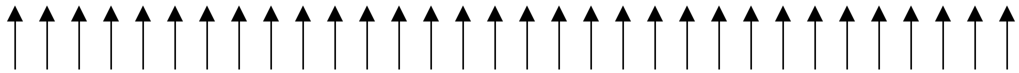
Magnons, plasmons, polaritons

Magnons



Energy of the Heisenberg term involving spin p

$$-2J\vec{S}_p \cdot \left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

The magnetic moment of spin p is

$$\vec{\mu}_p = -g\mu_B \vec{S}_p$$

$$-\vec{\mu}_p \cdot \left(\frac{-2J}{g\mu_B}\right)\left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

This has the form $-\mu_p \cdot B_p$ where B_p is

$$\vec{B}_p = \left(\frac{-2J}{g\mu_B}\right)\left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

Magnons

$$\vec{\mu}_p = -g\mu_B \vec{S}_p \qquad\qquad \vec{B}_p = \left(\frac{-2J}{g\mu_B}\right)\left(\vec{S}_{p+1} + \vec{S}_{p-1}\right)$$

The rate of change of angular momentum is the torque

$$\hbar\frac{d\vec{S}_p}{dt} = \vec{\mu}_p \times \vec{B}_p = 2J\left(\vec{S}_p \times \vec{S}_{p+1} + \vec{S}_p \times \vec{S}_{p-1}\right)$$

If the amplitude of the deviations from perfect alignment along the z-axis are small:

$$\hbar\frac{dS_p^x}{dt} = 2J\left|S\right|\left(S_{p+1}^y - 2S_p^y + S_{p-1}^y\right)$$

$$\hbar\frac{dS_p^y}{dt} = 2J\left|S\right|\left(S_{p+1}^x - 2S_p^x + S_{p-1}^x\right)$$

$$\hbar\frac{d\vec{S}_p^z}{dt} = 0$$

Magnons

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$$\hbar \frac{dS_p^z}{dt} = 0$$

These are coupled linear differential equations. The solutions have the form:

$$\begin{pmatrix} S_p^x \\ S_p^y \end{pmatrix} = \begin{pmatrix} u_k^x \\ u_k^y \end{pmatrix} \exp\left[i\left(kpa - \omega t\right)\right]$$

$$-i\hbar\omega u_k^x e^{ikpa} = 2J\left|S\right|\left(-e^{ik(p+1)a} + 2e^{ikpa} - e^{-ik(p-1)a}\right)u_k^y$$

$$-i\hbar\omega u_k^y e^{ikpa} = -2J\left|S\right|\left(-e^{ik(p+1)a} + 2e^{ikpa} - e^{-ik(p-1)a}\right)u_k^x$$

Cancel a factor of e^{ikpa}.

Magnons

$$-i\hbar\omega u_k^x = 2J\left|S\right|\left(-e^{ika}+2-e^{-ika}\right)u_k^y$$

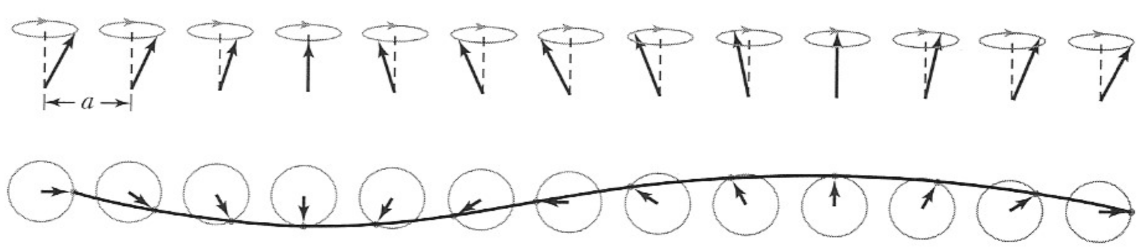
$$-i\hbar\omega u_k^y = -2J\left|S\right|\left(-e^{ika}+2-e^{-ika}\right)u_k^x$$

These equations will have solutions when,

$$\begin{vmatrix} i\hbar\omega & 4J\left|S\right|\left(1-\cos(ka)\right) \\ -4J\left|S\right|\left(1-\cos(ka)\right) & i\hbar\omega \end{vmatrix} = 0$$

