Physics of Semiconductor Devices

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Project:
A Physicists Guide To The Exam

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Abstract

This article should provide answers to important questions for the exam of the lecture 'Physics of Semiconductor Devices'. It is mainly based on the questions given on the course homepage at the end of each chapter in the section 'For The Exam'. Additional information from the 'Problems' also given on the course homepage, have been used to complete this collection. However, I have to state clearly here that it is not enough to just study with this article. For this reason, for many questions references to the course notes are given, as well as references to Singh - the textbook I worked with - where more (detailed) information can be found. I gave some general notes at the beginning of each chapter, where information for the whole chapter can be found and some more explicit references at each question. Concerning the books, as stated above, I worked with Singh and hence only references to Singh are given in this article. However, three books have been suggested for the lecture:

- 'Sze - Physics of semiconductor devices' ([3]), which is pretty hard to get hold of, since there are no copies to be found in the lirbrary
- 'Thuselt - Physik der Halbleiterbauelemente' ([4]) a detailed semiconductor book in German, which can also be downloaded as an ebook.
- 'Singh - Semiconductor Devices' ([2]) a quite easy to read book about semiconductors, which does not go too much into detail, but provides all the information needed.

I hope this article provides some help to effectively study for this lecture.
Good luck! Guenter Krois

Chapter 1

Introduction

1.1 You need to know that everything moves like a wave and exchanges energy and momentum like a particle. The correspondence between the wave nature and the particle nature of something is given by the Einstein relation, $E = hf$, and the de Broglie relation, $p = h/\lambda$.

...

1.2 You need to be familiar with the solutions of Schroedinger equation for the three-dimensional potential well.

Literature References:Lecture Notes, Oct 06 2009/p17; see also Problem 1.3
You can make the answer to this question arbitrarily long and difficult, let's keep it simple. Basically we need to know the Schroedinger equation for one state:

$$i\hbar\frac{d}{dt}\psi_{(t)} = \hat{H}\psi_{(t)} \tag{1.1}$$

With the stationary solution

$$\hat{H}\psi_{(t_0)} = E\psi_{(t_0)}$$

where $\psi_{(t_0)} = e^{-\frac{i}{\hbar}E(t-t_0)}\psi_t$
Using the Hamiltonian for a 3D potential well: $\hat{H} = \frac{\hat{P}^2}{2m} + V(x) \overset{P=i\hbar\nabla}{\rightarrow} \hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + V_{(x)}$ the Schroedinger equation above becomes:

$$i\hbar\frac{d}{dt}\psi_{(t)} = (\frac{-\hbar^2}{2m}\nabla^2 + V_{(x)})\psi_{(t)} \tag{1.2}$$

The solution for this differential equation is:

$$\psi_{(t)} = \frac{\sqrt{2}}{L_x}\frac{\sqrt{2}}{L_y}\frac{\sqrt{2}}{L_z}e^{-i\omega t}\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi x}{L_y}\sin\frac{n_z\pi x}{L_z} \tag{1.3}$$

$$E_{n_x n_y n_z} = \hbar\omega = \frac{\hbar^2\pi^2}{2m}(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}) \tag{1.4}$$

Chapter 2

Crystals

General literature references: lecture notes Oct 6th 13th and 20th 2009; Singh Chapter 1

2.1 For the exam you need to know that the allowed electron energies in a crystal are arranged in bands. You need to know what the Fermi energy is and must be able to explain the difference between a metal, a semiconductor, and an insulator in terms of the energy bands and the Fermi energy.

Literature references: lecture notes Oct 6th 2009/p26-29; Singh p33-37
See scans of Singh p36/37 in Appendix A, fig A.1.

2.2 You must know what the Fermi function is and how to calculate whether an electron state is occupied.

Literature references: Problem 2.1 and 2.4; Singh p32
The Fermi function is a distribution function which gives the probability that an allowed energy level at is occupied. At $T = 0$ the fermi-function is a simple step function. Hence, in order to calculate whether an energy state is occupied or not, we simply need the equation for the fermi function (or more exactly the Fermi-Dirac distribution function):

$$f_E = \frac{1}{1 + e^{\frac{E-E_F}{k_B T}}} \tag{2.1}$$

2.3 A three dimensional potential well is called a quantum dot if the sides of the well are small. If the sides are large, it is also known as the free electron model. You must be able to express the energies in this case in terms of the wavenumbers k.

Literature references: Lecture Notes Oct 13th/p17-23; Problem 1.3
The equation for the energy has already been given above - see equation 1.4. Some relationships which might be helpful here are:

Wavevector: $k_x = \frac{2\pi}{\lambda} = \frac{n_x \pi}{L_x}$
De Broglie: $p = \frac{h}{\lambda} \rightarrow \vec{p} = \hbar\vec{k}$
If there is a question asking for the energy it takes to excite an electron from one energy level to another, make sure you calculate the energy difference $\Delta E = E_{n_2} - E_{n_1}$ between the two given levels.

2.4 You must know what the density of states is and the form of the density of states for the free electron model.

Literature references: Lecture Notes Oct 6th/p21+; Problem 2.3 and 2.4; Singh p25, derivation p509
'Density of states is the number of available elecronic states per unit volume per unit energy around an enery E' [2] The 1D-density of states is:

$$N_E = \frac{\sqrt{2} m^{3/2} E^{1/2}}{\pi^2 \hbar^3} \tag{2.2}$$

As can be seen the form of the 1D density of states is $y = a\sqrt{x}$ (see also bottom right of fig A.10 in Appendix A)
For the whole derivation see scans of Singh pages 509-513 in Appendix A - (fig A.9 and fig fig:Singh512DoSDerivat

2.5 In the expression $n = \int_{E_0}^{E_1} N_{(E)} F_{(E)} dE$ n is the density of electrons in the conduction band, N(E) is the density of states, F(E) is the Fermi function, E0 is the bottom of the conduction band and E1 is the top of the conduction band. You should be able to explain how the Fermi energy can be determined from this expression.

Literature references: Lecture Notes Oct 6th/p21+; Problem 2.3 and 2.4; Singh p25, derivation p509
The equations for the fermi function (2.1) and the density of states (2.2) are given, plugging the fermi-function in, we see that for T=0K it just changes the limits of the integral to $E_0 = 0$ and $E_1 = E_f$. The integral over the density of states is quite trival.
See Problem 'Average electron energy in a metal' in Appendix B.

2.6 You should now what Miller indicies are.

Literature references: Lecture Notes Oct 13th/p15; Problem 2.5; Singh p24
'Miller Indices are a scheme used to describe latice planes, directions and points. The following procedure is used to the the Miller indices of plane:

1. Define the x,y,z-axis
2. Take the intercepts of the plane along the axes in units of lattice constants.
3. Take the reciprocal of the intercepts and reduce them to the smallest integers, h,k, and l. The resulting numbers are called Miller indices.

The notation (hkl) denotes a family of paralles plaens, while the notation hkl denotes a family of equivalent planes.' [2]

2.7 You should be able to explain what the diffraction of electron waves has to do with the allowed electron energies in a semiconductor.

If an electron with a forbidden energy for a certain material is shot at such a material, it is reflected out again. This happens due to quantum effects and is explained by the solution for the Schroedinger equation. Sloser explanations can be found in Solid State Physics books such as 'Kittel - Solid State Physics'.

Chapter 3

Intrinsic Semiconductors

General literature references: lecture notes Oct 20th 2009; Singh Chapter 2

3.1 You need to be able to look at a bandstructure diagram for a semiconductor and be able to identify the conduction band, the valence band, the energy gap, whether the semiconductor is direct or in direct, and to be able to determine the effective masses of electrons and holes. See problems 1 and 2.

Literature references: Lecture Notes Oct 13th/p3+; Problem 3.5, 3.6;
See copy of problem 'Band structure' in Appendix B.
As for the effective mass, it can be easily derived, knowing $E = \frac{\hbar^2 k^2}{2m}$. Just derivate E twice with respect to k, since the effective mass is given by the curvature of E over k.

3.2 You need to be able to explain what a hole is.

Make sure you understood the concept of a hole well and that you are able to explain it with your own words. One well formulated answer would be: 'An electron hole is the conceptual and mathematical opposite of an electron, useful in the study of physics, chemistry, and electrical engineering. The concept describes the lack of an electron at a position where one could exist in an atom or atomic lattice' [5]

3.3 You need to be able to calculate the density of electrons in the conduction band, the density of holes in the valence band, and the position of the Fermi energy for an intrinsic semiconductor. See problems 3 and 4.

Literature references: Lecture Notes Oct 13th/p3+; Problem 3.5, 3.6; Singh 64-68
Density of electrons in the conduction band:

$$n = N_C e^{-\frac{E_C - E_F}{k_B T}}; N_C = 2(\frac{m_e^* k_B T}{2\pi\hbar^2})^{3/2} \tag{3.1}$$

Density of holes in the valence band:

$$n = N_V e^{-\frac{E_F - E_V}{k_B T}}; N_V = 2(\frac{m_h^* k_B T}{2\pi\hbar^2})^{3/2} \tag{3.2}$$

N_C and N_V respectively are the effective densities of states for electrons and holes. Knowing that in an intrinsic semiconductor the number of holes is equal to the number of electrons (since each electron in the condcution band leaves a hole in the valence band):

$$n = p = n_i^2 = N_C N_V e^{\frac{-E_g}{k_B T}}$$

one can easily calculate the position of the Fermi energy:

$$E_F = \frac{E_C + E_V}{2} + \frac{k_B T}{2} ln(\frac{N_V}{N_C}) \tag{3.3}$$

For a derivation of the first two equations see Singh or the lecture notes given in the references.

Chapter 4

Extrinsic Semiconductors

General literature references: lecture notes Oct 27th 2009; Singh Chapter 2

4.1 Be able to explain under what conditions donor and acceptor atoms become ionized.

Literature references: lecture notes Oct 27th/14; Singh p79
'Explanation for n-type: At very low temperatures all of the electrons are confined to the donor atom and thus can not carry any current. With rising temperature electrons are excited into the conduction band ('freezout range', linear increment of the electron density over temperature) until all of the donors are ionized - 'saturation range'. At some point the temperature gets high enough so that the intrinsic carrier density exceeds the donor density, hence the carrier density is further increased. ' [2]. See figure 4.1.

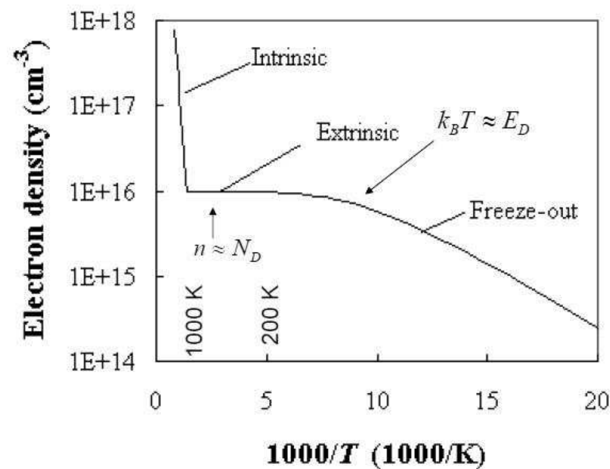

Figure 4.1: Electron density as a function of temperature.

