Carrier transport
pn - junctions

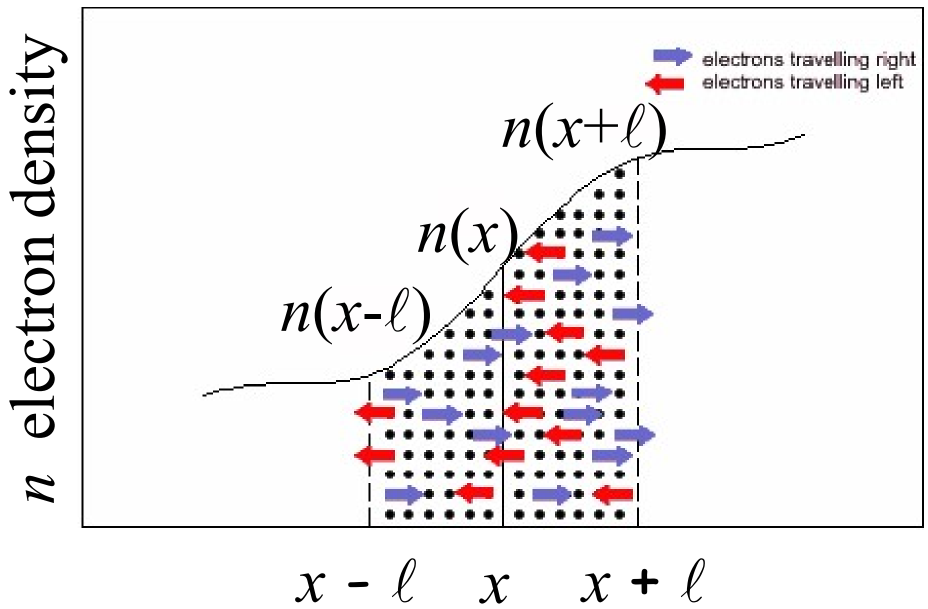
Carrier Transport

Ballistic transport
Drift
Diffusion
Tunneling

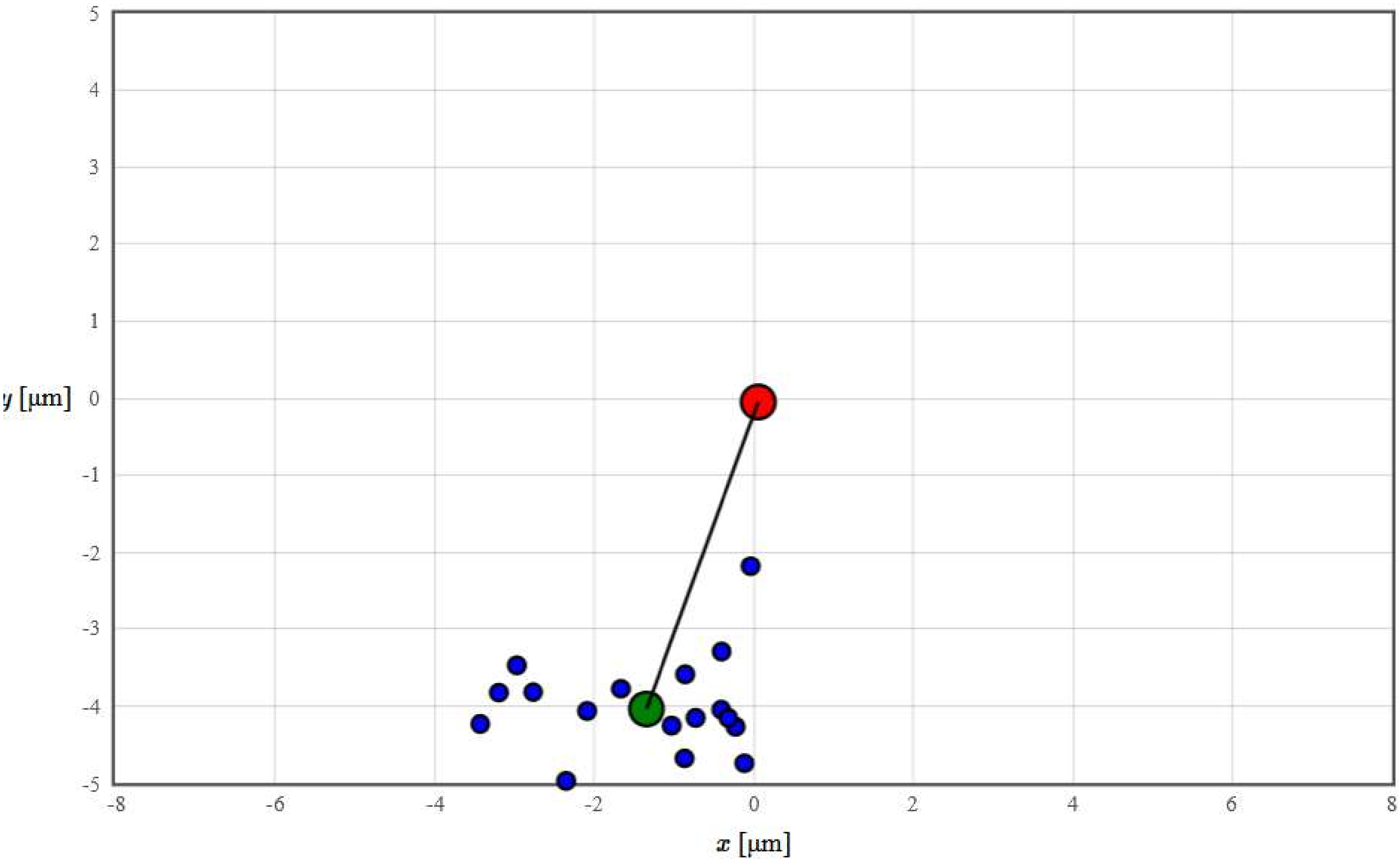
Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$



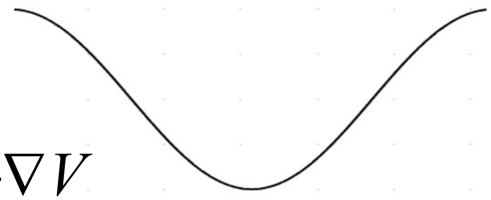
Diffusion is from high concentration to low concentration.



http://lampx.tugraz.at/~hadley/psd/L5/drude.php

Einstein relation

$\vec{E} = -\nabla V$



$$n = A\exp\left(\frac{-eV}{k_B T}\right)$$ Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T} A\exp\left(\frac{-eV_{pot}}{k_B T}\right)\nabla V = -\frac{ne}{k_B T}\nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD\frac{ne\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen

Current Density Equations

Drift

Diffusion

$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$

$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

In Equilibrium

$$\vec{j}_n = en\mu_n\vec{E} + eD_n\nabla n$$

The electric field is proportional to the gradient of the conduction band edge

$$e\vec{E} = \nabla E_c$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

If the Fermi energy is constant,

$$\nabla n = -\frac{\nabla E_c}{k_B T} N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = -\frac{\nabla E_c}{k_B T} n$$

$$\vec{j}_n = n\nabla E_c \left(\mu_n - \frac{eD_n}{k_B T}\right)$$

This means that the current density in a semiconductor where the Fermi energy is constant is zero.

