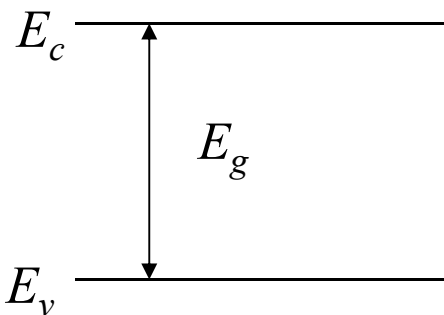


4. Extrinsic Semiconductors Carrier Transport

Oct. 24, 2018

Review

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$


$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

Intrinsic semiconductors

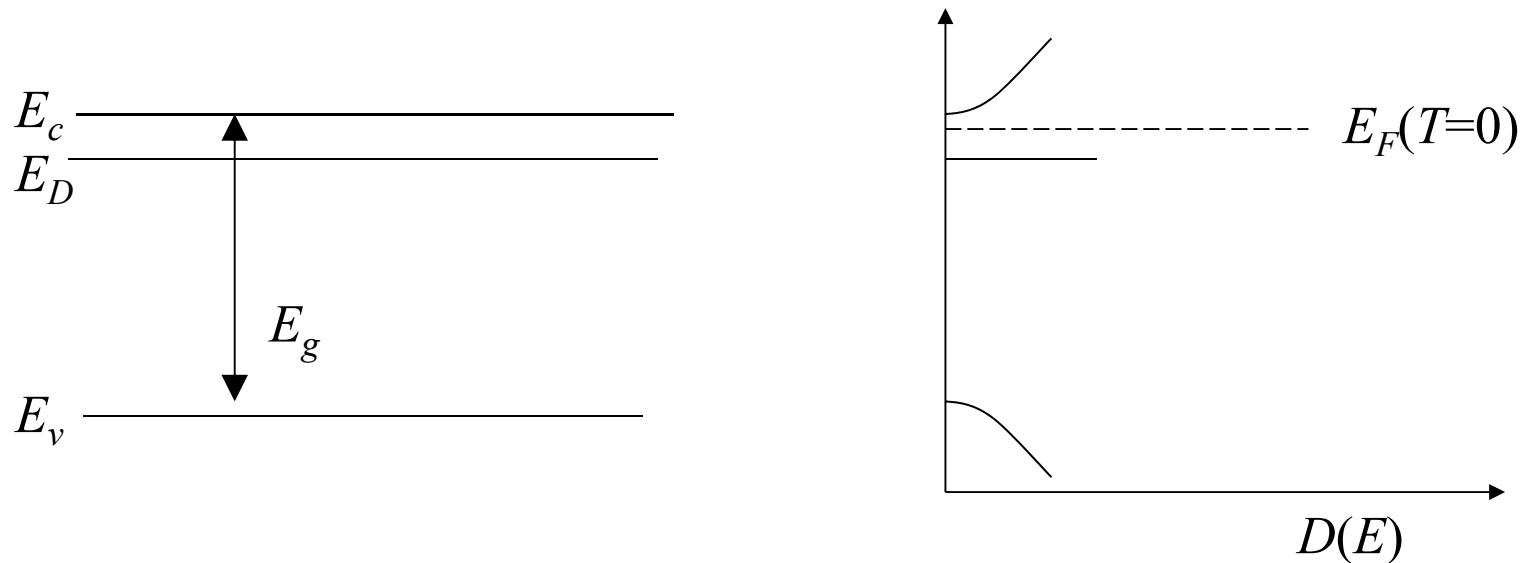
$$n = p = n_i = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2k_B T}\right)$$

$$E_F = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln\left(\frac{N_v}{N_c}\right)$$