4.2 You need to know how to calculate the concentration of electrons and holes for an extrinsic semiconductor.

Literature references: Lecture Notes Oct 27th/p27; Nov 11th/p7
See section 3.3

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D - N_A}$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A - N_D}$$

4.3 You need to know how to calculate the Fermi energy in an extrinsic semiconductor as a function of doping and temperature.

Literature references: Lecture Notes Oct 27th/p27; Nov 11th/p7
See section 3.3

$$E_F = E_V + k_B T ln(\frac{N_V}{N_A - N_D})$$

$$E_F = E_N + k_B T ln(\frac{N_V}{N_D - N_A})$$

These equations can be easily derived from the equations for n and p given in section 3.3 with the equations given in section 4.2.

4.4 Know what a degenerate semiconductor is.

A semiconductors so highly doped that they have conductivity characteristics of metals.
(http://en.wikipedia.org/wiki/Semiconductor)

Chapter 5

Transport

General literature references: lecture notes Nov 03th 2009; Singh Chapter 3 (p90-124)

5.1 Be able to calculate the response of electrons and holes to small electric fields.

Literature references: Lecture Notes Nov 3rd/p3-7,14-18,22-25; Singh p93-94, 104-107, 120-124
Several equations are important in this section:

Drift: Charge carrier movement due to electric fields.
At small electric fields material properties such as mobility and conductivity can be related to microscopic properties such as scattering rate or relaxation time. One way to do this is the Drude model, which makes the following assumptions:

- Electrons don't interact with each other.
- τ_{sc} is the mean time between successive collisions of electrons.
- In between collsisions, the electrons move according to the free electron equation: $\hbar dk/dt = \vec{F}$

'After a collision the electrons lose all their exces energy (on the average) so that the electron gas is essentially at thermal equilibirium. This assumption is really valid only at very low elecric fields.' [2] This means that an electron is only gaining speed in between collisions, due to the external field, which gives for the average velocity: (be carful, Singh denotes the electric field as $\vec{F}$, here it is denoted as $\vec{E}$)

$$\vec{v_F} = -e\vec{E} = m^* a = m^* \frac{d\vec{v_{avg}}}{dt}$$

$$\vec{v_{avg}} = -\frac{e\vec{E}\tau_{sc}}{m^*} = \vec{v_d}$$

with the **drift velocity** $\vec{v_d}$. For the current density we get:

$$\vec{J} = -ne\vec{v_d} = \frac{ne^2\tau_{sc}}{m^*}\vec{E}$$

comparing to Ohm's law $\vec{J} = \sigma\vec{E}$, we get for the conductivity:

$$\sigma = \frac{ne^2\tau_{sc}}{m^*}$$

and with the definition of mobility $\vec{v_d} = -\mu\vec{E}$, we get for the mobility:

$$\mu = \frac{e\tau_{sc}}{m^*}$$

Watch out for the signs, the hole drift velocity is $\vec{v_{d,p}} = \mu_p\vec{E}$ and the electron drift velocity is $\vec{v_{d,n}} = -\mu_n\vec{E}$.
If there are holes and electrons in the material, we get for the current density:

$$\vec{J} = -ne\vec{v_{d,n}} + ne\vec{v_{d,p}} = (ne\mu_n + ne\mu_p)\vec{E} = \sigma\vec{E}$$

Another important relationship is 'Matthiessen's Rule' which describes the relationship between the scattering times of the lattice and the impurities:

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

Diffusion: Charge carrier movement due to concentration gradients.
For a derivation see lecture notes of Nov 3rd p14.
For the current density of diffusion we simply get:

$$\vec{J_{n,diff}} = |e|\, D_n \frac{dn}{dx}$$

$$\vec{J_{p,diff}} = -\, |e|\, D_p \frac{dp}{dx}$$

where D_n and D_p are the diffusion constants for electrons and holes

$$D_n = \frac{k_B T \mu_n}{e};\ D_p = \frac{k_B T \mu_p}{e}$$

Summing up the effects of drift and diffusion we get the following current density equations:

$$\vec{J_n} = -ne\mu_n\vec{E} + eD_n\frac{dn}{dx}$$

$$\vec{J_p} = ne\mu_p\vec{E} - eD_p\frac{dp}{dx}$$

It is very important to understand the principles of drift and diffusion in order to understand the processes in a pn-junction, which is in turn essential to understand any semiconductor device.

5.2 Know that the drift velocity saturates at fields above about 1000 V/cm.

see next question

5.3 Know when the low field formulas are not valid and what processes occur at high electric field strengths.

Literature references: Lecture Notes Nov 3rd/p1; Singh p93

For low electric fields the carrier movement is described by the Drude model (doesn't take effective mass into account though). The assumptions taken for this model are givenin section 5.1

'According to these assumptions, immediately after a collision the electron velocity is the same as that given by the thermal equilibrium conditions. This average velocity is thus zero, after collisions. The electron gains a velocity in between collisions, i.e., only for the time τ_{sc}.' [2]

Singh p98 High Field Transport: At high electric field ($F\ 1 - 100kV/cm$) the electrons acquire a high average energy. The drift velocities are also quite high. As the charge carriers gain energy from the field, they suffer greater rates of scattering, i.e., τ_{sc} decreases. The mobility thus starts to decrease. Hence, the drift velocity becomes saturated at high electric fields.

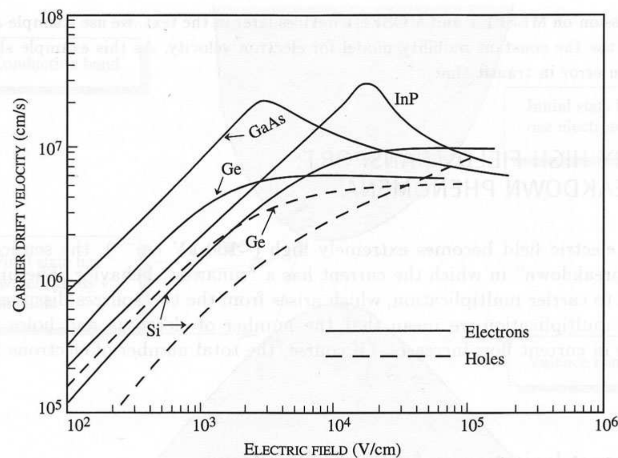

Figure 5.1: Velocity-field realations for several semiconductors at 300K. Note how the drift velocity of silicon slowly saturates, while the drift velocity of GaAs peaks first and saturates after decreasing again. (from [2])

Singh p100 Very High Field Transport - Breakdown Phenomena: At $E > 100kV/cm$; the semiconductor suffers a 'breakdown' caused by two effects which multiply the carrier density:
Impact Ionization: Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.
Band to Band Tunneling: Zener Tunneling: At strong electric fields, the electrons in the valence band can tunnel into an unoccupied state in the conduction band.

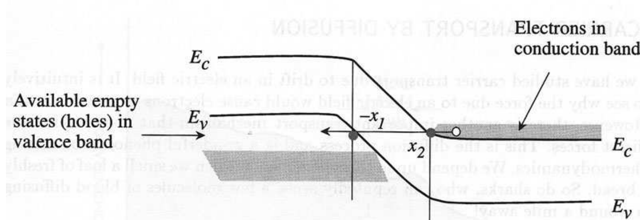

Figure 5.2: Zener tunneling in a pn-junction (from [2])

5.4 Be able to explain drift and diffusion. Know the transport equations where the current density is described by a drift term and a diffusion term.

Literature references: Singh p92-94,104
For the equations see section 5.1

Chapter 6

p-n junction

General literature references: lecture notes Nov 10th and 17th 2009; Singh Chapter 5 (p159-179)
Make sure that you really understand the pn-junction and there processes therein and that you can explain in detail. It is very important for the oral exam, since you will need it to explain any semiconductor device properly. The processes in a pn-junction are way easier to explain if you are able to draw the corresponding graphs properly. Have a look at Singh p161/162 (included in Appendix A - fig A.2)

6.1 Be able to calculate the contact potential.

Literature references: lecture notes Nov 10th/10,11; Singh p163
See lecture notes and Singh for derivation - you should know this derivation!

$$eV_{bi} = k_B T ln(\frac{N_D N_A}{n_i^2})$$

6.2 Know the field, the potential, and the charge distribution in the depletion model of a p-n junction.

Literature references: lecture notes Nov 10th 10-20; Singh 160+; Problem 6.1

All these parameters can be derived by the depletion approximation discussed in the references given above. Since the derivation is very important, I included the scans of the derivation (see figurees 6.1, 6.2 and 6.3). Please make sure that you also understand the theory behind this approximation. The depletion approximation simplifies the calculation of important properties of the pn-junction such as depletion width, charge distribution, electric field, etc. The following assumptions are taken in this approximation:

1. 'The physical junction is abrupt and each side is uniformly doped.' [2]
2. The mobile charge density in the depletion region is very small and thus will be assumed zero, of course only for the solution of the Possion equations, not for the calculation of the current flow.

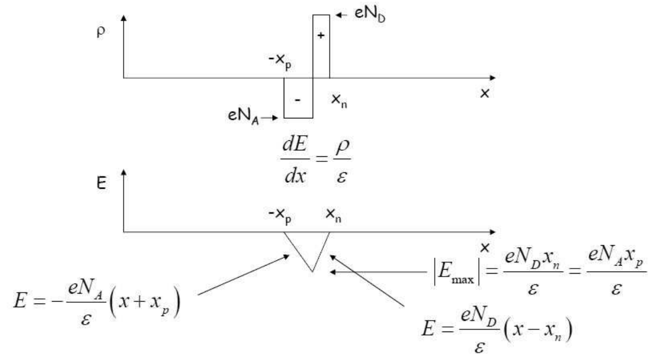

Figure 6.1: Depletion approximation 1 - Charge distribution and electric field in a pn-junction; Lecture notes of Nov 10th p15

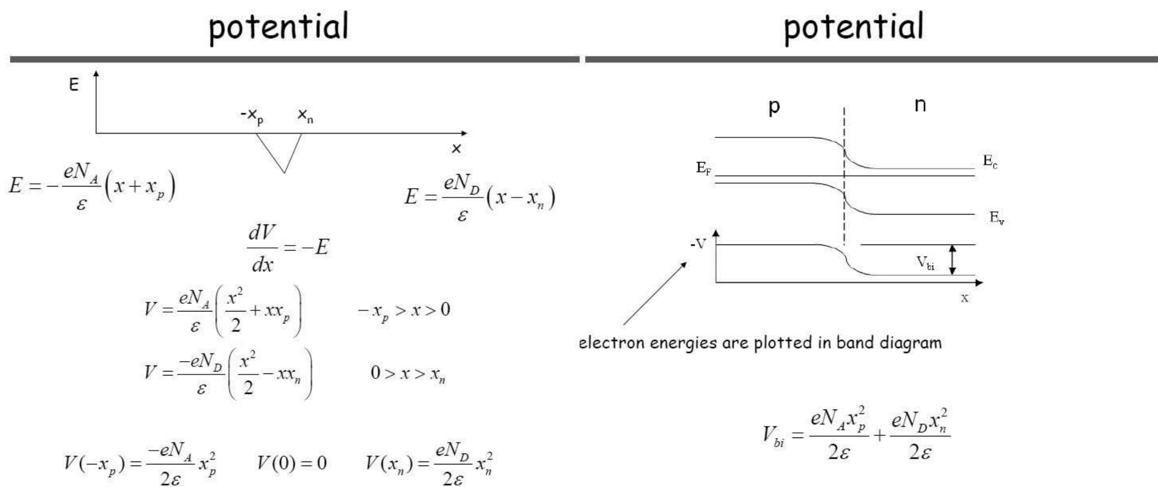

Figure 6.2: Depletion approximation 2 - Potential in a pn-junction; Lecture notes of Nov 10th p18/19

6.3 Be able to calculate the width of the depletion layer.

This question basically belongs to the one above, since the depletion width is 'covered' by the derivation of the depletion approximation.