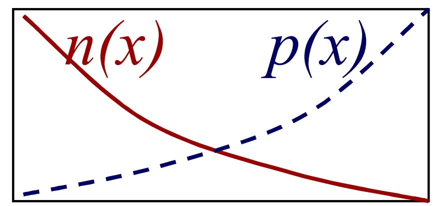
Current Density Equations

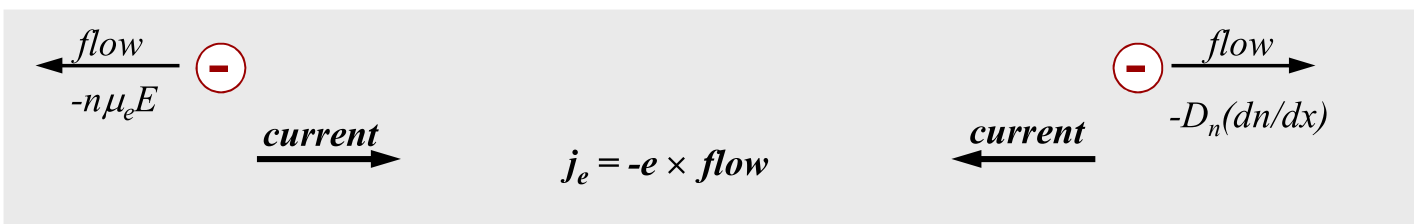
note: electron and hole currents have same direction

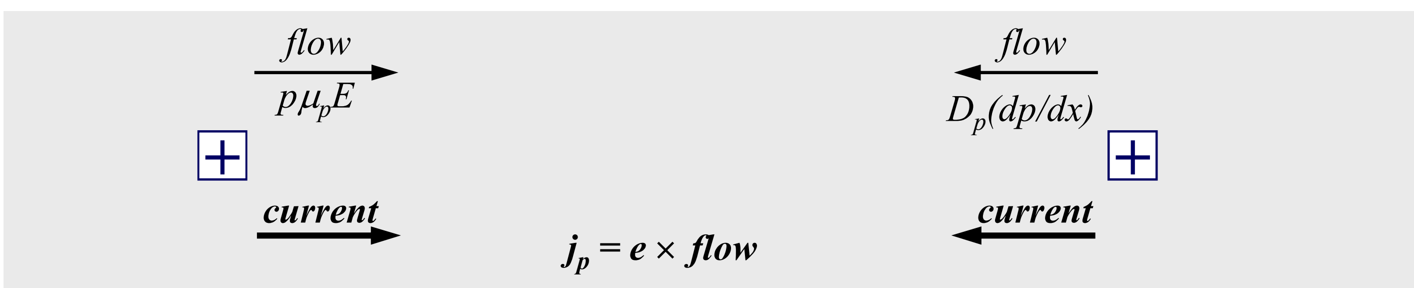
electric current = charge × particle flow

drift

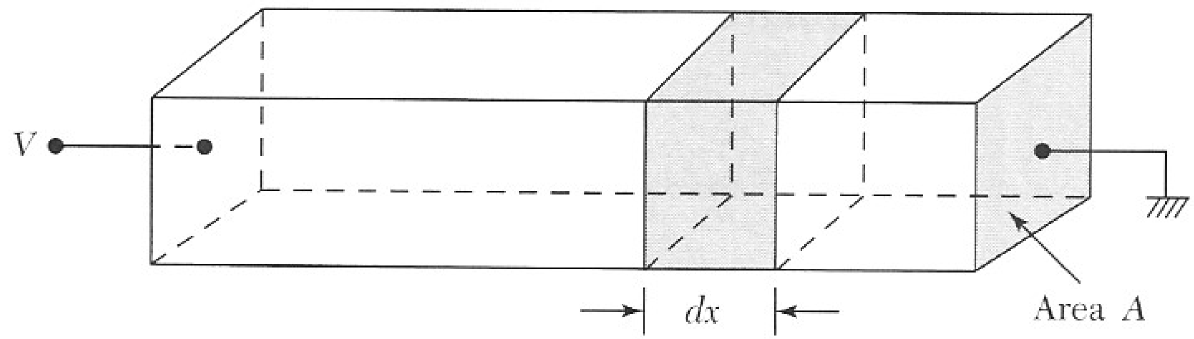
diffusion







Continuity equations

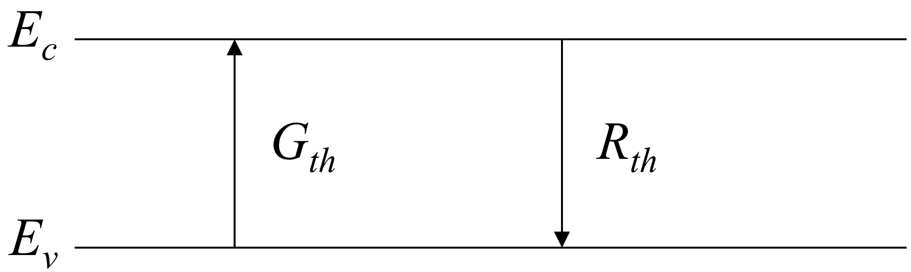


$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\nabla \cdot \vec{j}_p + G_p - R_p$$

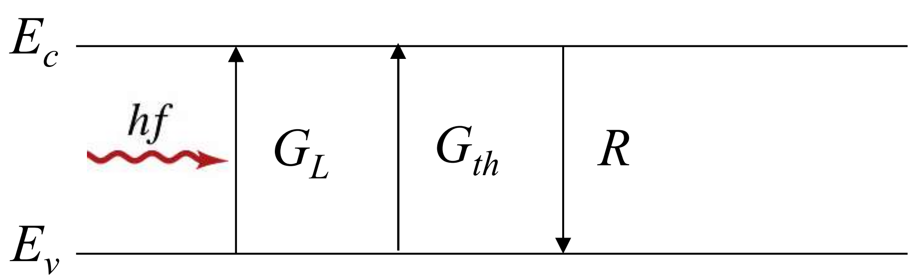
j_n and j_p consist of drift and diffusion terms

Generation and Recombination



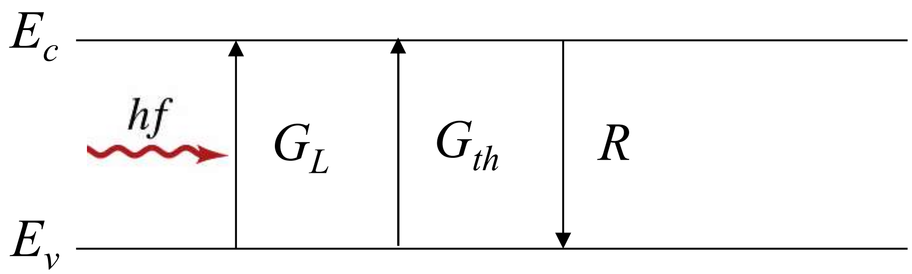
thermal equilibrium

Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



non equilibrium

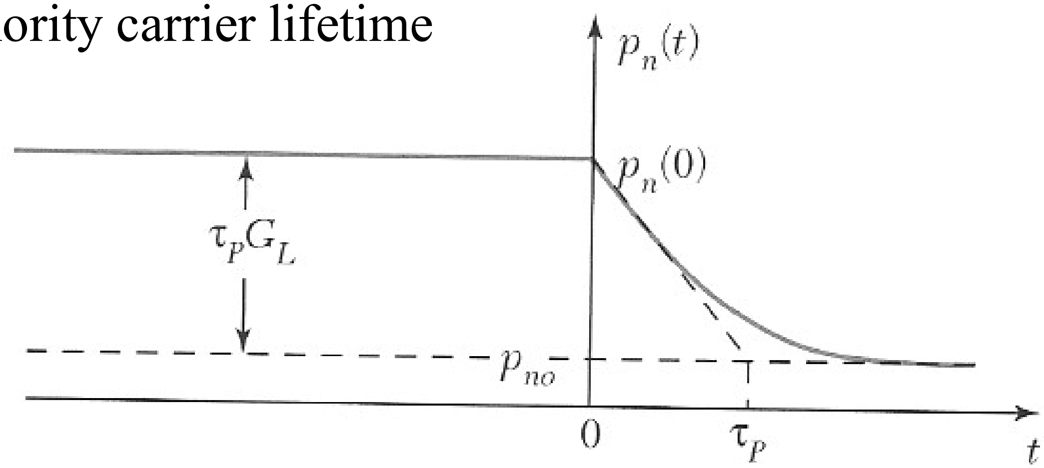
Recombination



$$R - R_{th} = \frac{p_n - p_{n0}}{\tau_p}$$

Recombination rate is limit by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration
p_{n0} (or n_{p0}) = equilibrium minority carrier concentration
τ_p = minority carrier lifetime



minority carrier lifetimes

p-type

$$n_p(t) = n_{excess} \exp(-t / \tau_n) + n_{p0}$$

minority carrier lifetimes

n-type

$$p_n(t) = p_{excess} \exp(-t / \tau_p) + p_{n0}$$

Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

drift: $\vec{j}_n = -ne\mu_n \vec{E}$ $\qquad \nabla \cdot \vec{j}_n = -en\mu_n \nabla \cdot \vec{E} - e\nabla n \mu_n \vec{E}$