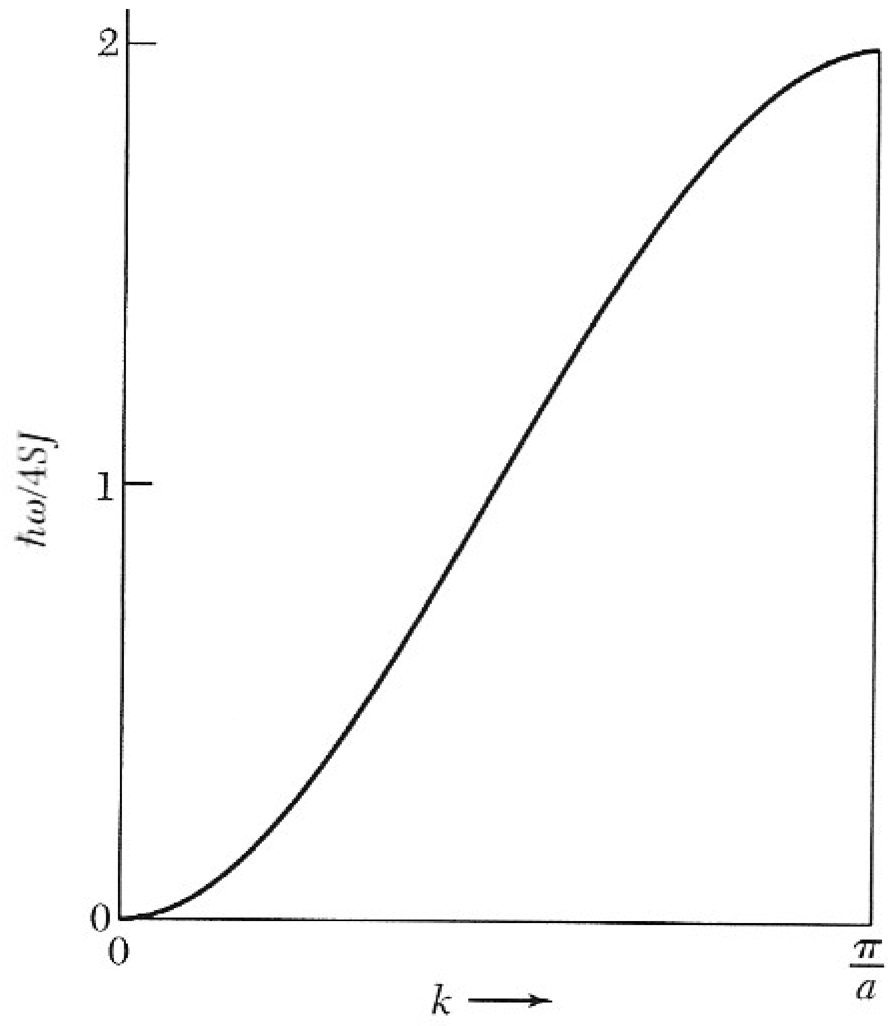
The dispersion relation is:

$$\hbar\omega = 4J\left|S\right|\left(1-\cos(ka)\right)$$



Magnon dispersion relation

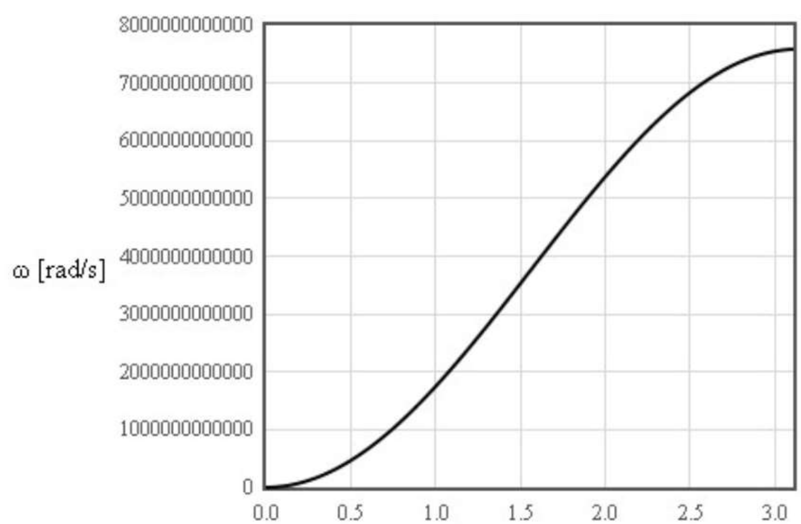
$$\hbar\omega = 4JS\left(1-\cos(ka)\right)$$