Donors

Five valence electrons: P, As

States are added in the band gap just below the conduction band



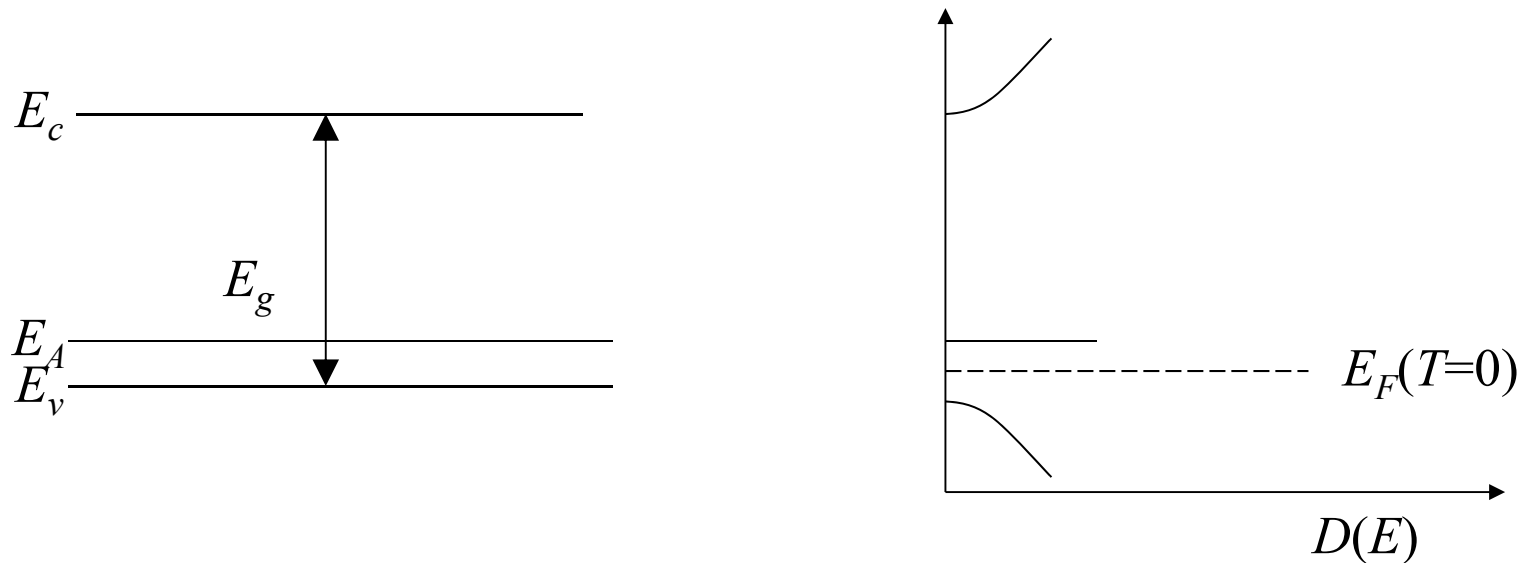
n-type: $n \sim N_D$ Many more electrons in the conduction band than holes in the valence band.

majority carriers: electrons; minority carriers: holes

Acceptors

Three valence electrons: B, Al, Ga

States are added in the band gap just above the valence band



p-type: $p \sim N_A$ Many more holes in the valence band than electrons in the conduction band.

majority carriers: holes; minority carriers: electrons

Donor and Acceptor Energies

Semiconductor	Donor	Energy (meV)
Si	Li	33
	Sb	39
	P	45
	As	54
Ge	Li	9.3
	Sb	9.6
	P	12
	As	13
GaAs	Si	5.8
	Ge	6.0
	S	6.0
	Sn	6.0