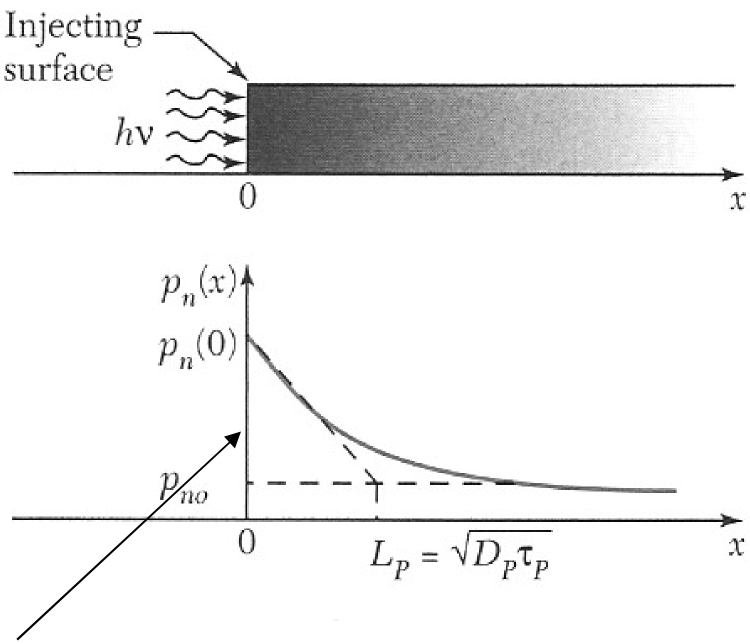
diffusion: $\vec{j}_{n,diff} = |e| D_n \nabla n$ $\qquad \nabla \cdot \vec{j}_{n,diff} = |e| D_n \nabla^2 n$

$$\frac{\partial n}{\partial t} = n\mu_n \nabla \cdot \vec{E} + \nabla n \mu_n \vec{E} + D_n \nabla^2 n + G_n - \frac{n - n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p\mu_p \nabla \cdot \vec{E} - \nabla p \mu_p \vec{E} + D_p \nabla^2 p + G_p - \frac{p - p_0}{\tau_p}$$

Diffusion Length

n-type



Steady state

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + \left(p_n(0) - p_{n0} \right) \exp\left(\frac{-x}{L_p} \right)$$

Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion Length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \qquad \Leftarrow \qquad p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

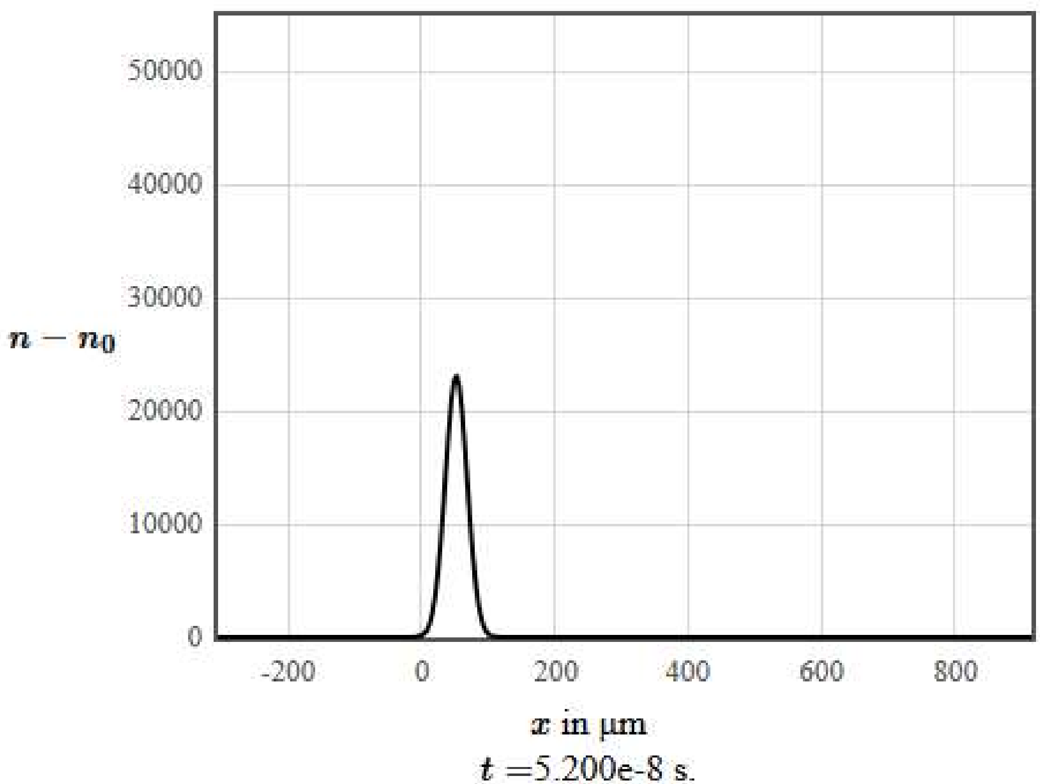
$$0 = \frac{D_p\left(p_n(0) - p_{n0}\right)}{L_p^2}\exp\left(\frac{-x}{L_p}\right) - \frac{\left(p_n(0) - p_{n0}\right)}{\tau_p}\exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

Haynes Shockley experiment

$$n_p(x,t) = \frac{n_{generated}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{(x-\mu_n E t)^2}{4 D_n t}\right) \exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



$\tau =$ 1E-6 [s]

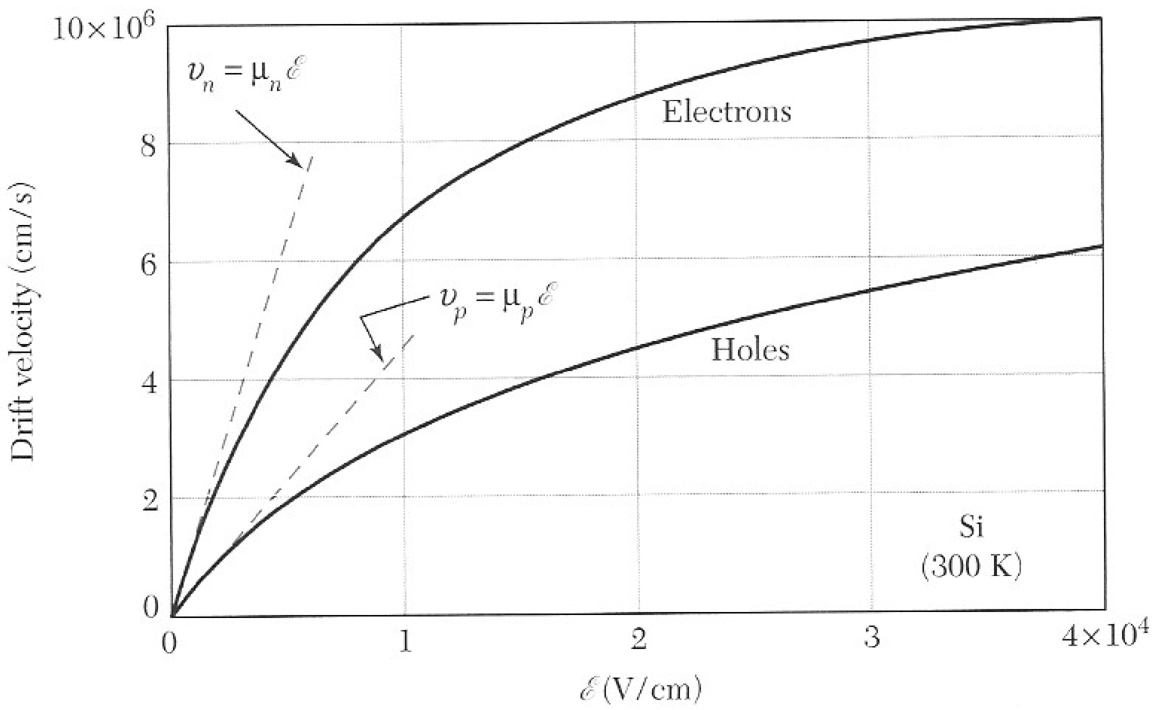
$E =$ 100 [V/cm]