A phonon dispersion relation
would be linear at the origin

Table of magnon properties

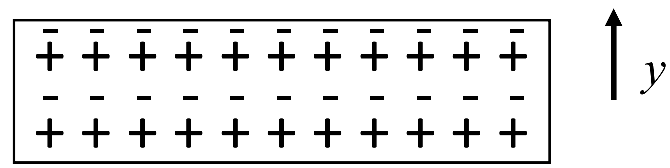
Magnons

	1-D ferromagnetic magnons
Equations of motion in mean field theory	$\hbar \frac{dS_p^x}{dt} = 2JS(2S_p^y - S_{p-1}^y - S_{p+1}^y)$ $\hbar \frac{dS_p^y}{dt} = -2JS(2S_p^x - S_{p-1}^x - S_{p+1}^x)$ $\hbar \frac{dS_p^z}{dt} = 0$
Eigenfunction solutions	$\begin{pmatrix} S_p^x \\ S_p^y \end{pmatrix} = \begin{pmatrix} u_k^x \\ u_k^y \end{pmatrix} \exp\left[i\left(kpa - \omega t\right)\right]$
Dispersion relation	$\hbar\omega = 4JS\left(1 - \cos(ka)\right)$ 

Longitudinal plasma waves

$$nm\frac{d^2y}{dt^2} = -neE$$

$$E = \frac{ney}{\varepsilon_0}$$



$$nm\frac{d^2y}{dt^2} = -\frac{n^2e^2y}{\varepsilon_0}$$

$$\frac{d^2y}{dt^2} + \omega_p^2 y = 0$$

Plasma frequency $\qquad \omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$

There is no magnetic component of the wave.

Plasma waves can be quantized like any other wave

Transverse optical plasma waves

The dispersion relation for light

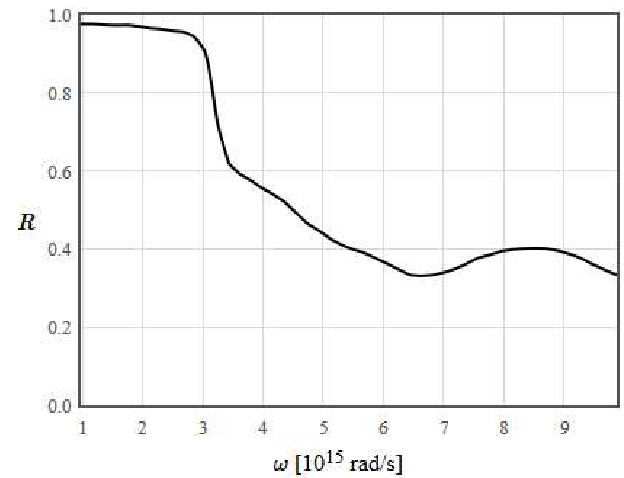
$$\varepsilon\varepsilon_0\mu_0\omega^2 = \frac{\varepsilon\omega^2}{c^2} = k^2$$

For a free electron gas

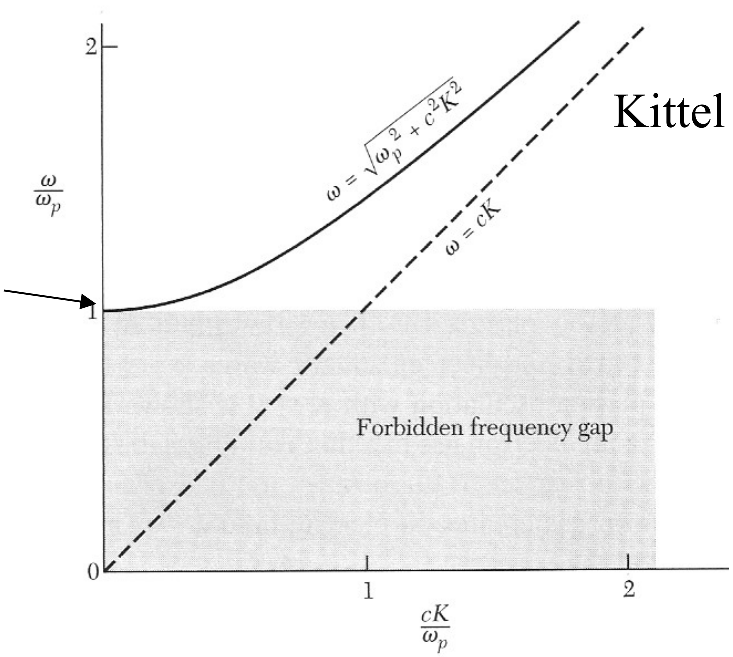
$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\left(1 - \frac{\omega_p^2}{\omega^2}\right)\omega^2 = c^2k^2$$