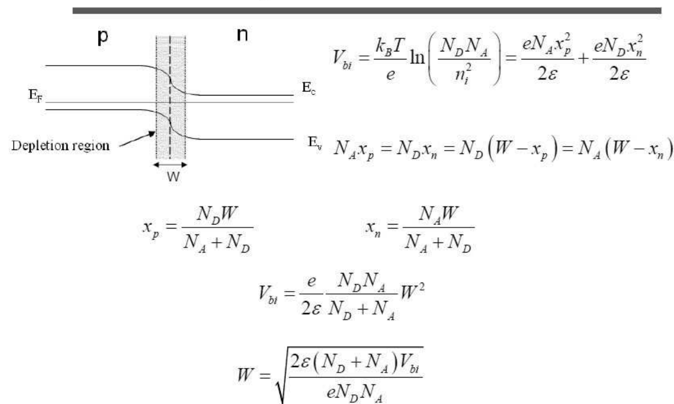

Figure 6.3: Depletion approximation 3 - Depletion width in a pn-junction; Lecture notes of Nov 10th p20

6.4 Be able to explain carrier generation and recombination in a pn-diode. Know the equations for drift and diffusion of electrons and holes.

Literature references: lecture notes Nov 17th 10-12; Singh 113

The processes how carriers are generated and how they recombine should be very clear to you anyway. The necessary equations are given in the next question.

6.5 Be able to derive the diode equation $I = I_0(e^{\frac{eV}{k_B T}} - 1)$ and know when this approximation is not valid.

Literature references: lecture notes Nov 17th p10-14; Singh 174-178

This is another rather long derivation which should be known. However, as with every derivation, as soon as the basic principles are clear it is pretty easy. I found it pointless to type the whole thing, so I just included the according copies of the lecture notes and added some notes for a few lines which might be unclear.

We start the whole derivation with calculating the diffusion current for minority carriers in a n-type semiconductor (the calculation for the p-type is completely analogue)

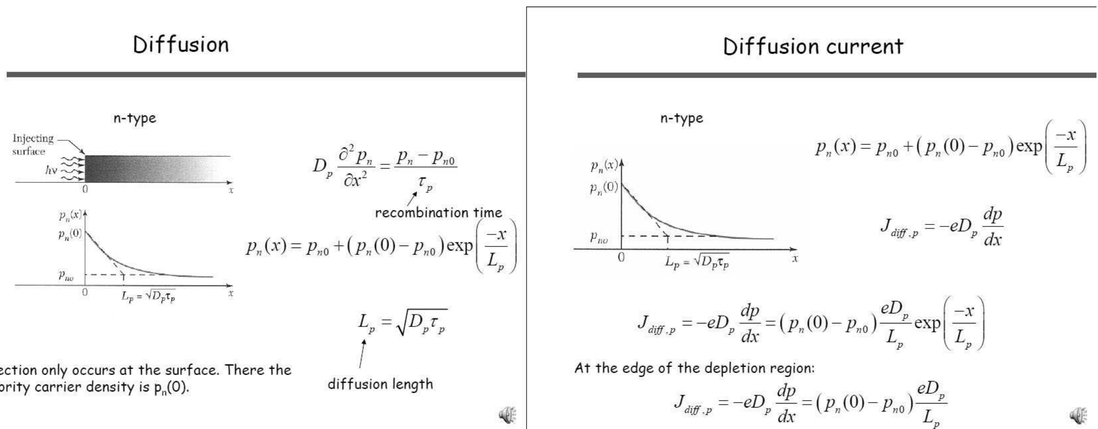

Figure 6.4: Derivation of the diode current 1; Lecture notes of Nov 17th p10,11

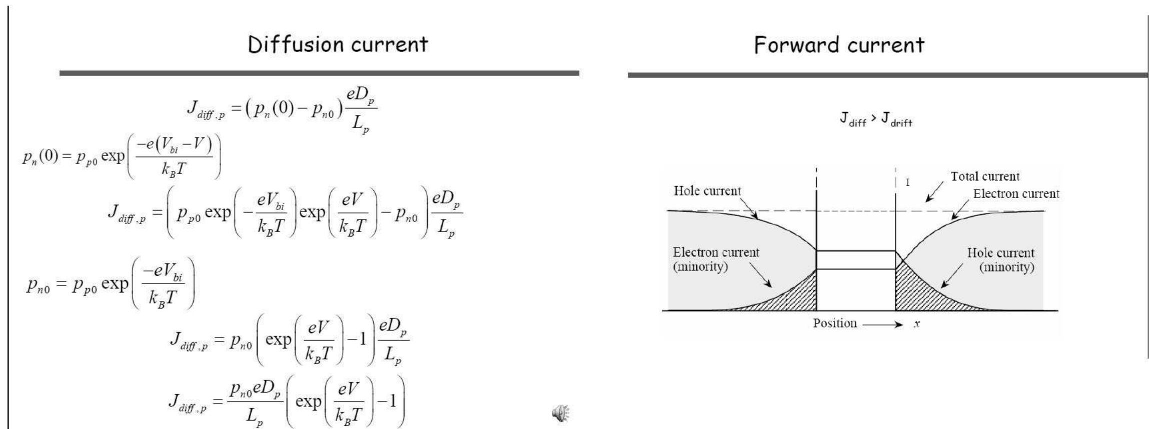

Figure 6.5: Derivation of the diode current 2; Lecture notes of Nov 17th p11,12

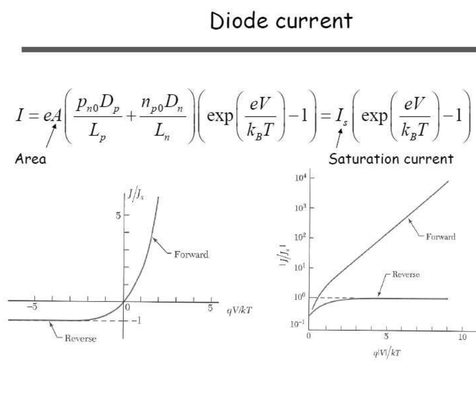

Figure 6.6: Derivation of the diode current 3; Lecture notes of Nov 17th p14

In the left part of figure 6.5 it might be unclear where the $p_n(0)$ and p_{n0} parts come from. They simply come from our derivation of the built-in potential. As given in section 6.1 the built-in

potential is:

$$eV_{bi} = k_B T ln(\frac{N_D N_A}{n_i^2})$$

Knowing the law of mass action: $n_n n_p = p_n p_p = n_i^2$ we can rewrite the built in potential to:

$$V_{bi} = \frac{k_B T}{e} ln(\frac{p_p}{p_n});\ V_{bi} = \frac{k_B T}{e} ln(\frac{n_n}{n_p})$$

and hence:

$$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{\frac{eV_{bi}}{k_B T}}$$

For our derivation we need to know that $p_n(0) = p_p e^{\frac{eV_{bi}-V}{k_B T}}$ since there is a voltage applied to the pn-jnction and $p_{n0} = p_p e^{\frac{eV_{bi}}{k_B T}}$ for the intrinsic minority carrier density.

6.6 Be able to explain what happens to the depletion widths, and the drift and diffusion currents for forward and reverse bias.

Literature references: lecture notes Nov 17th p7,8; Singh 170,171
This is also a very important question for the understanding of a pn-junction. It is again easier to understand and explain this question when you are able to draw the accoring graph (see references).

Chapter 7

Contacts

General literature references: lecture notes Nov 24th 2009; Singh Chapter 6

7.1 Be able to describe a metal-semiconductor junction. Known under what conditions a Schottky barrier is formed, when an ohmic contact is formed, and when a tunnel contact is formed.

Literature references: lecture notes Nov 24th p3; Singh p227-236,240-243
Here it is again important for the understanding (and to be able to explain it properly) to be able to draw the band diagrams with all important parameters properly.

7.2 Be able to explain how a junction can be more transparent for electrons than for holes.

7.3 Know the relationship between current and voltage in a Schottky contact.

Literature references: lecture notes Nov 24th p15; Singh p235-236
Basically we get the same relationship as width diodes in section 6.5

$$I = I_{sm} + I_{ms} = I_s(e^{\frac{eV}{k_B T}})$$

7.4 Be able to explain the role of interface states.

Literature references: lecture notes Nov 24th p12; Singh 228
'According to the discussion so far, the Schottky barrier height for n- or p-type semiconductors depends upon the metal and the semiconductor properties. However, it is found experimentally that the Schottky barrier height is usually independent of the metal employed. This can be understood qualitatively in terms of a model based upon nonideal surfaces. In this model the metal-semiconductor interface has a distribution of interface states that may arise from the presence of chemical defects (e.g., an oxide film) or broken bonds, etc. The defect region leads to a distribution of electronic levels in the bandgap at the interface. The distribution may be characterized by a neutral level ?0

having the properties that states below it are neutral if filled and above it are neutral if empty. If the density of bandgap states near ?0 is very large, then addition or depletion of electrons to the semiconductor does not alter the Fermi level position at the surface and the Fermi level is said to be pinned. In this case, the Schottky barrier height is e [e?b= Eg - e?0] and is almost independent of the metal used. The model discussed above provides a qualitative understanding of the Schottky barrier heights. However, the detailed mechanism of the interface state formation and Fermi level pinning is quite complex.' [2]

Chapter 8

JFET

General literature references: lecture notes Dec 1st 2009; Singh Chapter 8 (p338-349)

8.1 Be able to describe the operation principle of JFETs and MESFETs.

Literature references: lecture notes Dec 1st; Singh p338-342

8.2 Know how to calculate the pinch-off voltage.

Literature references: lecture notes Dec 1st p2,3; Singh p341

$$V_p = \frac{eN_D h^2}{2\epsilon}$$

If you compare that with the results for the field in the derivation of section 6.2 you will realize that V_p is just the field at $x_n = h$. The depletion width is, as above:

$$x_n = \sqrt{\frac{2\epsilon(V_{bi} - V)}{eN_D}}$$

8.3 Be able to calculate the drain current for a JFET or MESFET. Know the what the linear regime and the saturation regimes are.

Literature references: lecture notes Dec 1st p9-12; Singh Chapter 8 (p346-348)
This is a quite long derivation. I talked with Prof. Hadley about it and he said we do not have to know the whole derivation by heart, but we should know what is going on in the derivation and be able to make use of it. So have a look at it, either in the lecture notes or in Singh and try do understand what is happening and how the derivation works, you don't have to remeber all the equations. The principle is pretty simple actually.