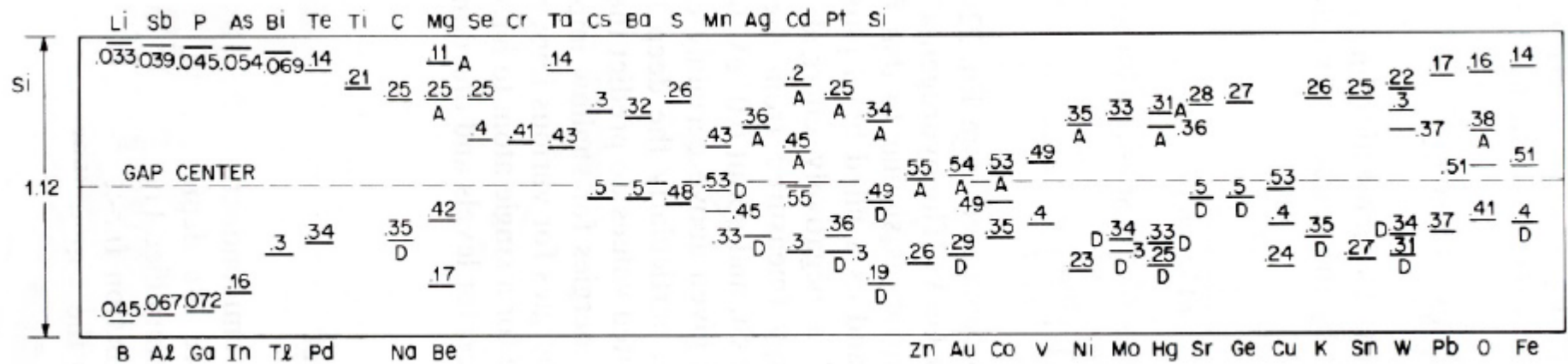
Energy below the conduction band



Semiconductor	Acceptor	Energy (meV)
Si	B	45
	Al	67
	Ga	72
	In	160
Ge	B	10
	Al	10
	Ga	11
	In	11
GaAs	C	26
	Be	28
	Mg	28
	Si	35

Energy above the valence band





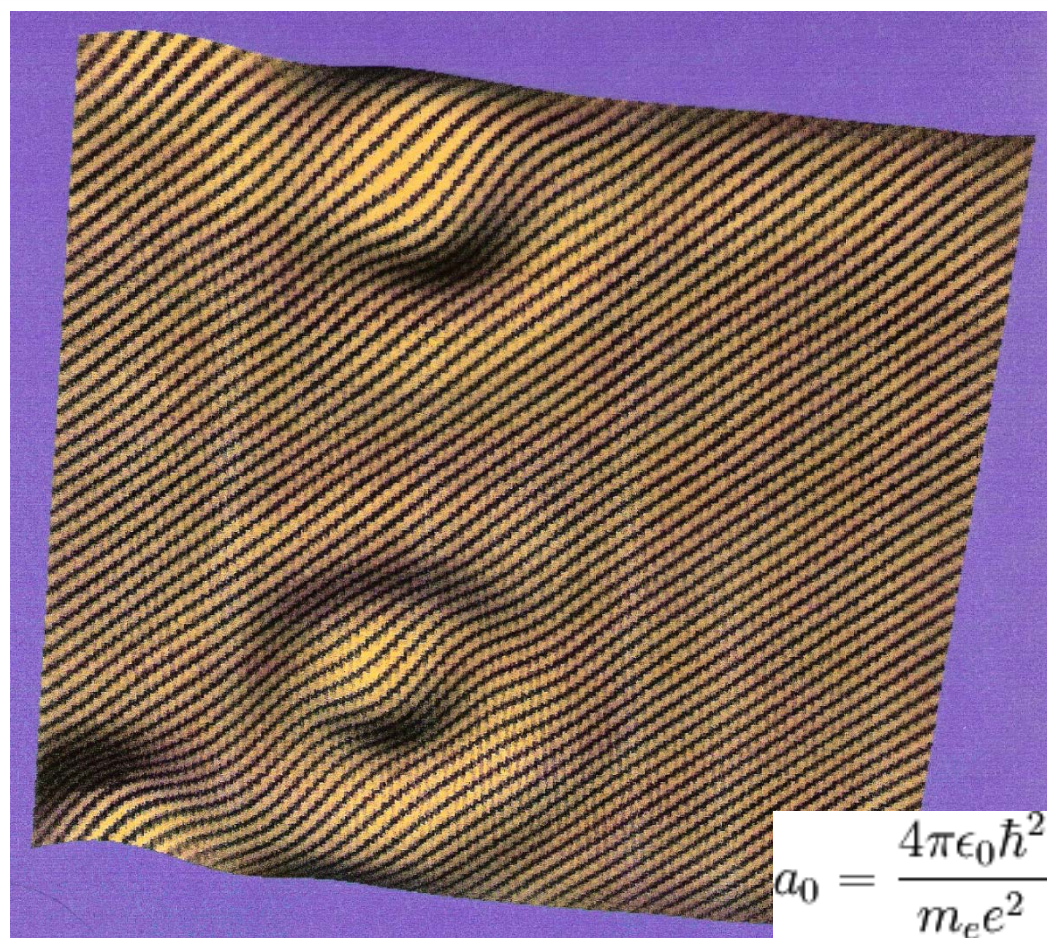
Source: Semiconductor Devices Physics and Technology, S.M. Sze, 1985