$$\omega^2 = \omega_p^2 + c^2k^2$$



Plasmons



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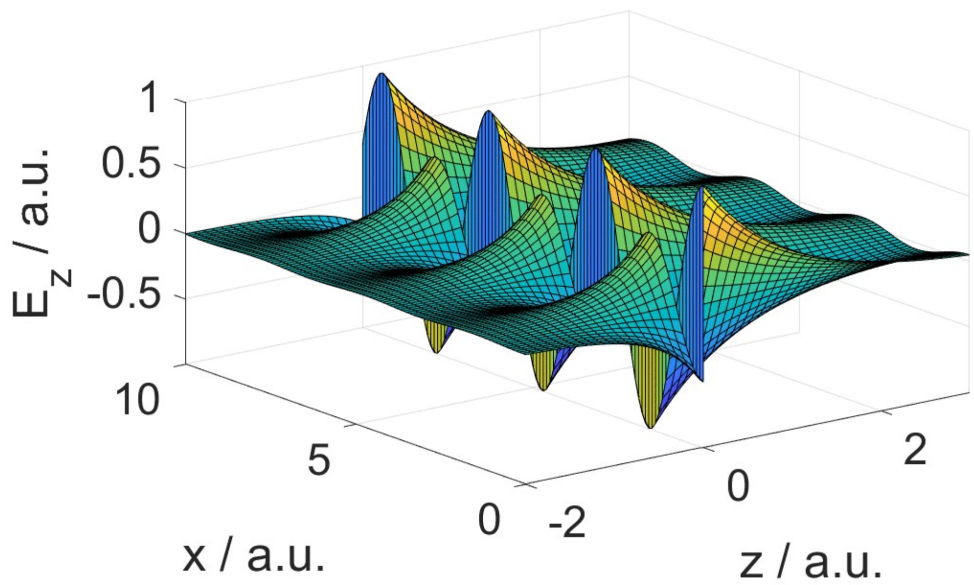
Surface Plasmons

Waves in the electron density at the boundary of two materials.

Surface plasmons have a lower frequency that bulk plasmons. This confines them to the interface.