Chapter 9

MOSFET

General literature references: lecture notes Dec 15th 2009 and Jan 12th 2010; Singh Chapter 9 (p374-384; 396-400)

9.1 Be able to describe a MOS capacitor in terms of the flatband voltage, threshold voltage, acuumulation, depletion, and inversion.

Literature references: lecture notes Dec 15th; Singh p374-384

9.2 Draw the charge density, electric field, and electrostatic potential as a function of position in a MOS capacitor.

Literature references: lecture notes Dec 15th p14-18; Singh p381
See Scans of Singh Appendix A, fig A.8

9.3 Be able to draw the band diagrams for a nMOS capacitor or a pMOS capacitor.

Literature references: lecture notes Dec 15th p5-9; Singh p378,380
See Scans of Singh Appendix A, fig A.6 and fig A.7.
This page might also prove helpful:
http://jas.eng.buffalo.edu/education/mos/mosCap/biasBand10.html - Java Animation of the band diagrams

9.4 Know how the equation for the drain current in the linear regime is derived.

Literature references: lecture notes Jan 12th p6-9; Singh p396,397
As in section 8.3 it is important to understand how the derivation works, in this case how the gradual channel approximation works, not to learn the whole derivation by heart.

What the gradual channel approximation is, is explainend in Singh p395:
'In the absence of any source-drain bias, the depletion width is simply given by the 1D model we developed fo the pn diode. However, strictly speaking, when there is a source-sraind bias, one has to solve a 2D problem to find the depletion width and, subsequently, the current flow. In the gradual channel approximation, we assume that the field in the direction from the gate to the substrate is much stronger than from the source to the drain, i.e, the potential varies 'slowly' along the channel as compared to the potential variation in the direction from the gate to the substrate. Thus the depletion width, at a point along the channel, is given by the potential at that point using the simlpe 1D results. This approxmation is good if the gate length L is larger than the channel depth, which is typically a few hundred angstroms.'

9.5 Be able to explain 'pinchoff' and what controls the drain current in the saturation regime. Know the equation for the drain current in the saturation regime.

Literature references: lecture notes Jan 12th p10; Singh p396,397
This question belongs to the one above.

9.6 Know why the simple model for MOSFETs is inadequate to describe very small MOSFETs.

Literature references: Singh p395
With all the current calculations (for JFET, MESFET and MOSFET) two approximations have been made, firstly the mobility of the electrons is regarded as independent of the electric field, as explained in section 5.3. This is, however, only valid for very small electric fields. As modern MOSFETs are very small, the fields acting in these devices are quite high, so this approximation is not valid anymore.
The second approximation is the gradual channel approximation, as explained in the section 9.4.

9.7 Be able to draw the electric field as a function of position along a cross section from the gate, through the oxide and into the substrate for various body voltages. Know how changing the body voltage modifies the drain current.

Literature references: Singh p381,399,400

Chapter 10

Bipolar

General literature references: lecture notes Jan 19th 2010; Singh Chapter 7 (p260-278)

10.1 Know the forward active, reverse active, cut-off, and saturation operation modes of a bipolar transistor.

Literature references: lecture notes Jan 19th p17; Singh p266

10.2 Know how the emitter, base, and collector are doped and why.

Literature references: Singh p260-266

10.3 Be able to explain how a bipolar transistor works and why the base must be thin.

Literature references: Singh p260-266

Chapter 11

Opto-electronics

General literature references: lecture notes Jan 26th 2010; Singh Chapter 11

11.1 Be able to describe how a light emitting diode, solar cell, and a laser diode work. Know what determines the color of a LED or laser.

Literature references: lecture notes Jan 26th 2010; Singh Chapter 11
If you thoroughly understood the pn-junction, you will only have to learn a few more details to be able to explain all of these 3 devices. Detailed information can be found in Singh:

- Optical processes: p458-463
- Solar cell: p465-468
- LED: p472-479
- Semiconductor laser: p482-491

11.2 Be able to explain total internal reflection and how it is used in an optical fiber and why it can be a problem coupling light out of an LED.

Literature references: lecture notes Jan 26th p9
Total internal reflection is the effect of light being completely reflected back if it travels from a material with a high index of refraction to one with a lower index of refraction (e.g. from water to air). This effect is used to keep light in a fibre wire, however for LEDs it is a disadvantage since, the photons which are produced in the LED are spread out in all possible directions, many travel back in the semiconductor material, create some electron-hole pairs in the base material and their light is transformed to heat energy and thus lost. Other photons reach the surface of the LED, but total internal reflection causes them to be reflected back into the material. In order to make the efficiency of an LED better several steps can be taken. Firstly the surface of the LED can be roughened up, in order to decrease the probability for total internal reflection to occur. Secondly the base material should be chosen to have a higher band gap than the material producing the light, thus photons

reflected back will not be able to be absorbed by the material, if some reflective materials are applied at the very base of the LED the photons can even be reflected back out again.
For more information on total internal reflection see:
http://en.wikipedia.org/wiki/Total_internal_reflection for more information

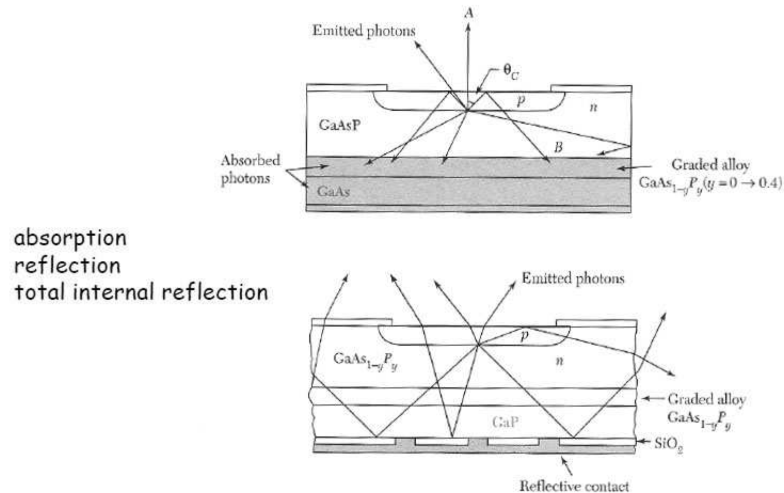

Figure 11.1: Lecture notes of Jan 26th p13, reflection, absorption and total internal reflection in a LED.

11.3 Know the difference between spontaneous emission and stimulated emission. Know why a laser diode has a threshold current.

Literature references: Singh p481,490
The first part should be known, the treshold current occurs, because if the current is too low 'the number of electrons and holes injected is small. As a result, the gain in the device is too small to overcome the cavity loss.' [2]

11.4 Be able to describe light absorption in a semiconductor and what this has to do with the bandgap.

Literature references: Singh p458-463

Chapter 12

Appendix

A Very important pages of Singh - Scans

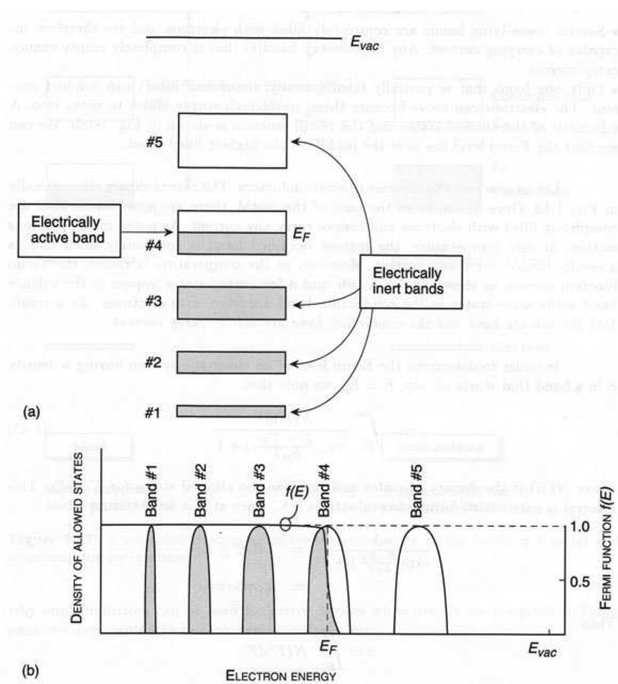

Figure 1.18: (a) Allowed energy bands and filling of bands by electrons in metals. (b) A schematic of allowed bands and density of states in a metal. In the example shown, Band #4 is the conduction band.

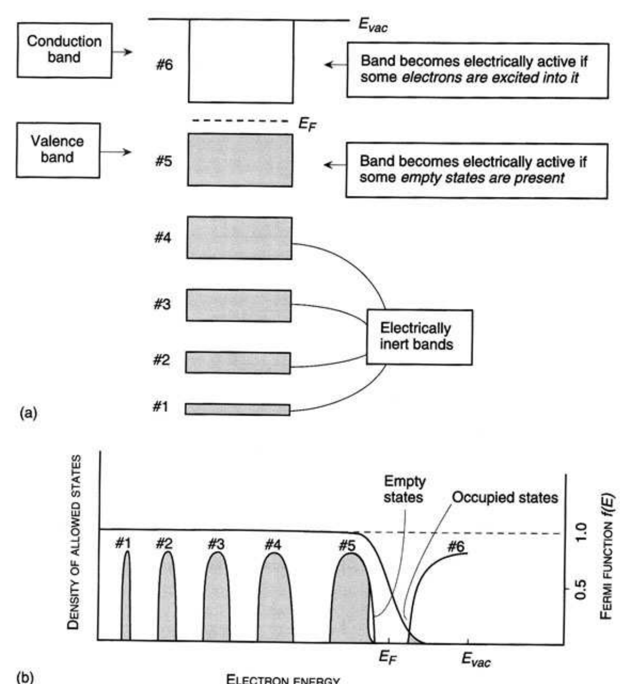

Figure 1.19: (a) Allowed energy bands and filling of bands in a semiconductor. (b) Density of allowed states in a semiconductor. Here, Band #5 is the valence band and Band #6 is the conduction band.