Direct Observation of Friedel Oscillations around Incorporated Si_{Ga} Dopants in GaAs by Low-Temperature Scanning Tunneling Microscopy

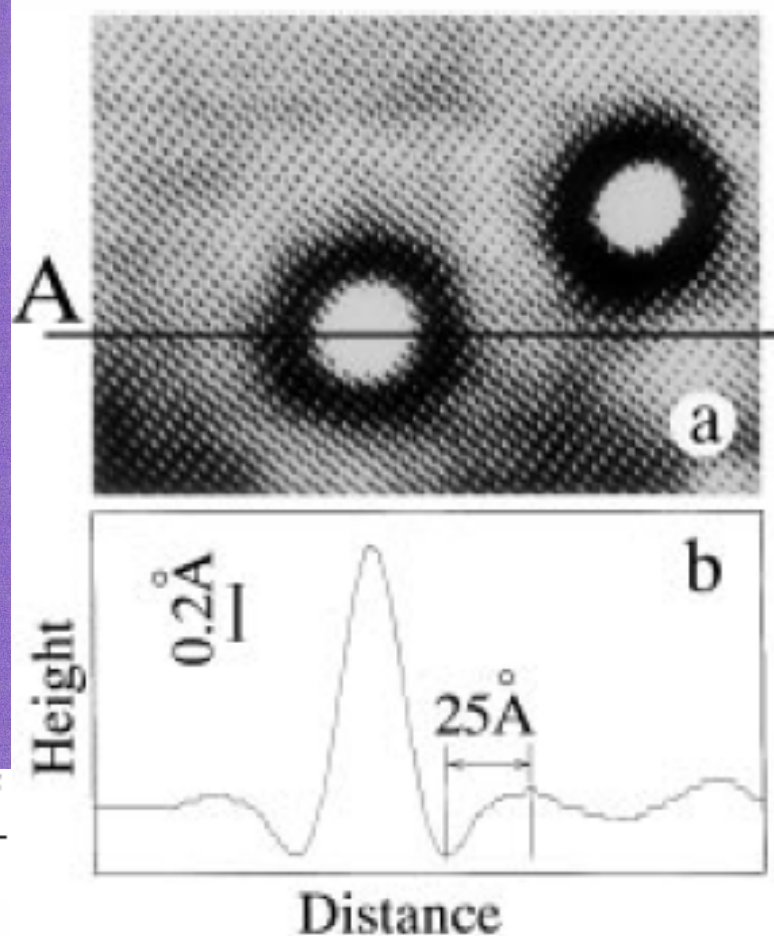
M. C. M. M. van der Wielen, A. J. A. van Roij, and H. van Kempen

Research Institute for Materials, University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands

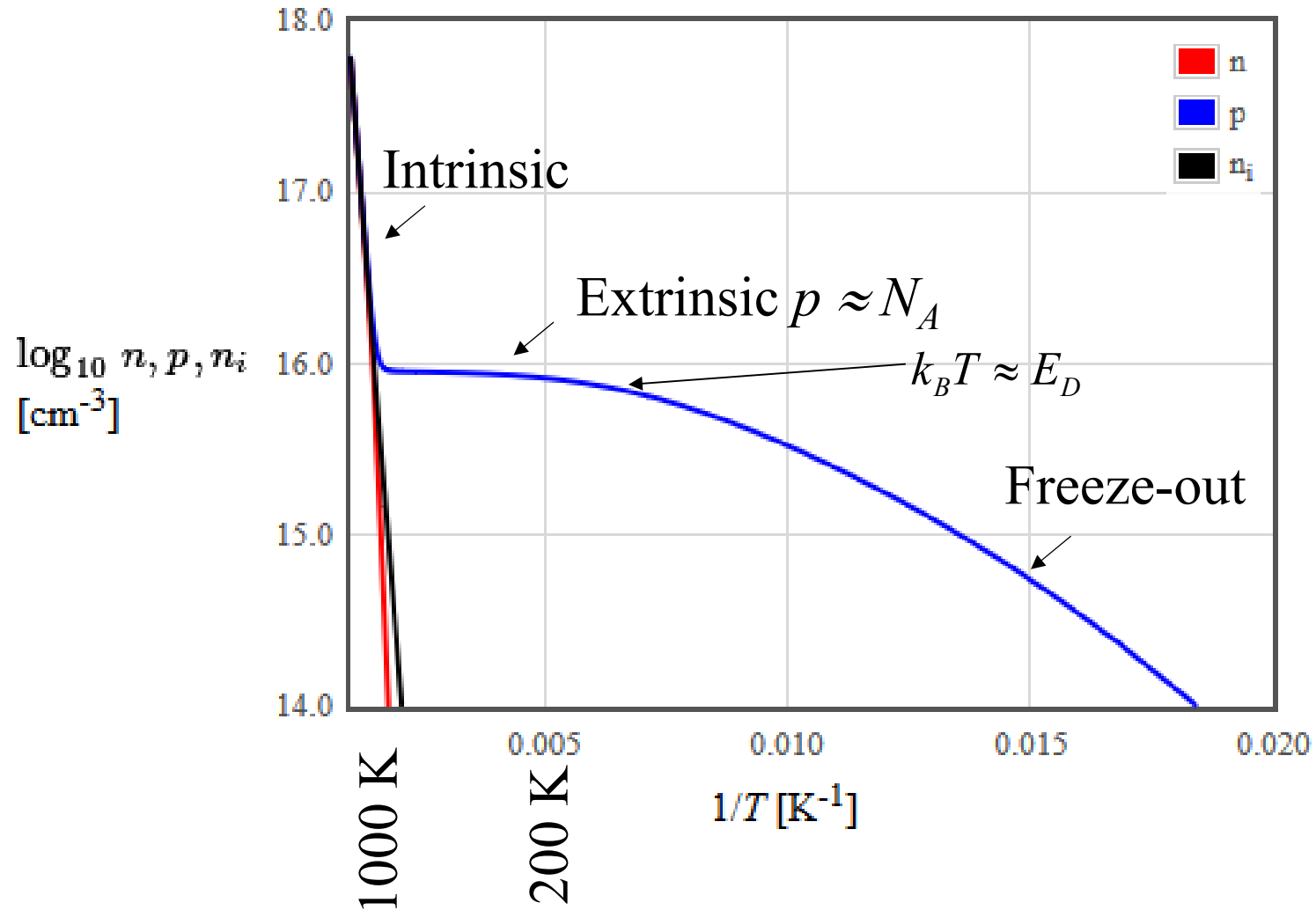
(Received 25 July 1995)



$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$$



Temperature dependence



[Return to problem list](#)
[Login](#)