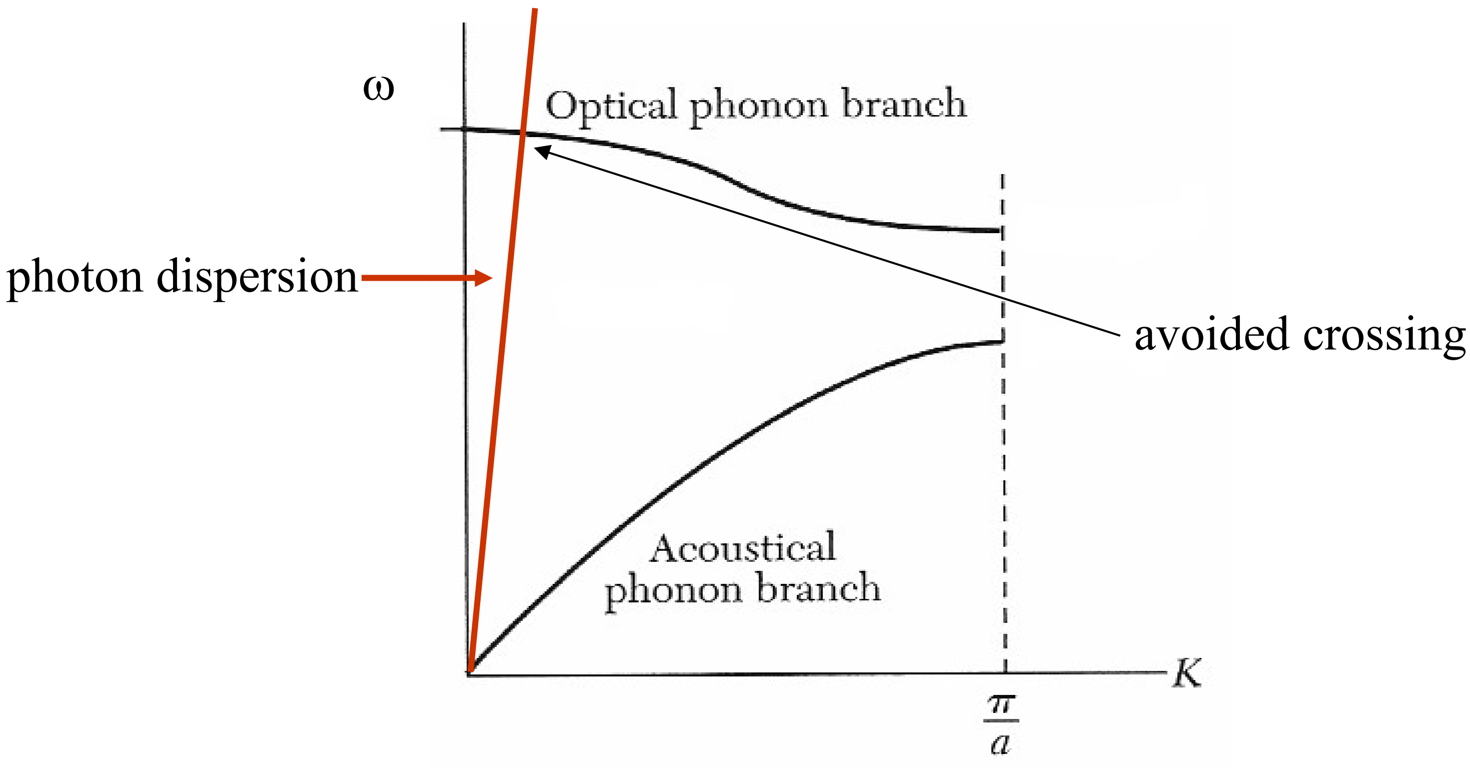
Surface Plasmons

Surface plasmons on nanoparticles are efficient at scattering light.

Green and blue require different sized particles.

Polaritons

Transverse optical phonons will couple to photons with the same ω and *k*.



Light Bragg reflects off the sound wave; sound Bragg reflects off the light wave.