Figure A.1: Left: Banddiagram and Fermi Energy for a metal. Right: Banddiagram and Fermi Energy for a insulator/semiconductor. (from [2])

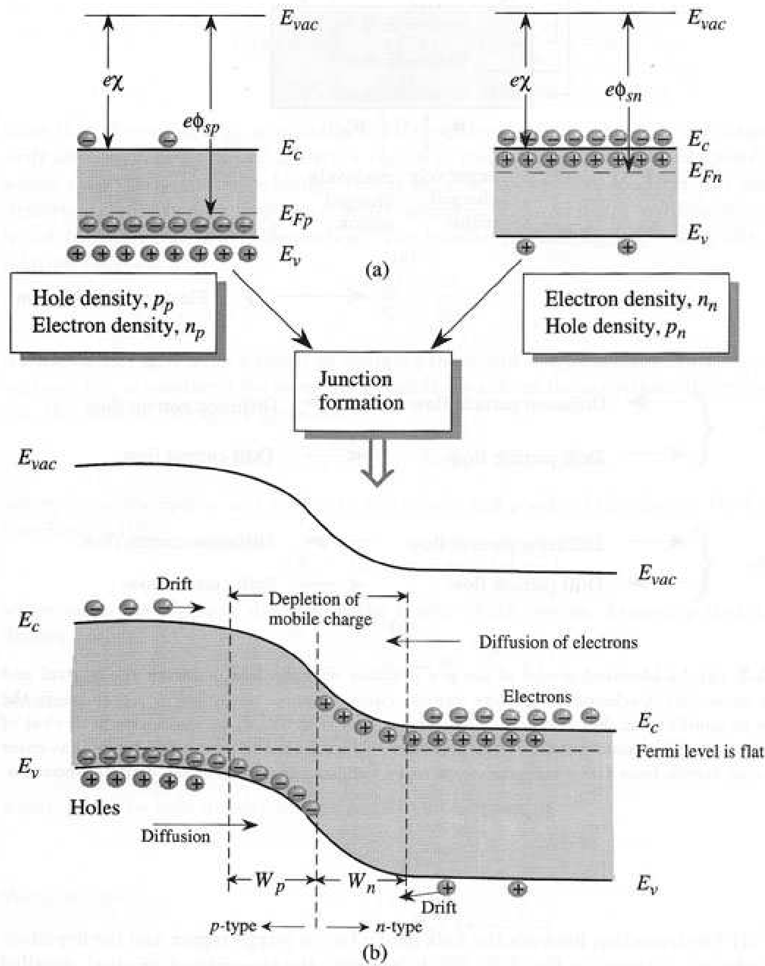

Figure 5.1: (a) The p- and n-type regions before junction formation. The electron affinity $e\chi$ and work functions $e\phi_{sp}$ and $e\phi_{sn}$ are shown along with the Fermi levels. (b) A schematic of the junction and the band profile showing the vacuum level and the semiconductor bands. The Fermi level is "flat" in the absence of current flow.

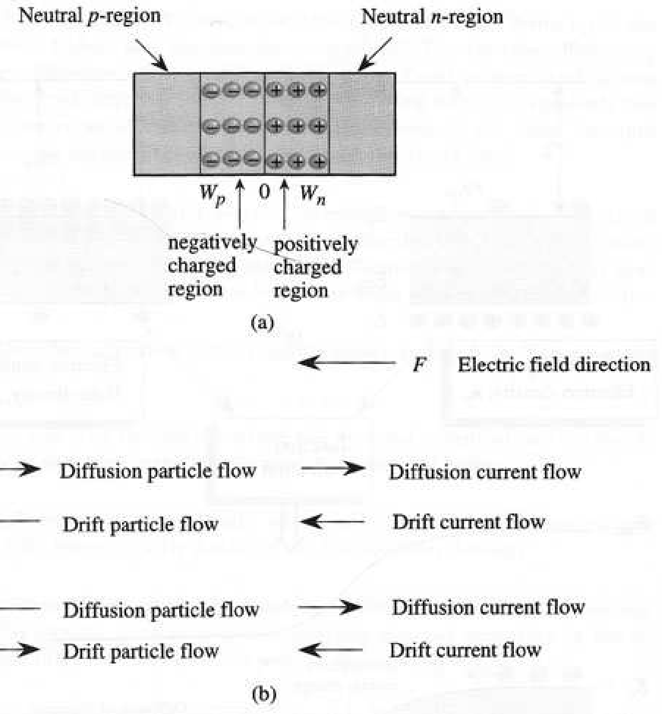

Figure 5.2: (a) An idealized model of the p-n junction without bias, showing the neutral and depletion areas. (b) A schematic showing various current and particle flow components in the p-n diode at equilibrium. For electrons, the current flow is in the direction opposite to that of the particle flow. Electrons that enter the depletion region from the p-side and holes that enter the depletion region from the n-side are swept away and are the source of the drift components.

Figure A.2: Left: Band diagrams for a pn junction. Right: Direction of the diffusion and drift currents for electrons and holes. Remeber these two diagrams well! (from [2])

As shown in Fig. 5.3, as a result of bringing the p and n type semiconductors, a ***built-in voltage***, V_{bi}, is produced between the n and the p side of the structure. Referring to Fig. 5.3, the built-in voltage is given by

$$eV_{bi} = E_g - (E_c - E_F)_n - (E_F - E_v)_p$$

where the subscripts n and p refer to the n-side and p-side of the device. We know that (see Eqn. 2.19)

$$(E_c - E_F)_n = -k_B T \ell n(\frac{n_n}{N_c})$$

where n_n is the electron density on the n-side of the device. Assuming that all of the donors are ionized,

$$n_n = N_d$$

Similarly (see Eqn. 2.22),

$$(E_F - E_v)_p = -k_B T \ell n(\frac{p_p}{N_v})$$

where p_p is the hole density on the p-side and is given by

$$p_p = N_a$$

We now have

$$eV_{bi} = E_g + k_B T \ell n(\frac{n_n p_p}{N_c N_v})$$

Using the relation (Eqn. 2.24)

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{k_B T}\right)$$

we get

$$V_{bi} = \frac{k_B T}{e} \ell n(\frac{n_n p_p}{n_i^2}) \qquad (5.3)$$

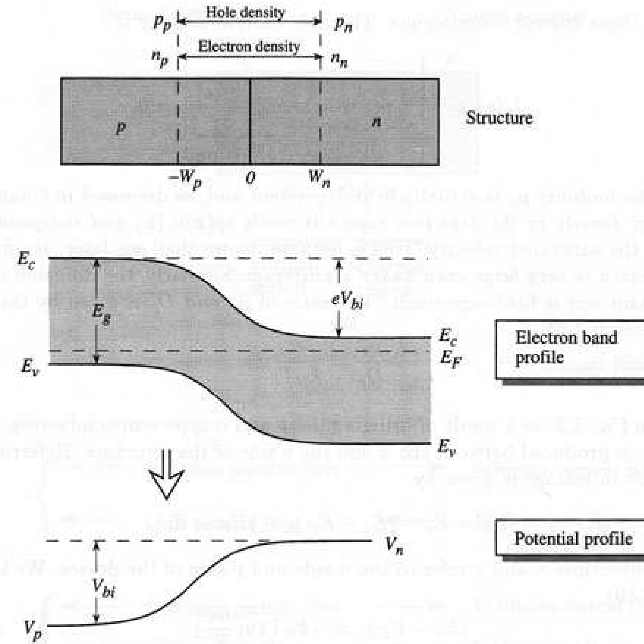

Figure 5.3: A schematic showing the p-n diode and the potential and band profiles. The voltage V_{bi} is the built-in potential at equilibrium. The expressions derived in the text can be extended to the cases where an external potential is added to V_{bi}.

If n_n and n_p are the electron densities in the n-type and p-type regions, the law of mass action (i.e., the product np is constant) tells us that

$$n_n p_n = n_p p_p = n_i^2 \qquad (5.4)$$

Thus, the contact potential or built-in potential, $V_{bi} = V_n - V_p$, is

$$\boxed{V_{bi} = \frac{k_B T}{e} \; \ell n \; \frac{p_p}{p_n}} \qquad (5.5)$$

or

$$\boxed{V_{bi} = \frac{k_B T}{e} \; \ell n \; \frac{n_n}{n_p}} \qquad (5.6)$$

We can thus write the following equivalent expressions:

$$\frac{p_p}{p_n} = e^{eV_{bi}/k_B T} = \frac{n_n}{n_p} \qquad (5.7)$$

Figure A.3: Derivation of the built in voltage (from [2])

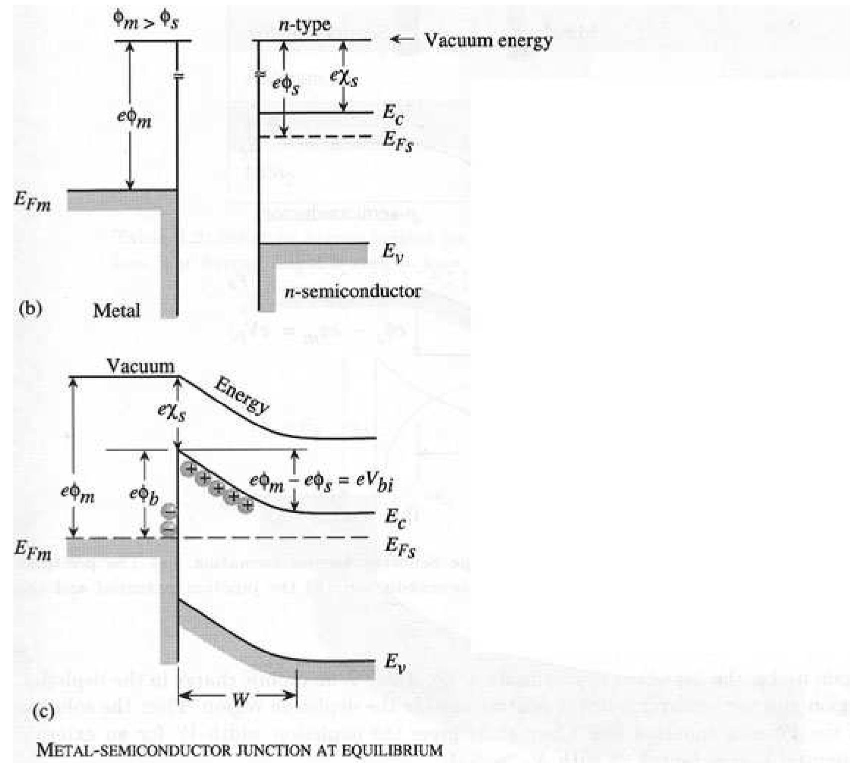

Figure 6.2: (a) A schematic of a Schottky barrier junction. (b) The various important energy levels in the metal and the semiconductor with respect to the vacuum level. (c) The junction potential produced when the metal and semiconductor are brought together. Due to the built-in potential at the junction, a depletion region of width W is created. Also shown in the inset are the work functions and electron affinities of several systems.

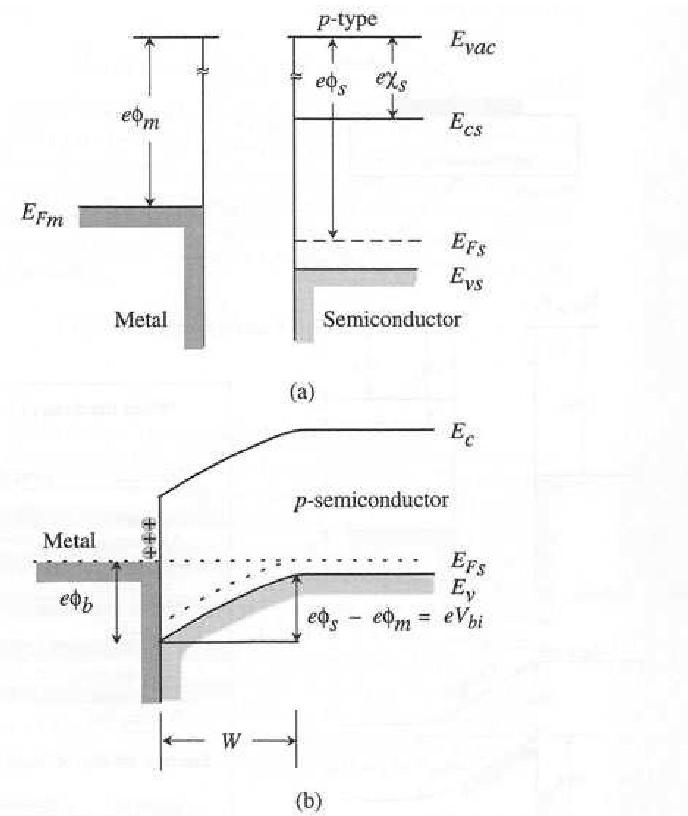

Figure 6.3: A schematic of the ideal p-type Schottky barrier formation. (a) The positions of the energy levels in the metal and the semiconductor; (b) the junction potential and the depletion width.