$\mu =$ 1000 [cm²/V s]

$D = \mu k_B T/e =$ 0.00258 [m²/s]

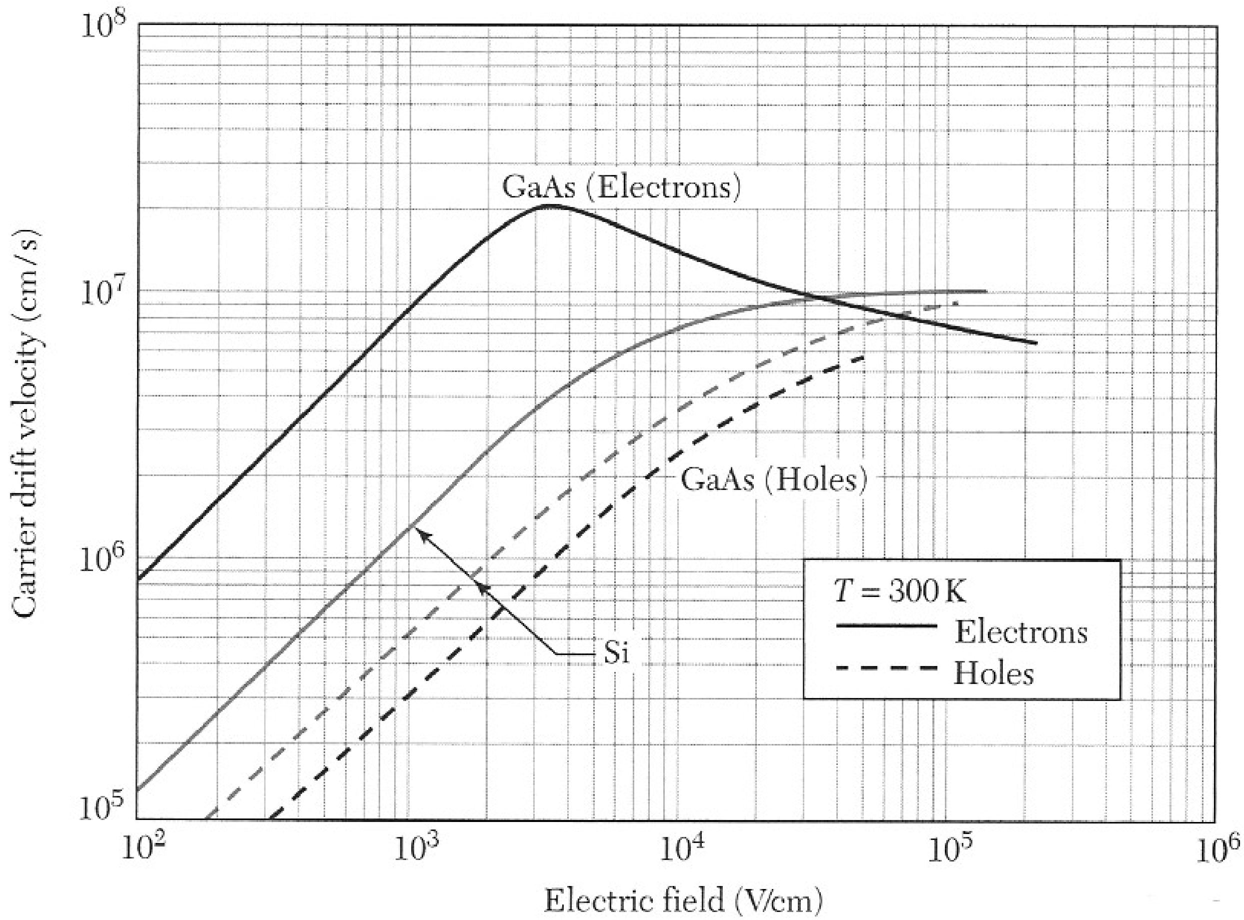
$L = \sqrt{D\tau} =$ 50.8 [µm]

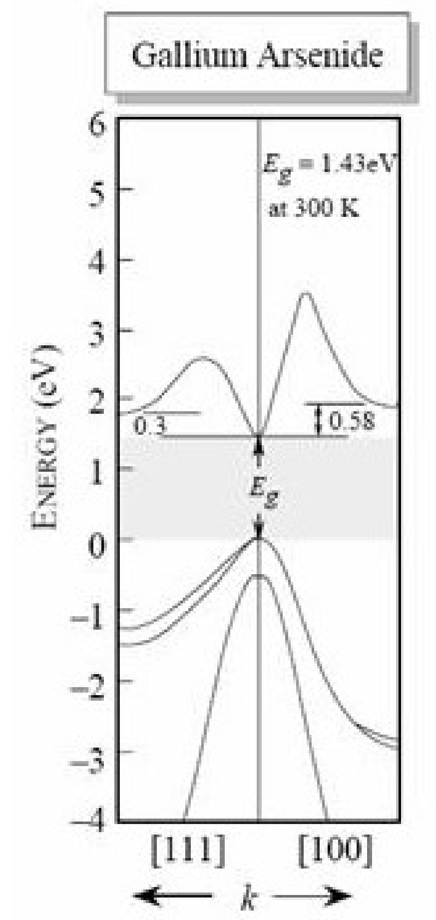
High Fields



Silicon

High Fields



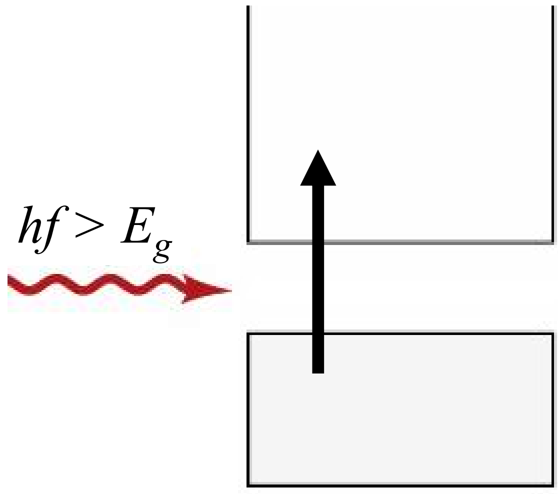


GaAs

Impact ionization

Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.