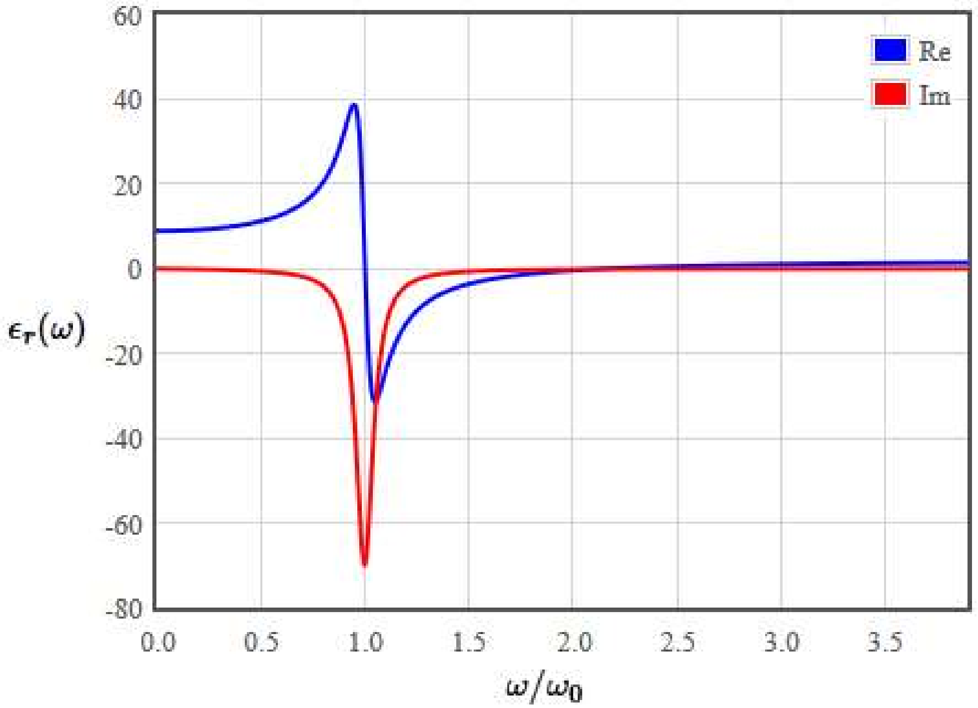
Polaritons

The dispersion relation for light

$$\varepsilon\varepsilon_0\mu_0\omega^2 = \frac{\varepsilon\omega^2}{c^2} = k^2$$

For an insulator

$$\epsilon_r(\omega) = \epsilon(\infty) + \frac{\omega_0^2(\epsilon(0)-\epsilon(\infty))}{\omega_0^2-\omega^2+i\gamma\omega}$$



The description of polaritons is already built into the dielectric function.

Polaritons

Ignore the loss term $i\gamma\omega$

$$\varepsilon(\omega) = \varepsilon(\infty) + \frac{\omega_0^2\left(\varepsilon(0) - \varepsilon(\infty)\right)}{\omega_0^2 - \omega^2}$$

Use a common denominator

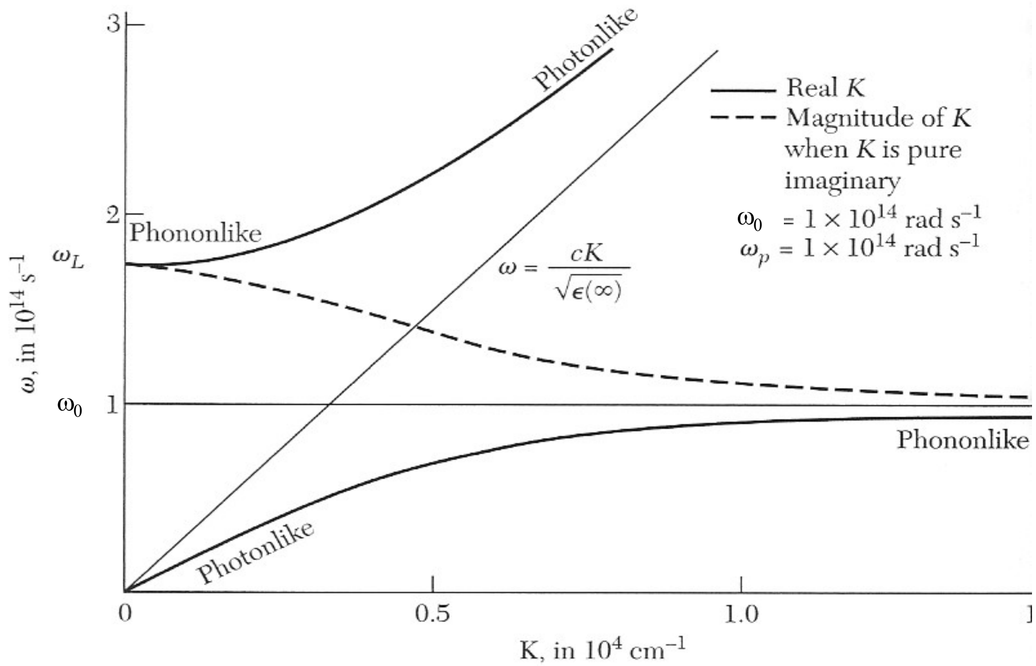
$$\varepsilon(\omega) = \frac{\varepsilon(\infty)\left(\omega_0^2 - \omega^2\right) + \omega_0^2\left(\varepsilon(0) - \varepsilon(\infty)\right)}{\omega_0^2 - \omega^2}$$

Define ω_L $\quad \omega_0^2\varepsilon(0) = \varepsilon(\infty)\omega_L^2$

$$\varepsilon(\omega) = \varepsilon(\infty)\frac{\omega_L^2 - \omega^2}{\omega_0^2 - \omega^2}$$