Figure A.4: Band diagrams for a Shottky contact, without bias. (from [2])

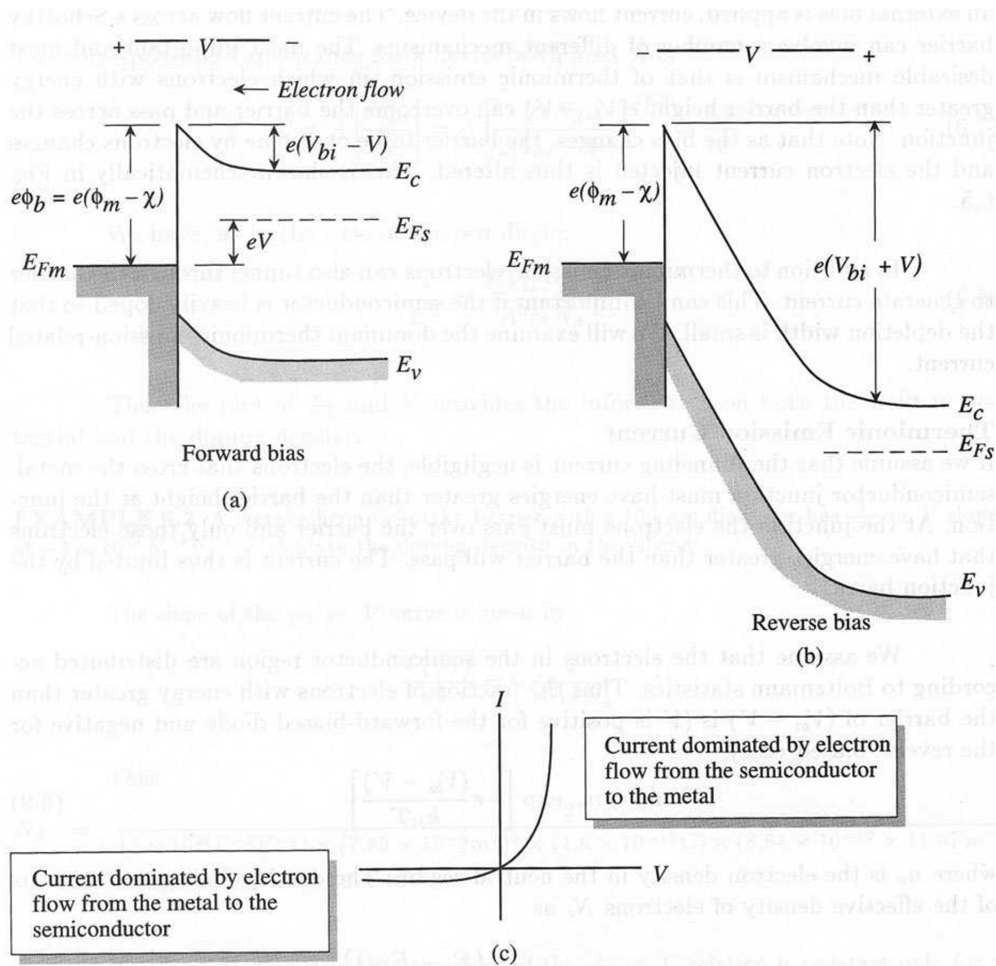

Figure 6.5: (a) The forward bias allows electrons to flow from the semiconductor to the metal side, increasing the current. (b) The reverse bias suppresses the electron flow from the semiconductor side while the flow from the metal side is unaffected. (c) The rectifying characteristics of the Schottky diode.

Figure A.5: Band diagrams for a Shottky contact, with a voltage applied. (from [2])

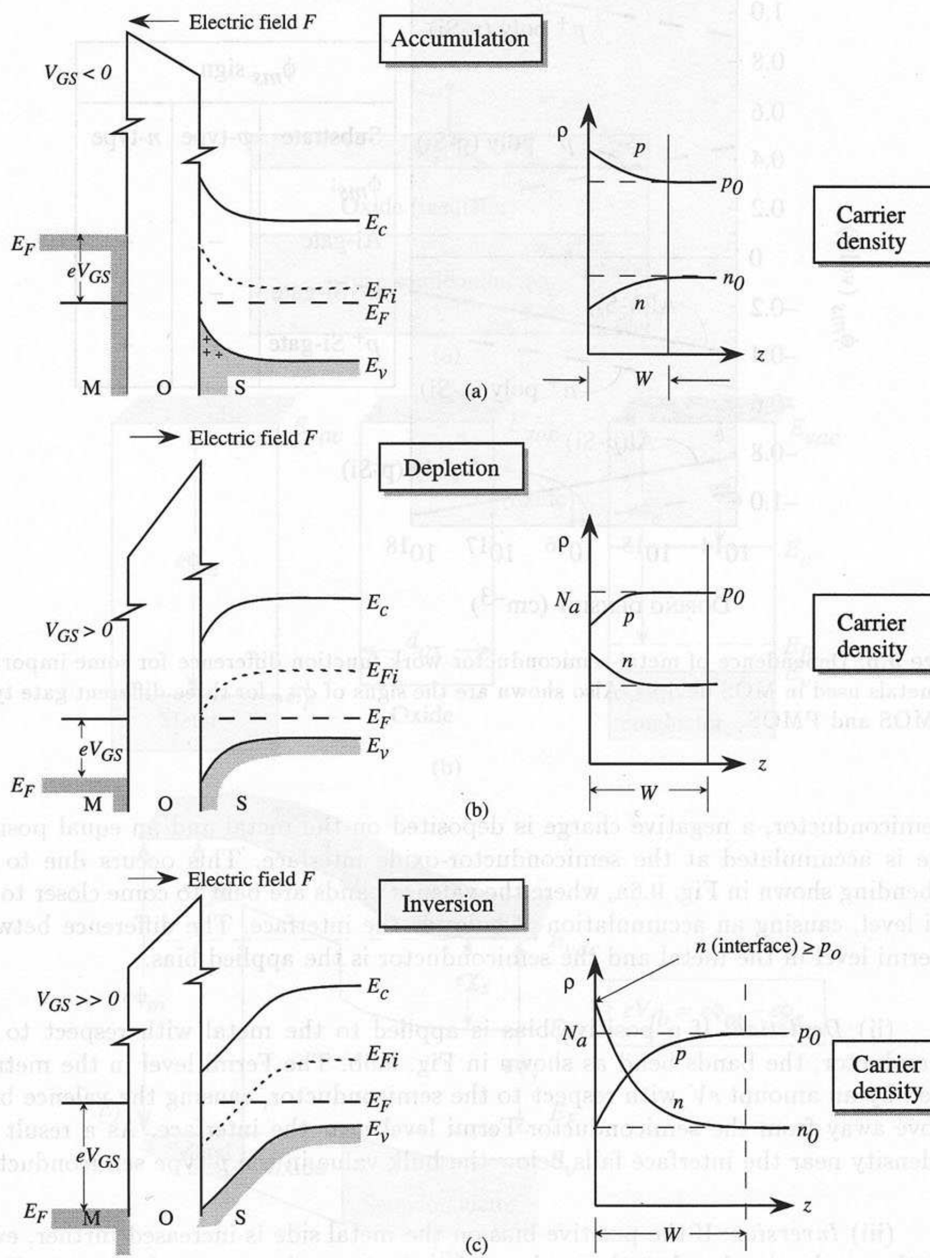

Figure 9.6: Effects of applied voltage on the ideal MOS capacitor: (a) negative voltage causes hole accumulation in the p-type semiconductor; (b) positive voltage depletes holes from the semiconductor surface; and (c) a larger positive voltage causes inversion—an "n-type" layer at the semiconductor surface. The figures also show the electron and hole distributions in each case. W represents the depletion width and N_a is the background acceptor concentration.

Figure A.6: Band diagrams for MOSFET. (from [2])

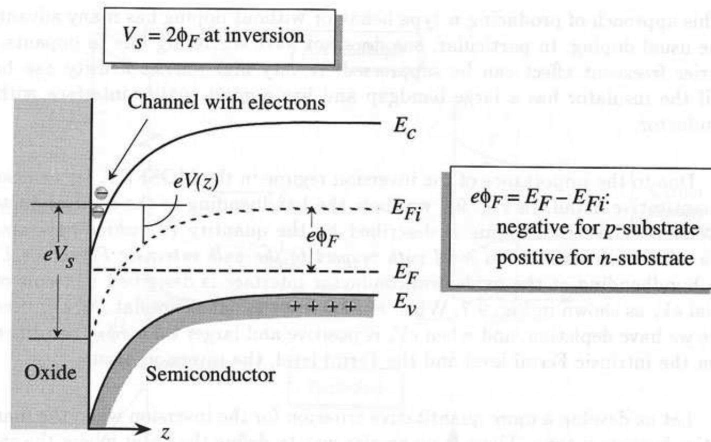

Figure 9.7: Bandbending of the semiconductor in the inversion mode. The interface potential is V_s. A simple criterion for inversion is that $V_s = 2\phi_F$.