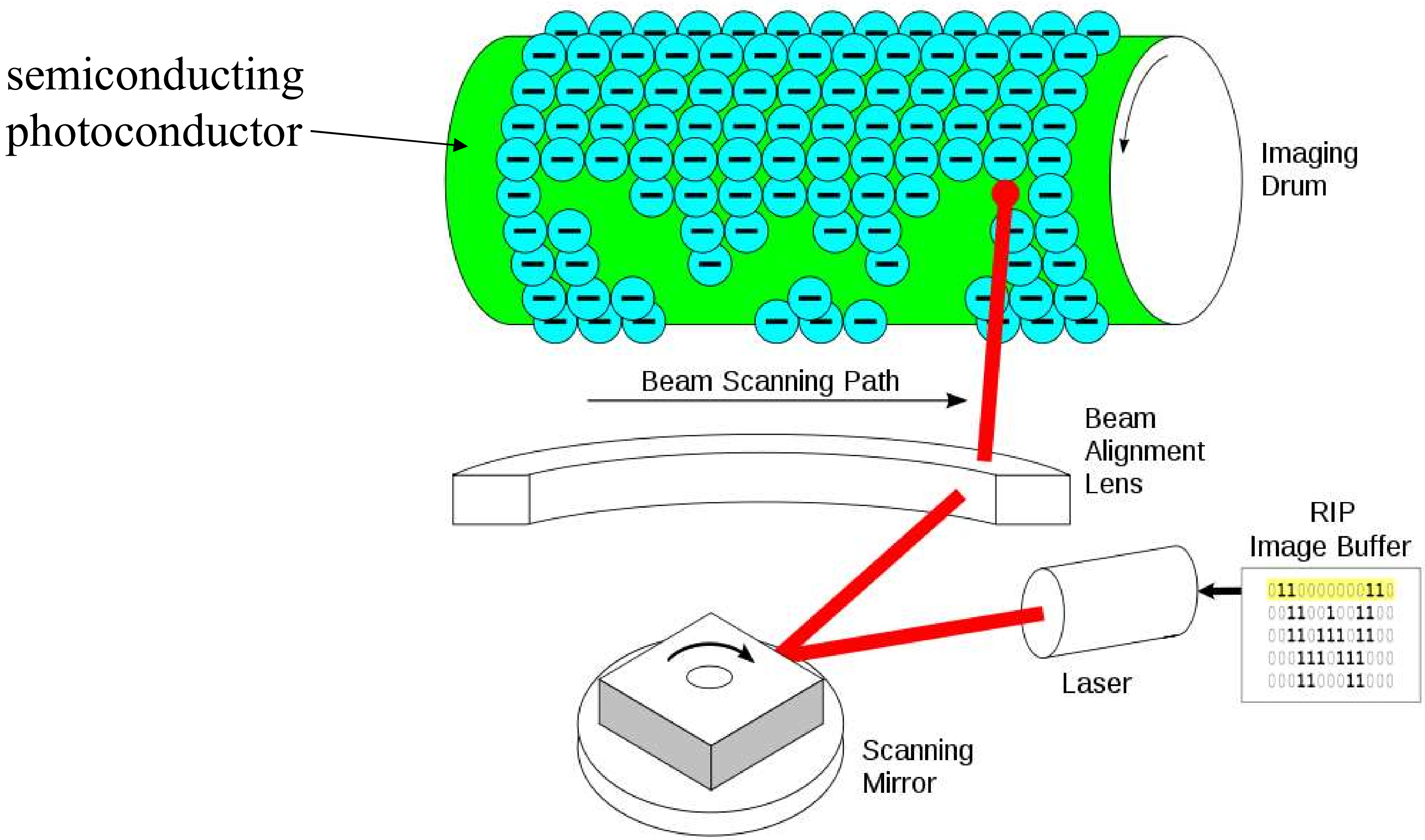
Photoconductivity



$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

Laser printer

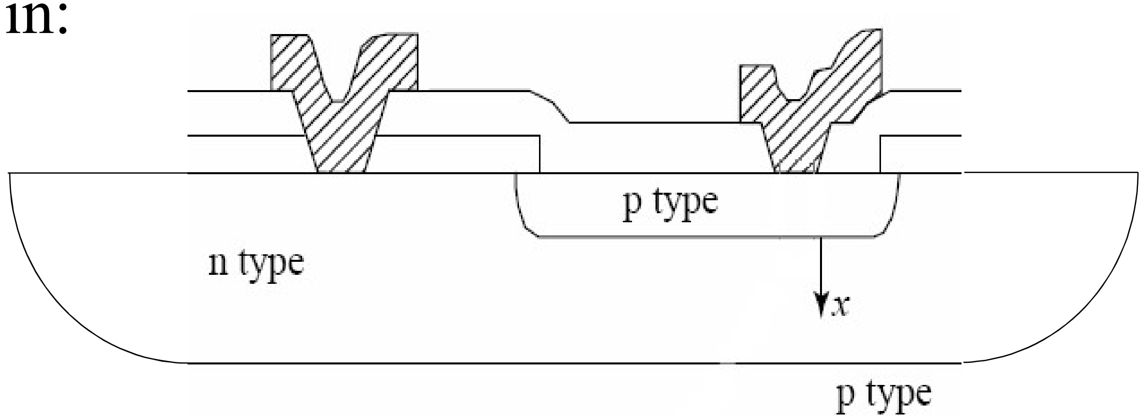


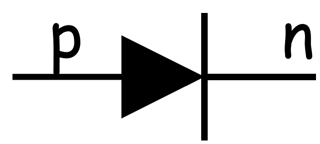
pn - Junctions

pn junctions

pn junctions are found in:

- diodes
- solar cells
- LEDs
- isolation
- JFETs
- bipolar transistors
- MOSFETs
- Lasers diodes

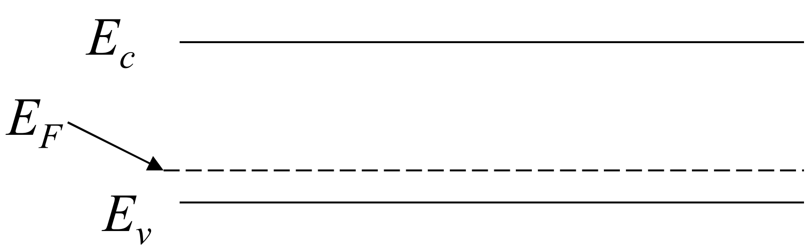




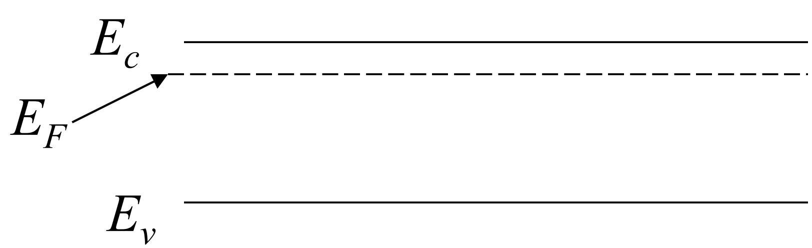
pn junction

isolated semiconductors

p-type



n-type



$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

valid for both n and p doping

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

pn junction

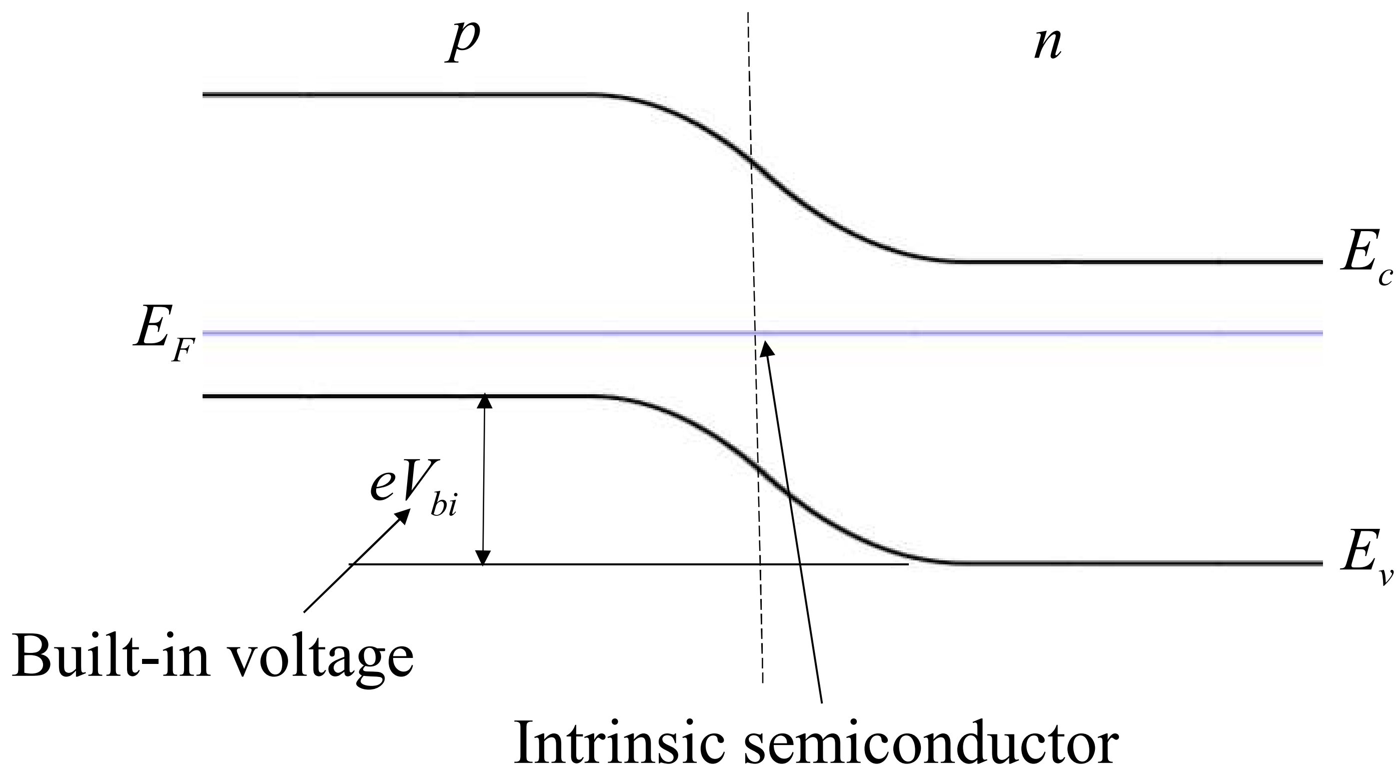
semiconductors in contact
electrons flow from n to p