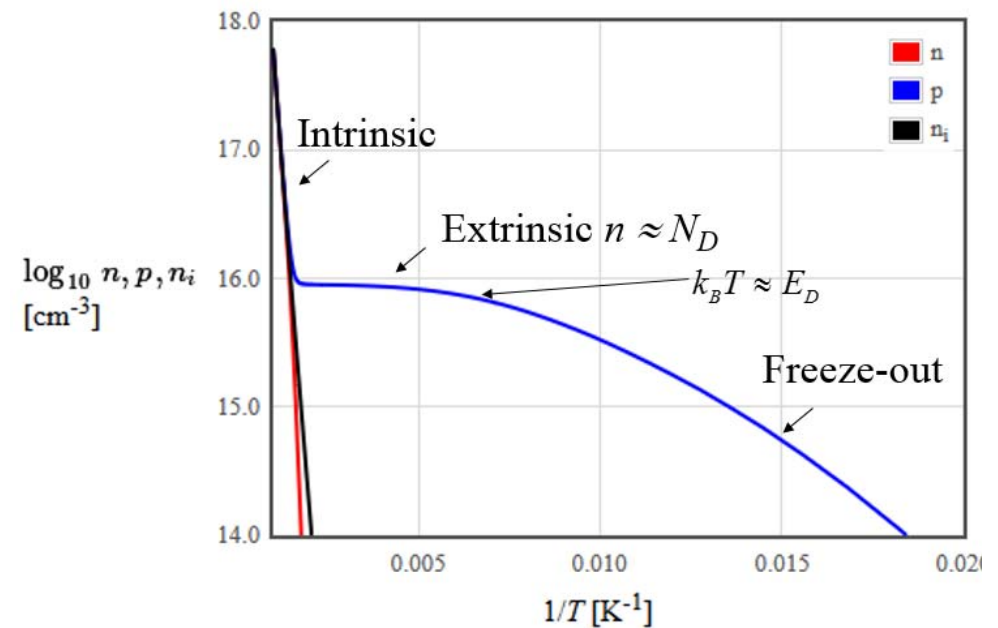
Temperature dependent conductivity

A doped semiconductor makes a transition from extrinsic behavior to intrinsic behavior when number of thermally activated charge carriers equals the number of dopants. What is this temperature for silicon doped with boron at $4E+17 \text{ cm}^{-3}$?

E_g is slightly temperature dependent but use $E_g = 1.12 \text{ eV}$, $N_c = 2.78 \times 10^{25} \text{ m}^{-3}$ and $N_v = 9.84 \times 10^{24} \text{ m}^{-3}$ to estimate the transition temperature.