Figure A.7: Bandbending of MOSFET at Inversion. (from [2])

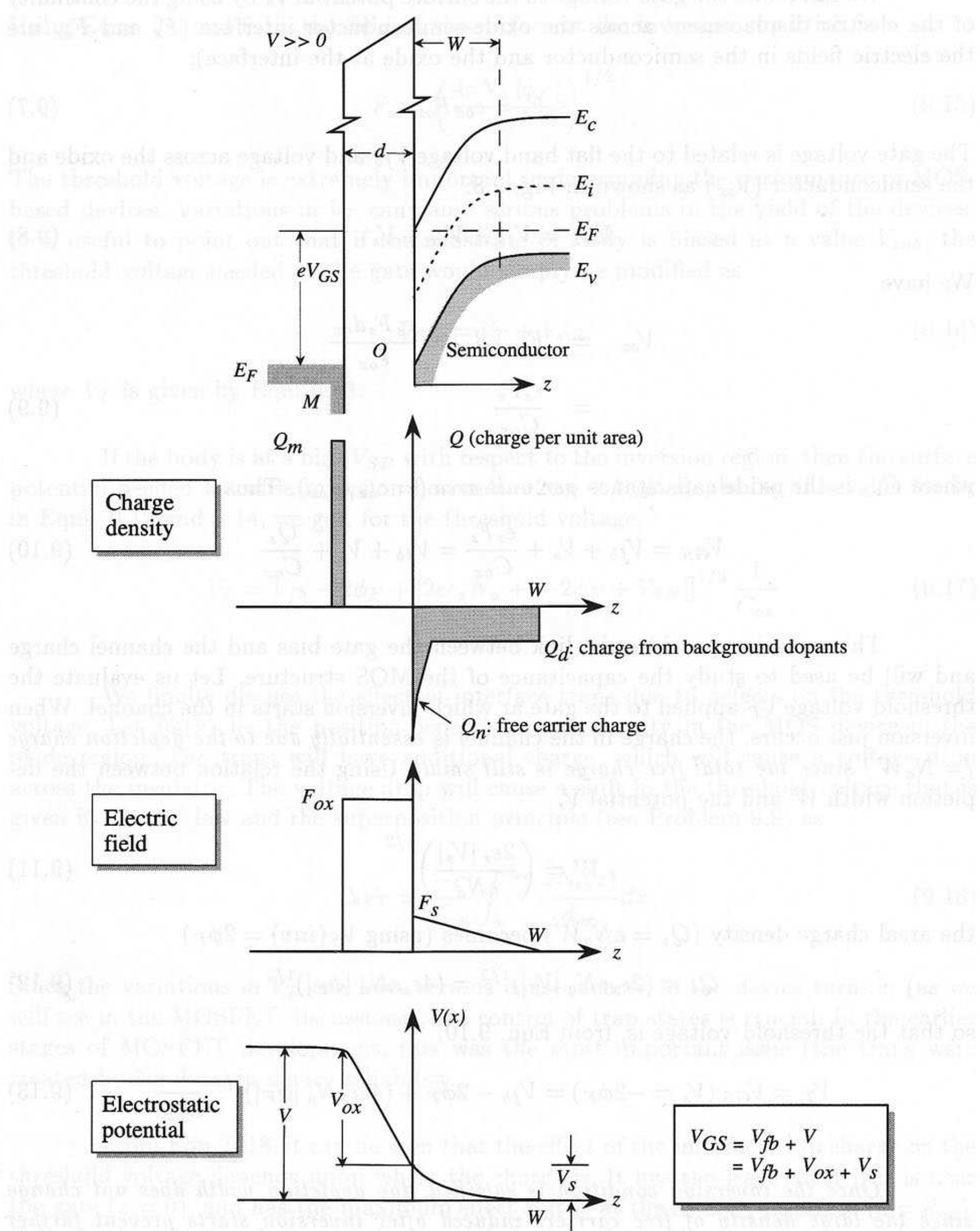

Figure 9.8: Approximate distributions of charge, electric field, and electrostatic potential in the ideal MOS capacitor in inversion. Once inversion begins, the depletion width W does not increase further because of the high mobile electron density at the interface region.

Figure A.8: Charge distribution, electric field and potential for a MOSFET. (from [2])

B.1 DENSITY OF STATES

We will start with the problem of density of states of electrons in free space or in perfectly crystalline materials. We have noted that in perfect semiconductors, electrons can be regarded as "free" electrons.

Let us consider the Schrödinger equation for free electrons. The time-independent equation is

$$\frac{-\hbar^2}{2m_o}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi(r)=E\psi(r) \tag{B.1}$$

A general solution of this equation is

$$\psi(r)=\frac{1}{\sqrt{V}}e^{\pm i k\cdot r} \tag{B.2}$$

where the factor $\frac{1}{\sqrt{V}}$ comes because we wish to have one electron per volume V, or

$$\int_V d^3r \mid \psi(r)\mid^2=1 \tag{B.3}$$

We assume that the volume V is a cube of side L.

The corresponding energy of the electron is obtained from Eqn. B.1 and is

$$E=\frac{\hbar^2k^2}{2m_o} \tag{B.4}$$

The momentum of the electron is (replacing p by the differential operator)

$$p\psi=-i\hbar\frac{\partial}{\partial \mathbf{r}}\psi\rightarrow\hbar\mathbf{k} \text{ or } \mathbf{p}=\hbar\mathbf{k} \tag{B.5}$$

while the velocity is

$$\mathbf{v}=\frac{\hbar\mathbf{k}}{m_o} \tag{B.6}$$

In classical mechanics the energy-momentum relation for the free electron is $E=p^2/2m_o$, and p can be a *continuous variable.* The quantity $\hbar k$ appearing above seems to be replacing p, in quantum mechanics. Due to the wave nature of the electron, the quantity k is not continuous but discrete. For describing moving electrons, the boundary condition used is known as a periodic boundary condition. Even though we focus our attention on a finite volume V, the wave can be considered to spread in all space as we conceive the entire space is made up of identical cubes of sides L, as shown in Fig. B.1. Then,

$$\begin{aligned}\psi(x,y,z+L) &= \psi(x,y,z)\\ \psi(x,y+L,z) &= \psi(x,y,z)\\ \psi(x+L,y,z) &= \psi(x,y,z)\end{aligned} \tag{B.7}$$

The boundary conditions impose certain restrictions in the k of the wavefunction.

Because of the boundary conditions the allowed values of k are (n are integers—positive and negative)

$$k_x=\frac{2\pi n_x}{L};\ k_y=\frac{2\pi n_y}{L};\ k_z=\frac{2\pi n_z}{L} \tag{B.8}$$

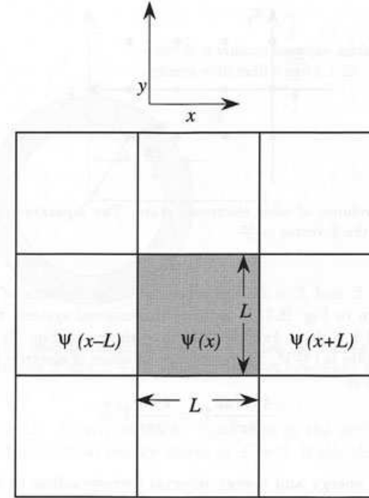

Figure B.1: A schematic showing how periodic boundary conditions are applied. A large volume is considered to be made up of identical cubic volumes.

If L is large, the spacing between the allowed k values is very small. It is useful to discuss the *volume in k-space that each electronic state occupies.* As can be seen from Fig. B.2, this volume is

$$\left(\frac{2\pi}{L}\right)^3=\frac{8\pi^3}{V} \tag{B.9}$$

If Ω is a volume of k-space, the number of electronic states in this volume is

$$\boxed{\frac{\Omega V}{8\pi^3}} \tag{B.10}$$

We will now use the discussion provided above to derive the concept of density of states. The concept of density of states is extremely powerful, and important physical properties such as optical absorption, transport, etc., are intimately dependent upon it. Density of states is the number of available electronic states *per unit volume per unit energy* around an energy E. If we denote the density of states by $N(E)$, the number of states in a unit volume in an energy interval dE around an energy E is $N(E)dE$.

Figure A.9: Derivation of the 1D density of states-1. (from [2])

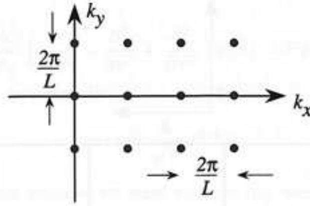

Figure B.2: k-space volume of each electronic state. The separation between the various allowed components of the k-vector is $\frac{2\pi}{L}$.

The energies E and $E + dE$ are represented by surfaces of spheres with radii k and $k + dk$ as shown in Fig. B.3. In a three-dimensional system, the k-space volume between vector k and $k + dk$ is $4\pi k^2 dk$. We have shown in Eqn. B.9 that the k-space volume per electron state is $(\frac{2\pi}{L})^3$. Therefore, the number of electron states in the region between k and $k + dk$ is

$$\frac{4\pi k^2 dk}{8\pi^3} V = \frac{k^2 dk}{2\pi^2} V \tag{B.11}$$

Denoting the energy and energy interval corresponding to k and dk as E and dE, we see that the number of electron states between E and $E + dE$ per unit volume is

$$N(E)dE = \frac{k^2 dk}{2\pi^2} \tag{B.12}$$

and since

$$E = \frac{\hbar^2 k^2}{2m_o} \tag{B.13}$$

$$k^2 dk = \frac{\sqrt{2} m_o^{3/2} E^{1/2} dE}{\hbar^3} \tag{B.14}$$

and

$$N(E)dE = \frac{m_o^{3/2} E^{1/2} dE}{\sqrt{2}\pi^2 \hbar^3} \tag{B.15}$$

An electron can have two possible states with a given energy. These states are called the spin states. The electron can have a spin state $\hbar/2$ or $-\hbar/2$. To take spin into account, the density of states obtained above is simply multiplied by 2,

$$\boxed{N(E) = \frac{\sqrt{2} m_o^{3/2} E^{1/2}}{\pi^2 \hbar^3}} \tag{B.16}$$

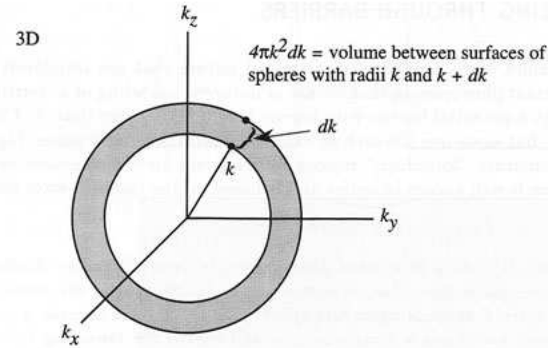

Figure B.3: Geometry used to calculate density of states. By finding the k-space volume in an energy interval between E and $E + dE$, one can find out how many allowed states there are.

In Fig. B.4, we show the density of states. Note that in the derivation given here, we have assumed that the electron energy starts at $E = 0$. If the electron energy is given by

$$E = \frac{\hbar^2 k^2}{2m_0} + V_0 \tag{B.17}$$

i.e., if there is a background potential V_0, the density of states becomes

$$N(E) = \frac{\sqrt{2} m_0^{3/2} (E - V_0)^{3/2}}{\pi^2 \hbar^3} \tag{B.18}$$

In this case, the density of states is zero for $E < V_0$.

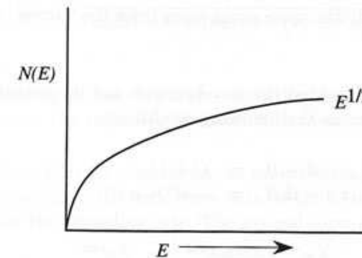

Figure B.4: Variation in the energy dependence of the density of states.

Figure A.10: Derivation of the 1D density of states-2. (from [2])

B Important problems - solved

B.1 Average electron energy in a metal

For a metal at a temperature of T = 0 K, the conduction electrons at the bottom of the band have an energy E = 0 and the conduction electrons with the highest energy have an energy E = Ef. Assuming that the density of states is given by $N_{(E)} = \frac{8\pi\sqrt{m^3 E}}{h^3}$, what is the average energy of the conduction electrons?
An expression for the average energy should be given in terms of Planck's constant h, the Fermi energy Ef, the effective mass m, pi = 3.14159... and numerical constants. The functions sin, cos, tan, asin, acos, exp, ln, and sqrt can be used. For instance, 2*sin(h*m)*exp(Ef) is a valid (but wrong) answer. Be sure to include a * to indicate multiplication; write 2*h*m not 2hm to multiply 2 times h times m.
Hint: Not all of the constants h, m, Ef, and pi appear in the answer.