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) \approx N_D$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right) \approx N_A$$

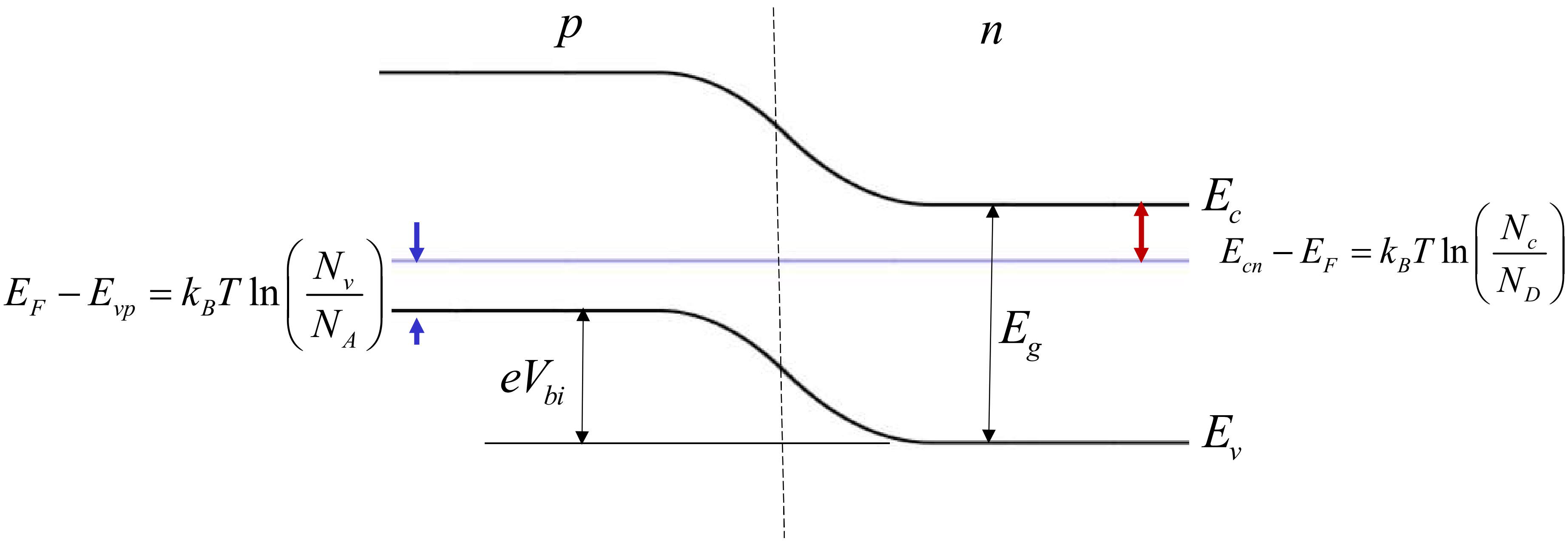
pn junction

semiconductors in contact



Abrupt junction: the doping changes abruptly from p to n

Built-in voltage V_{bi}



$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c}{N_{D,n} - N_{A,n}}\right) - k_B T \ln\left(\frac{N_v}{N_{A,p} - N_{D,p}}\right)$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}\right)$$

V_{bi}

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}\right)$$

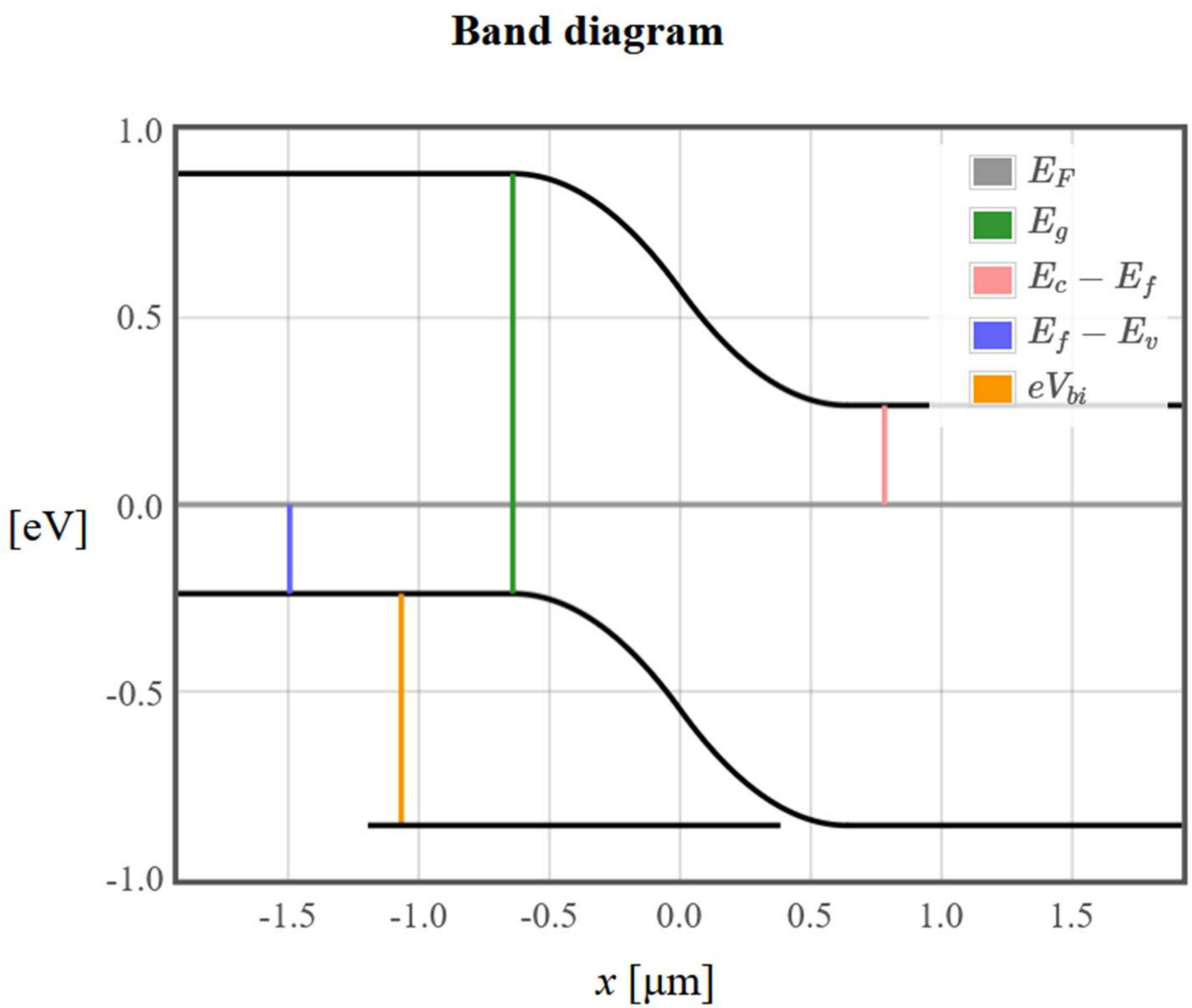
$$n_i^2 = N_v N_c \exp\left(\frac{-E_g}{k_B T}\right) \qquad E_g = -k_B T \ln\left(\frac{n_i^2}{N_v N_c}\right)$$