Polaritons

$$\varepsilon(\infty)\frac{\omega_L^2-\omega^2}{\omega_0^2-\omega^2}\frac{\omega^2}{c^2}=k^2$$

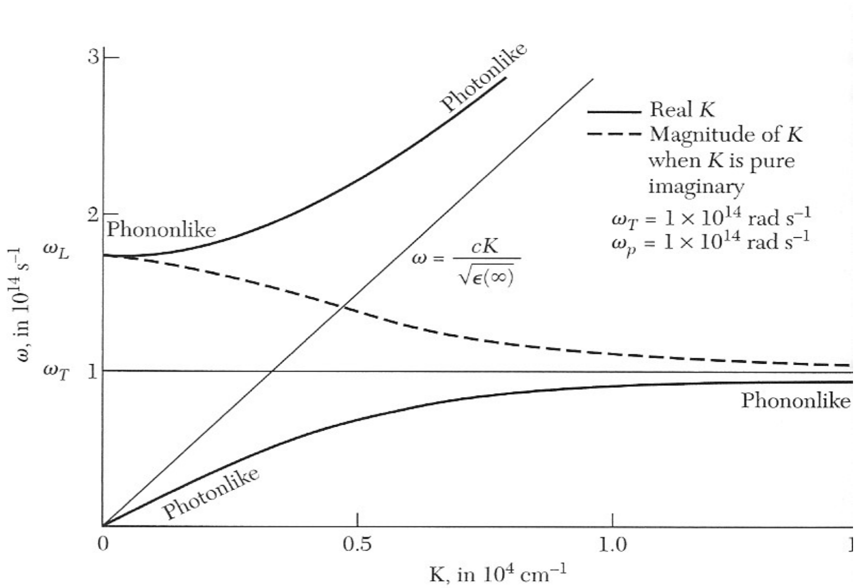


There are two solutions for every k, one for the upper branch and one for the lower branch.

A gap exists in frequency.

Polaritons are the normal modes near the avoided crossing.

Polaritons allow us to study the properties of phonons using optical measurements



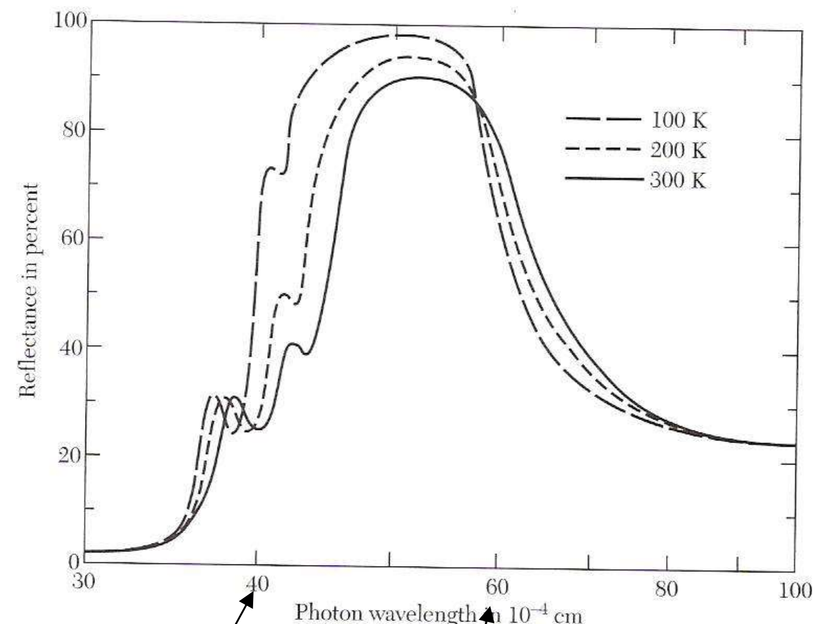


Figure 15 Reflectance of a crystal of NaCl at several temperatures, versus wavelength. The nominal values of ω_L and ω_T at room temperature correspond to wavelengths of 38 and 61 $\times 10^{-4}$ cm, respectively. (After A. Mitsuishi et al.)

ω = 4.7E13

ω = 3.1E13

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By looking at the reflectance in different crystal directions, you can determine the frequencies of the transverse optical phonons.