Answer: Here we need to mess around with the density of states again. However, the problem is pretty straight forward. The distribution of the density of states (which is in this case the 3-D density of states for a free electron Fermi gas with L=1), tells us how many states there are for electrons to occupy at a certain energy. Now to get the average energy we just have to calculate the sum over the energy of each single electron and divide it by the number of electrons in the band. We can get the number of electron by integrating over the density of states times the fermi function (which tells us how many states are occupied):

$$n = \int_0^\infty N_{(E)} f_{FD(E)} dE$$

since we do our calculation for $T = 0K$ the fermi function is a step function and its only effect in this case is that it changes the limits of the integral:

$$n = \int_0^{E_f} N_{(E)} dE = \frac{8\pi\sqrt{m^3}}{h^3} \int_0^{E_f} \sqrt{E} dE = \frac{8\pi\sqrt{m^3}}{h^3} (\frac{2}{3} E_f^{3/2})$$

To get the sum over the energy of each single electron we just have to integrate over the density of states times the fermi function times the energy:

$$E_{sum} = \int_0^\infty N_{(E)} f_{FD(E)} E dE$$

for $T = 0K$ we get:

$$E_{sum} = \int_0^{E_f} N_{(E)} E dE = \frac{8\pi\sqrt{m^3}}{h^3} \int_0^{E_f} E^{3/2} dE = \frac{8\pi\sqrt{m^3}}{h^3} (\frac{2}{5} E_f^{5/2})$$

To get the average we devide this result by the number of electrons we derived before:

$$\bar{E} = \frac{E_{sum}}{n} = \frac{\frac{8\pi\sqrt{m^3}}{h^3}(\frac{2}{3}E_f^{3/2})}{\frac{8\pi\sqrt{m^3}}{h^3}(\frac{2}{5}E_f^{5/2})} = \frac{(\frac{2}{5}E_f^{5/2})}{(\frac{2}{3}E_f^{3/2})} = \frac{5}{3} E_f^{5/2-3/2}$$

$$\bar{E} = \frac{5}{3} E_f$$

B.2 Band structure - Exam March 2007

The band structure of a semiconductor is shown below B.12. The zero of energy is chosen to be the top of the valence band.

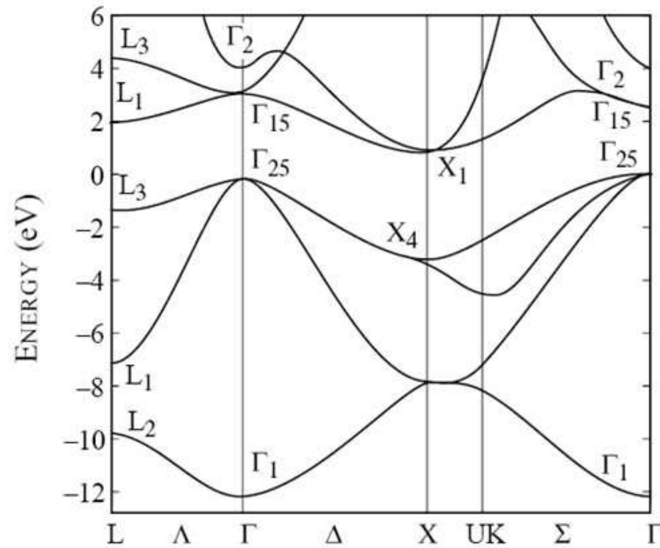

Figure B.11: Band structure of semiconductor

(a) Is this a direct or an indirect semiconductor? Why?

Answer: Indirect - The maximum of the valence band and the minimum of the conduction band are not located directly above each other

(b) What is the band gap?

Answer: approx. 1ev - take the difference between valence band maximum and conduction band minimum

(c) What are light holes and heavy holes? Explain how you can determine the effective mass of the holes from this diagram.

Answer: The effective mass is given by

$$m_x^* = \frac{\hbar^2}{\frac{d^2E(\vec{k})}{dk_x^2}}$$

Hence it is inversely proportional to the curvature of the Energy - the light holes are the ones with the higher curvature, the heavy holes are the ones with the smaller curvature.

One accurate but rather complicated possibility of approximation for the effective mass is via measuring the focal length: The equation for a parabola is $ax^2 + bx + c = 0$ if the zero point of the coordinate system is put at the crest of the parabola $b = c = 0$, derivating the remaining $y = ax^2$ twice gives $2a$ for the curvature. The focal length of a parabola is $f = 1/(4a)$, hence one can easily get the curvature once the focal length is known. The focal length is the distance between focal

point and crest of the parabola. The focal point can be found by letting some perpendicular light rays reflect of the surface of the parabola, where the angle of the incident and the reflected beam have to be the same (see B.12)

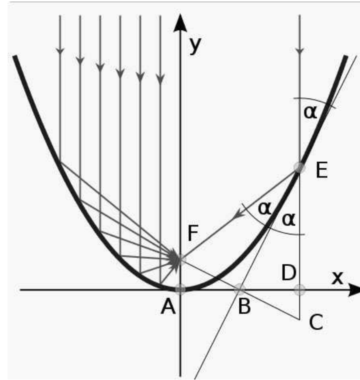

Figure B.12: Finding out the focal distance of a parabola

(d) When is a semiconductor degenerately doped?

Answer: (http://en.wikipedia.org/wiki/Semiconductor) Semiconductors so highly doped that they have conductivity characteristics of metals.

C Best of Equation - A Selection

This chapter should provide you with a short overview of all the equations you should know for the exam.
Physical constants:
$e = 1.6e-19C$; $h = 6.626e-34Js$; $k_B = 1.38e-23J/K$
Introduction:
Einstein relation: $E = hf$ De Broglie: $p = \frac{h}{\lambda} \rightarrow \vec{p} = \hbar\vec{k}$, Wavevector: $k_x = \frac{2\pi}{\lambda} = \frac{n_x\pi}{L_x}$
Schroedinger: $i\hbar\frac{d}{dt}\psi_t = \hat{H}\psi_t$ Hamiltonian 3D potential well: $\hat{H} = \frac{-\hat{\hbar}^2}{2m}\nabla^2 + V(x)$
Solution wavefunction $\psi_{(t)} = \frac{\sqrt{2}}{L_x}\frac{\sqrt{2}}{L_y}\frac{\sqrt{2}}{L_z}e^{-i\omega t}\sin\frac{n_x\pi x}{L_x}\sin\frac{n_y\pi x}{L_y}\sin\frac{n_z\pi x}{L_z}$
Energy: $E_{n_x n_y n_z} = \hbar\omega = \frac{\hbar^2\pi^2}{2m}(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2})$
Crystals:
Fermi-Dirac distribution function: $f_E = \frac{1}{1+e^{\frac{E-E_F}{k_BT}}}$
1D Density of states (know the derivation - see section 2.4) $N_E = \frac{\sqrt{2}m^{3/2}E^{1/2}}{\pi^2\hbar^3}$
Number of electrons: $n = \int_{E_0}^{E_1} N_{(E)}F_{(E)}dE$ (also have a look at Problem 2.6 in Appendix B)
Intrinsic Semiconductors:
Effective Mass: $m^* = \hbar^2(\frac{d^2E}{dk^2})^{-1}$
Density of electrons in the conduction band: $n = N_C e^{-\frac{E_C-E_F}{k_BT}}$; $N_C = 2(\frac{m_e^* k_BT}{2\pi\hbar^2})^{3/2}$
Density of holess in the valence band: $n = N_V e^{-\frac{E_F-E_V}{k_BT}}$; $N_V = 2(\frac{m_h^* k_BT}{2\pi\hbar^2})^{3/2}$
Law of mass action: $n = p = n_i^2 = N_C N_V e^{\frac{-E_g}{k_BT}}$ Fermi energy: $E_F = \frac{E_C+E_V}{2} + \frac{k_BT}{2}ln(\frac{N_V}{N_C})$
Extrinsic Semiconductors:
$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D-N_A}$; $n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A-N_D}$; $E_F = E_V + k_BTln(\frac{N_V}{N_A-N_D})$; $E_F = E_N + k_BTln(\frac{N_V}{N_D-N_A})$
Transport:
$\vec{v_F} = -e\vec{E} = m^*a = m^*\frac{d\vec{v_{avg}}}{dt}$; $\vec{v_{avg}} = -\frac{e\vec{E}\tau_{sc}}{m^*} = \vec{v_d}$; $\vec{J} = -ne\vec{v_d} = \frac{ne^2\tau_{sc}}{m^*}\vec{E}$
Ohm's law $\vec{J} = \sigma\vec{E}$; $\sigma = \frac{ne^2\tau_{sc}}{m^*}$; Holes $\vec{v_{d,p}} = \mu_p\vec{E}$ Electrons $\vec{v_{d,n}} = -\mu_n\vec{E}$
$\vec{J} = -ne\vec{v_{d,n}} + ne\vec{v_{d,p}} = (ne\mu_n + ne\mu_p)\vec{E} = \sigma\vec{E}$ Matthiessen's Rule $\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$
Diffusion: $\vec{J_{n,diff}} = |e|\,D_n\frac{dn}{dx}$; $\vec{J_{p,diff}} = -|e|\,D_p\frac{dp}{dx}$; Diffusion constants: $D_n = \frac{k_BT\mu_n}{e}$; $D_p = \frac{k_BT\mu_p}{e}$
Current density equations: $\vec{J_n} = -ne\mu_n\vec{E} + eD_n\frac{dn}{dx}$; $\vec{J_p} = ne\mu_p\vec{E} - eD_p\frac{dp}{dx}$
pn-junction:
Built in Potential (know the derivation, see section 6.1) $eV_{bi} = k_BTln(\frac{N_DN_A}{n_i^2})$ Depletion Approximation - know the derivation, see section 6.2, make sure you know the equations for the charge density, the electric field, the potential and the depletion width, or that you can derive them.
Know the diode current $I = I_0(e^{\frac{eV}{k_BT}} - 1)$ and be able to derive it, see section 6.5
Contacts:
Relationship between current and voltage in a Schottky contact: $I = I_{sm} + I_{ms} = I_s(e^{\frac{eV}{k_BT}})$
JFET MESFET:
Pinch-off voltage: $V_p = \frac{eN_Dh^2}{2\epsilon}$ Depletion width: $x_n = \sqrt{\frac{2\epsilon(V_{bi}-V)}{eN_D}}$ Have a look at the derivation of the drain current, see section 8.3
MOSFET:
Have a look at the derivation of the drain current, see section 9.4 (gradual channel approximation)
BJT:
Emitter efficiency $\gamma_E = \frac{I_{En}}{I_{En}+I_{Ep}}$
Base transport factor with $I_E = I_{En} + I_{Ep}$ and $I_C = I_{Cn} + I_{Cp}$: $\alpha_T = \frac{I_C}{I_{En}}$
Common-base current gain $\alpha_0 = \frac{I_C}{I_E}$

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