$$eV_{bi} = k_B T \ln\left(\frac{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}{n_i^2}\right)$$

for $N_{D,n}$ - $N_{A,n} = N_D$ and $N_{A,p}$ - $N_{D,p} = N_A$

$$\boxed{eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

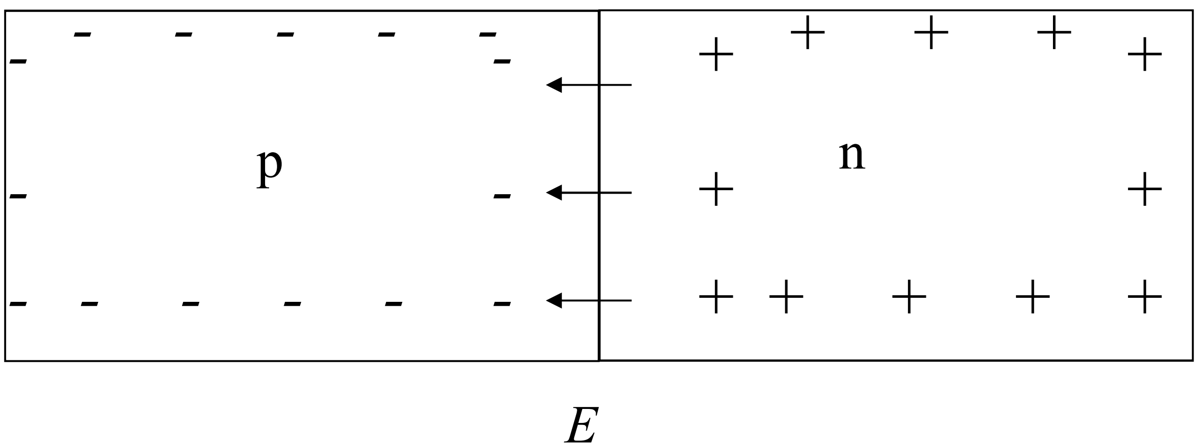
Depletion width



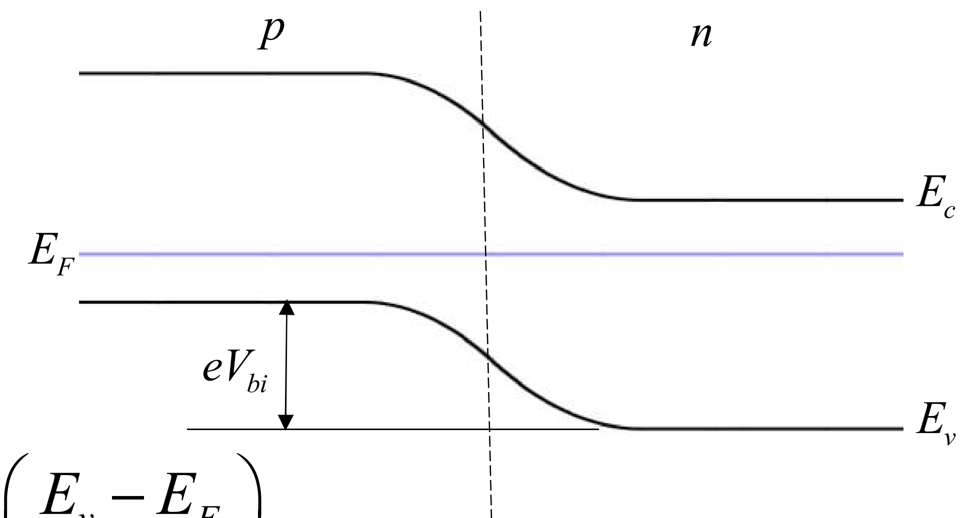
http://lampz.tugraz.at/~hadley/psd/L6/Vbi.html

V_{bi}

Can V_{bi} perform work?

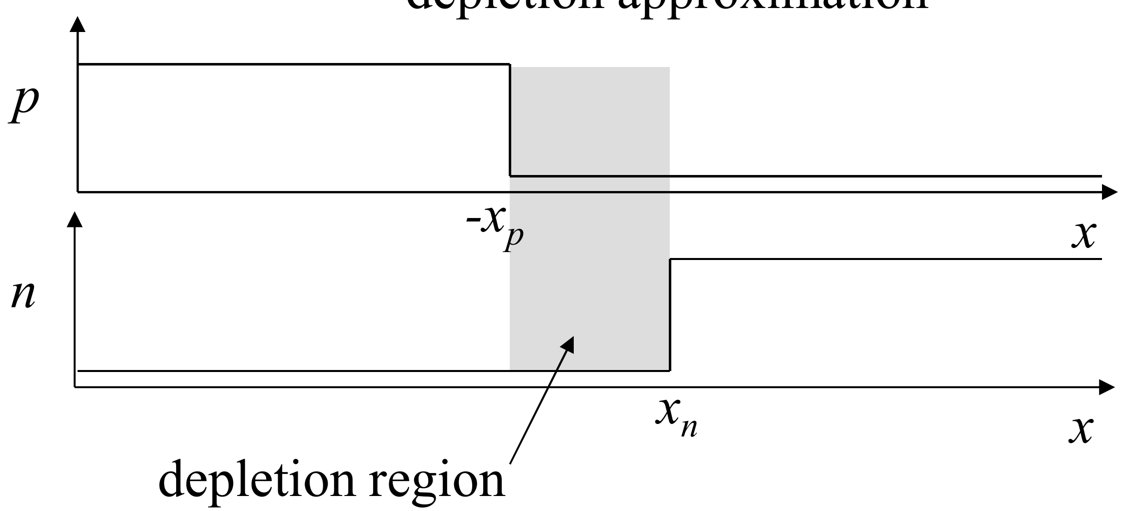


p and n profiles

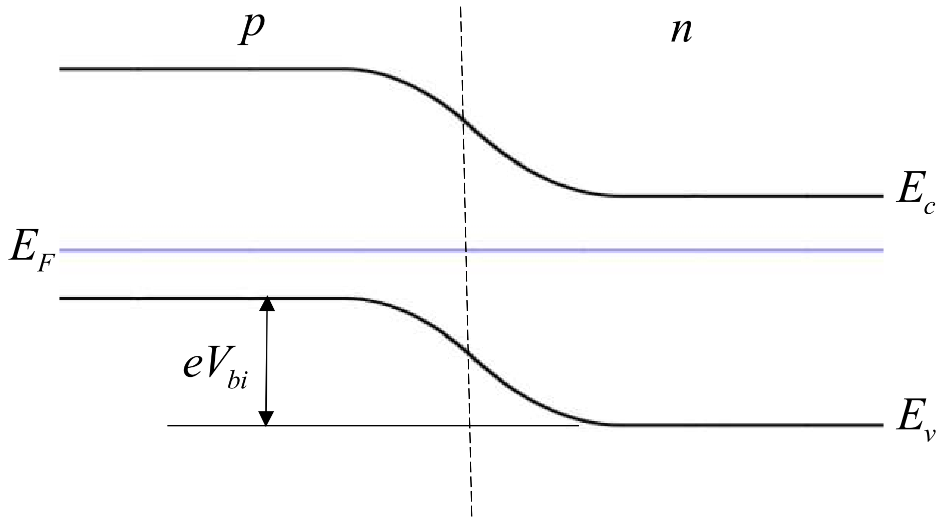


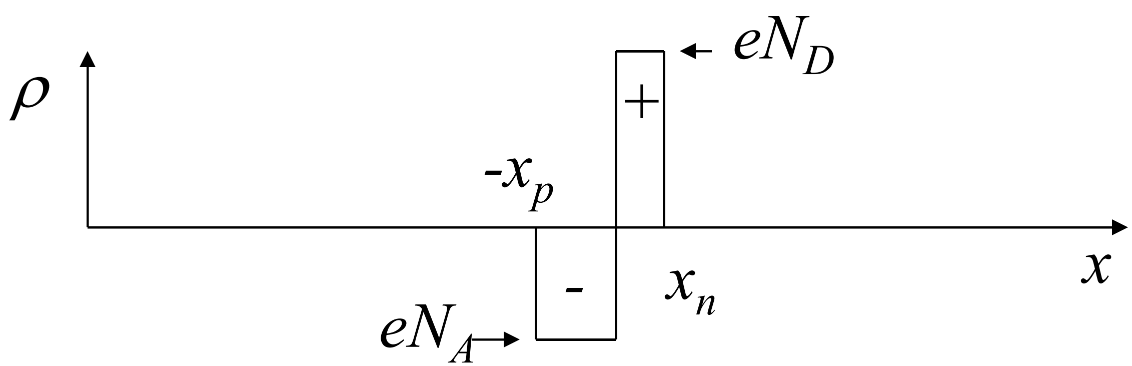
$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right) \qquad n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

