Fourier series of the form,

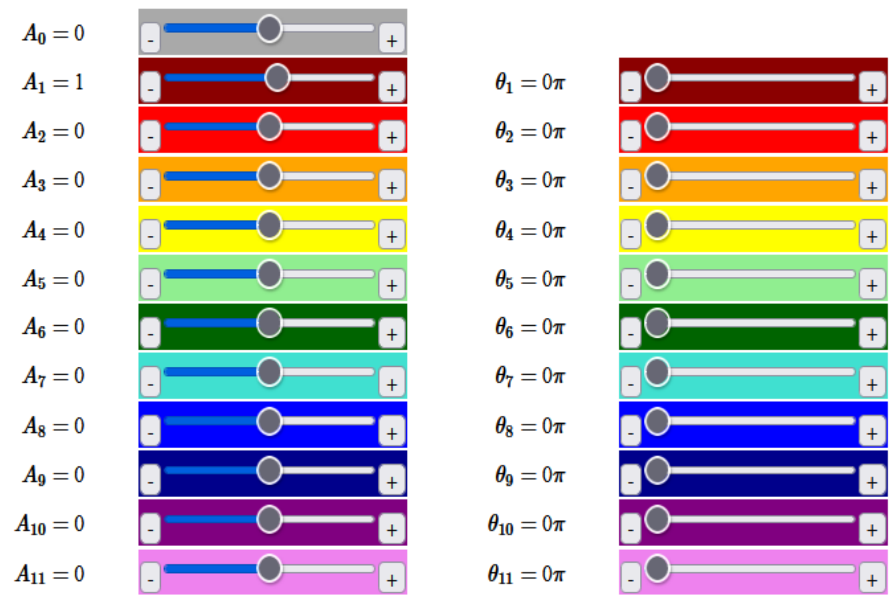
$$f(x) = A_0 + \sum_n A_n \left(\cos(\theta_n)\cos(2\pi n x/a) + \sin(\theta_n)\sin(2\pi n x/a)\right).$$



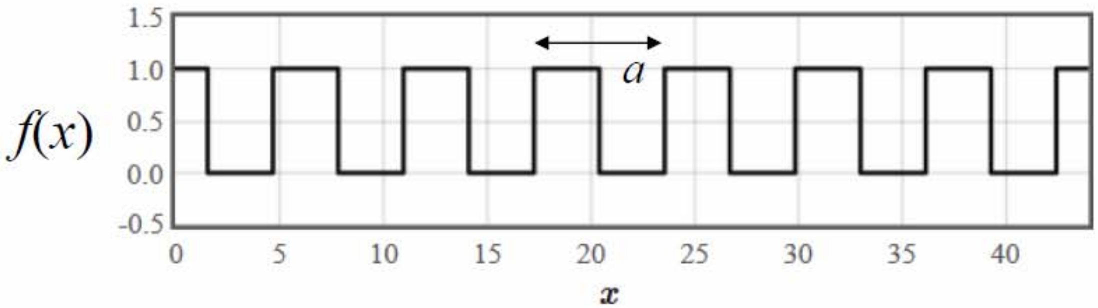




Number of periods displayed: 2



Expanding a 1-d function in a Fourier series



Any periodic function can be represented as a Fourier series.

