

The perpendicular components of B (magnetic flux density) and D (electric displacement field) and the parallel components of E (electric field) and H (magnetic field) are continuous through the interface of two dielectrics. A derivation for this relations can be found in the lecture notes of the Electrodynamics course (chapter 12).

This leads us to the continuity condition for our problem:

$$n \times E_R = n \times (E_I + E_T) \quad (1)$$

$$n \cdot \epsilon_2 E_R = n \cdot \epsilon_1 (E_I + E_T) \quad (2)$$

The components of the electromagnetic waves have the form $A_0 e^{i(kx - \omega t)}$. At the interface the phase of the incoming, the reflected and the transmitted wave is the same.

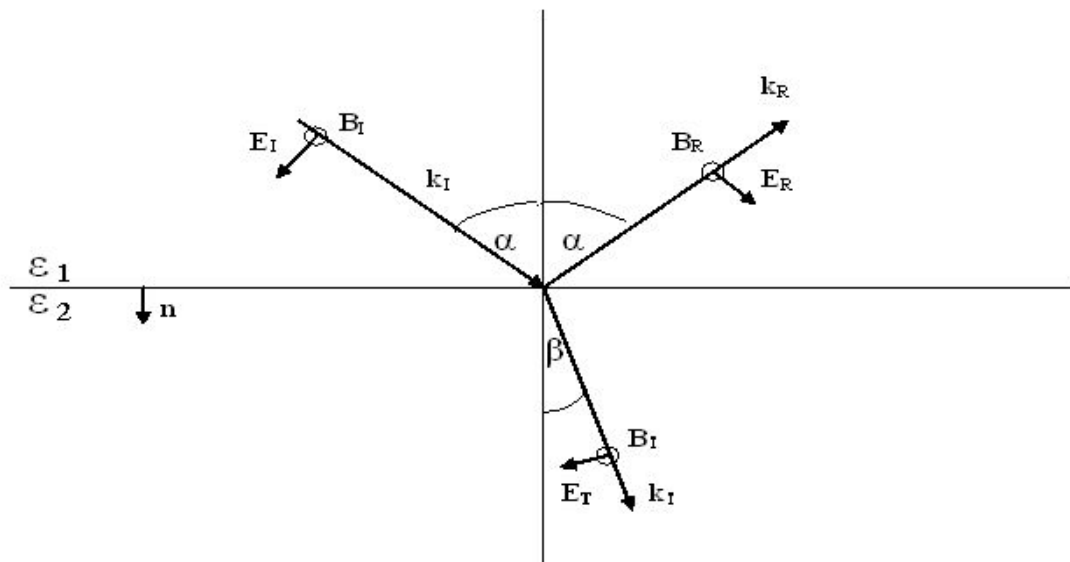


Abbildung 1: Refraction and Reflection of an EM Wave on an Interface.

From (2) we get

$$\begin{aligned} \epsilon_2 E_{0T} \cos\left(\frac{\pi}{2} - \beta\right) - \epsilon_1 (E_{0I} \cos\left(\frac{\pi}{2} - \alpha\right) + E_{0R} \cos\left(\frac{\pi}{2} - \alpha\right)) &= 0 \\ \Leftrightarrow \\ \epsilon_2 E_{0T} \sin \beta - \epsilon_1 (E_{0I} + E_{0R}) \sin \alpha &= 0 \end{aligned}$$

Snell's law of reflection says $n_1 \sin \alpha = n_2 \sin \beta$ and thus

$$\frac{\epsilon_2 n_1}{\epsilon_1 n_2} E_{0T} \sin \alpha - (E_{0I} + E_{0R}) \sin \alpha = 0 \quad (3)$$

From (1) we get

$$\begin{aligned} E_{0T} \sin\left(\frac{\pi}{2} - \beta\right) &= (E_{0I} - E_{0R}) \sin\left(\frac{\pi}{2} - \alpha\right) \\ \Leftrightarrow \\ E_{0T} \cos \beta &= (E_{0I} - E_{0R}) \cos \alpha \\ \Leftrightarrow \\ E_{0T} \frac{\cos \beta}{\cos \alpha} - E_{0I} + E_{0R} &= 0 \end{aligned} \quad (4)$$

We add eq.(3) and eq.(4) and get

$$E_{0T} \left(\frac{\cos \beta}{\cos \alpha} + \frac{\epsilon_2 n_1}{\epsilon_1 n_2} \right) = 2E_{0I}$$

If we assume that the permeability is the same in both materials, we get

$$\begin{aligned} \frac{E_{0T}}{E_{0I}} &= \frac{2}{\frac{\cos \beta}{\cos \alpha} + \frac{n_2}{n_1}} \\ \Leftrightarrow \\ \frac{E_{0T}}{E_{0I}} &= \frac{2 \cos \alpha}{\cos \beta + \frac{n_2}{n_1} \cos \alpha} \end{aligned} \quad (5)$$

We go back to eq.3 and divide on both sides by E_{0I} . We also use $\frac{\epsilon_2 n_1}{\epsilon_1 n_2} = \frac{n_2}{n_1}$

$$\frac{E_{0R}}{E_{0I}} = \frac{n_2 E_{0T}}{n_1 E_{0I}} - 1$$

Now we plug in eq.(5) and get

$$\begin{aligned} \frac{E_{0R}}{E_{0I}} &= \frac{n_2}{n_1} \frac{2 \cos \alpha}{\cos \beta + \frac{n_2}{n_1} \cos \alpha} - 1 \\ \Leftrightarrow \end{aligned}$$

$$\frac{E_{0R}}{E_{0I}} = \frac{n_2 \cos \alpha - n_1 \cos \beta}{n_2 \cos \alpha + n_1 \cos \beta} \quad (6)$$

We get our result - the complex reflectance coefficient - when we plug in the complex index of refraction $n_1 = \tilde{n}_1 + iK_1$ and $n_2 = \tilde{n}_2 + iK_2$

$$\frac{E_{0R}}{E_{0I}} = \frac{(\tilde{n}_2 + iK_2) \cos \alpha - (\tilde{n}_1 + iK_1) \cos \beta}{(\tilde{n}_2 + iK_2) \cos \alpha + (\tilde{n}_1 + iK_1) \cos \beta} \quad (7)$$

We can compare this result with the given solution in Kittel for light perpendicular to the surface. In this case $\alpha = \beta = 0$. Moreover $n_1 = 1$ (vacuum). Plugging that into eq.(7) leads us to the same result as in Kittel,

$$\frac{E_{0R}}{E_{0I}} = \frac{(\tilde{n}_2 + iK_2) - 1}{(\tilde{n}_2 + iK_2) + 1}$$