depletion approximation



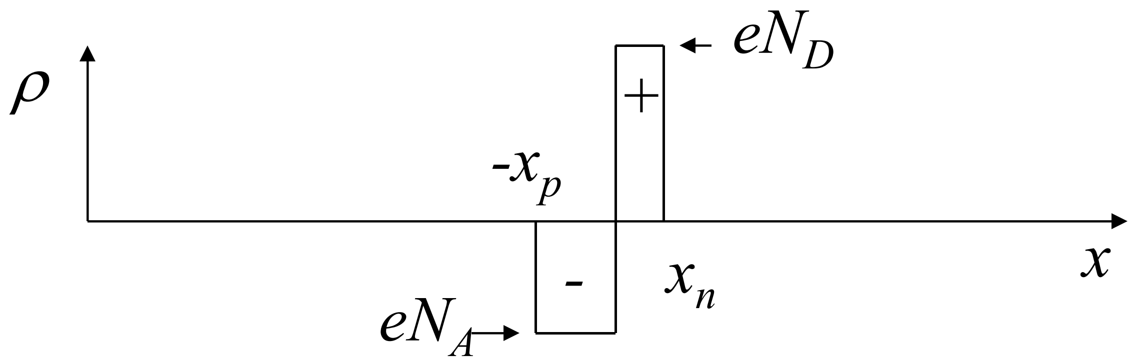
space charge



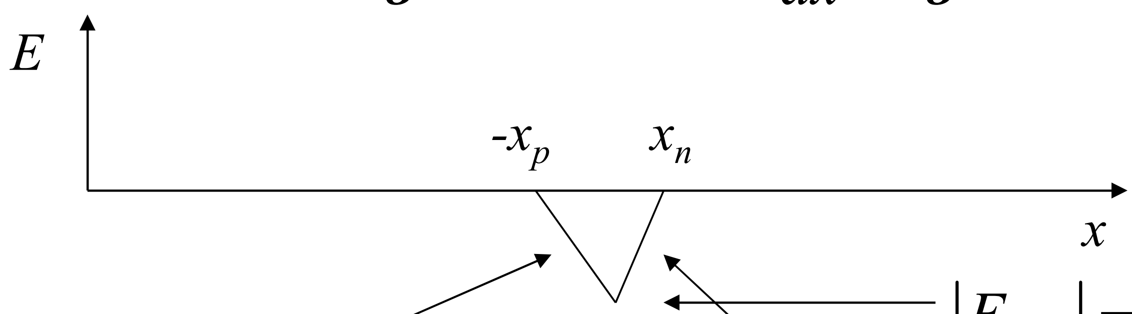


$$N_A x_p = N_D x_n$$

electric field



Gauss's law $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon}$ in 1-D is $\dfrac{dE}{dx} = \dfrac{\rho}{\varepsilon}$



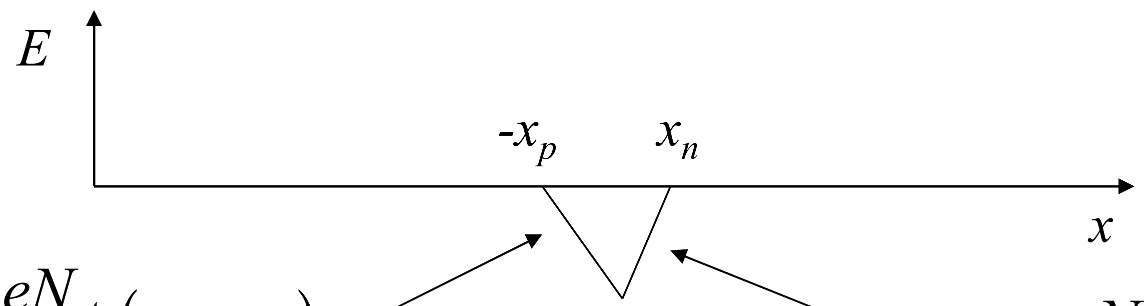
$$E = -\frac{eN_A}{\varepsilon}\left(x + x_p\right)$$

$$\left|E_{\max}\right| = \frac{eN_D x_n}{\varepsilon} = \frac{eN_A x_p}{\varepsilon}$$

$$E = \frac{eN_D}{\varepsilon}\left(x - x_n\right)$$

E pushes the holes towards *p* and the electrons towards *n*

potential



$$E = -\frac{eN_A}{\varepsilon}\left(x + x_p\right)$$

$$E = \frac{eN_D}{\varepsilon}\left(x - x_n\right)$$

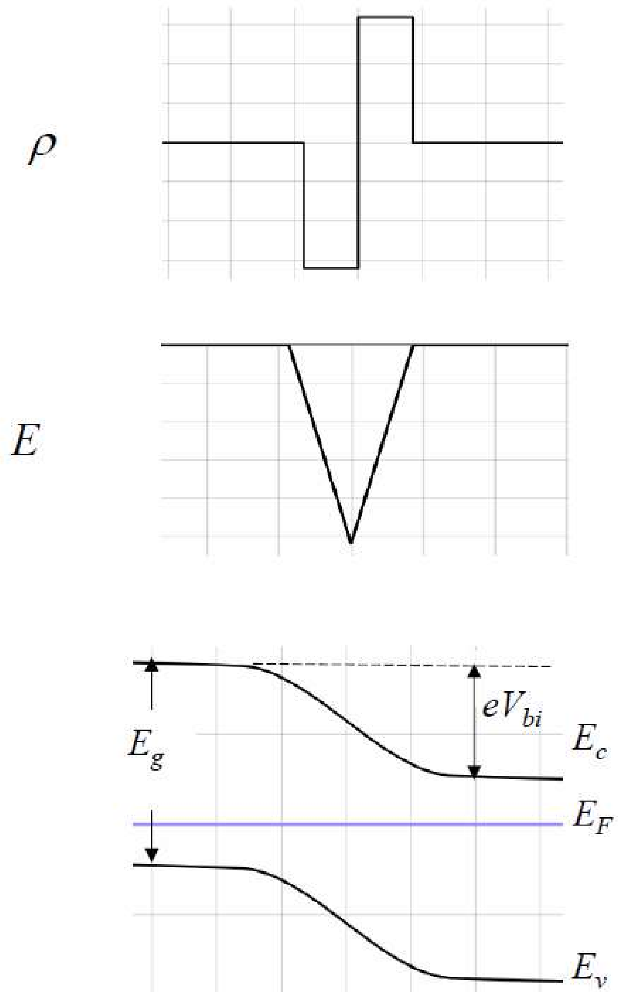
$$\frac{dV}{dx} = -E$$

$$V = \frac{eN_A}{\varepsilon}\left(\frac{x^2}{2} + xx_p\right) \qquad -x_p > x > 0$$

$$V = \frac{-eN_D}{\varepsilon}\left(\frac{x^2}{2} - xx_n\right) \qquad 0 > x > x_n$$

$$V(-x_p) = \frac{-eN_A}{2\varepsilon}x_p^2 \qquad V(0) = 0 \qquad V(x_n) = \frac{eN_D}{2\varepsilon}x_n^2$$

abrupt pn junction



$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= \frac{eN_A x_p^2}{2\varepsilon} + \frac{eN_D x_n^2}{2\varepsilon}$$

http://lampx.tugraz.at/~hadley/psd/L6/abrupt.html