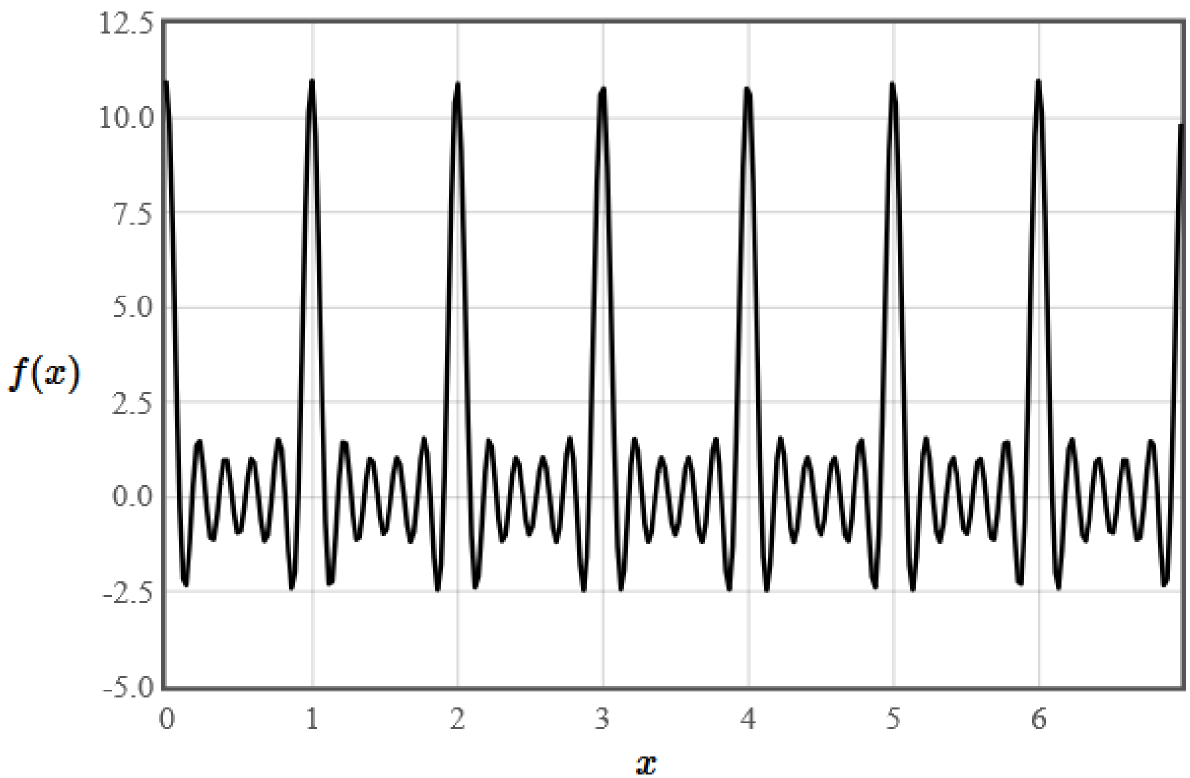
$$f(x) = f_0 + \sum_{p=1}^{\infty} c_p \cos(2\pi px/a) + s_p \sin(2\pi px/a)$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$f(x) = \sum_{G=-\infty}^{\infty} f_G e^{iGx} \qquad f_G = \frac{c_p}{2} - i\frac{s_p}{2} \qquad G = \frac{2\pi p}{a}$$

For real functions: $f_G^* = f_{-G}$

reciprocal lattice vector



square triangle sawtooth comb

$f_0 = 1$ - +

$f_1 = f^*_{-1} = 1$ - + $+i\,(0)$ - +

$f_2 = f^*_{-2} = 1$ - + $+i\,(0)$ - +

$f_3 = f^*_{-3} = 1$ - + $+i\,(0)$ - +

$f_4 = f^*_{-4} = 1$ - + $+i\,(0)$ - +

$f_5 = f^*_{-5} = 1$ - + $+i\,(0)$ - +

Fourier series in 1-D, 2-D, or 3-D

$$f(\vec{r}) = \sum_{\vec{G}} f_{\vec{G}} e^{i\vec{G}\cdot\vec{r}}$$

Reciprocal lattice vectors G
(depend on the Bravais lattice)

Structure factors
(complex numbers)

$$\vec{T}_{hkl} = h\vec{a}_1 + k\vec{a}_2 + l\vec{a}_3$$

$$\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij} \qquad \delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0, & \text{for } i \neq j \end{cases}$$

$$\vec{G} = \nu_1\vec{b}_1 + \nu_2\vec{b}_2 + \nu_3\vec{b}_3$$