$T =$ K

$$n_i = \sqrt{N_v N_c \exp\left(-\frac{E_g}{k_B T}\right)}$$



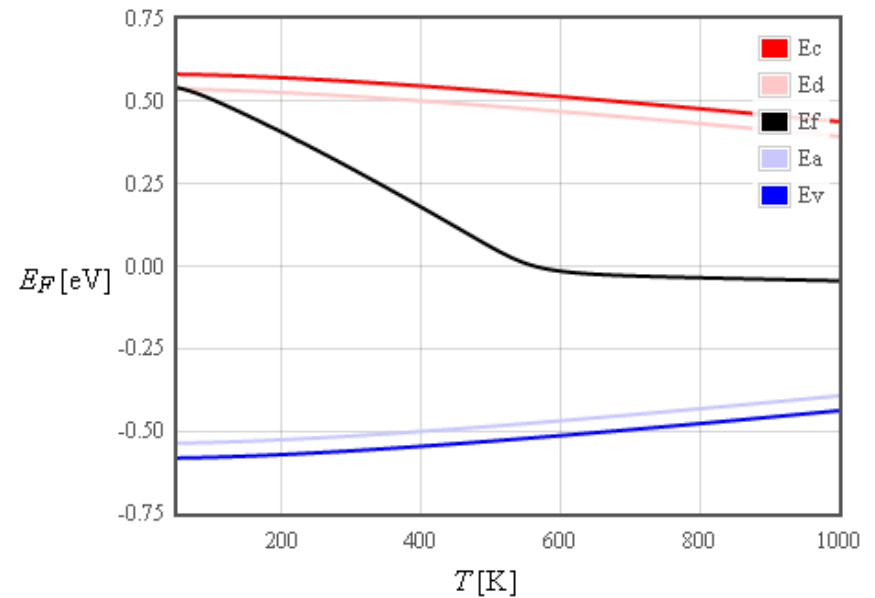
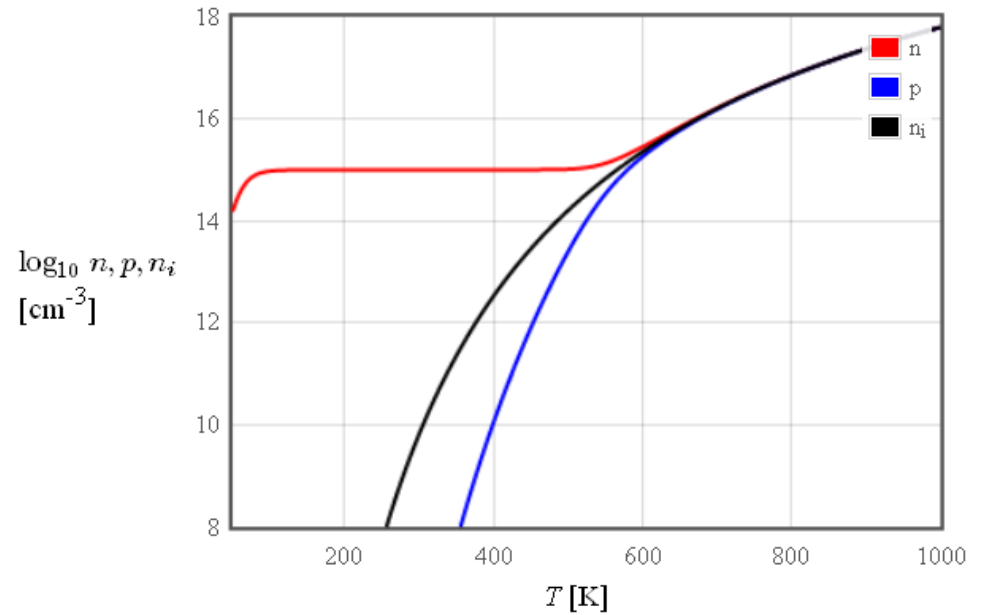
n-type

n-type $N_D > N_A$, $p \sim 0$

$$n = N_D = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

For n-type, $n \sim$ density of donors,
 $p = n_i^2 / n$



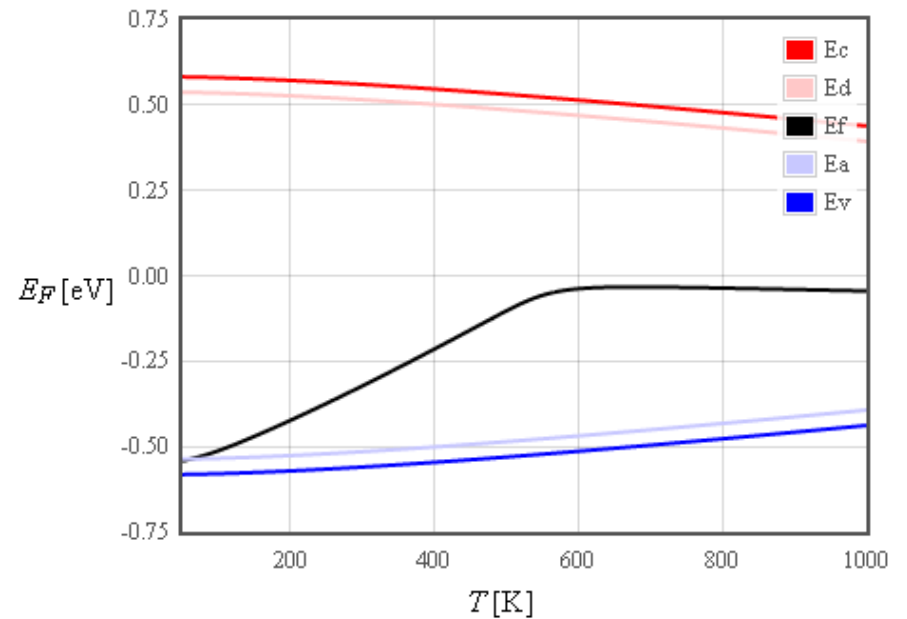
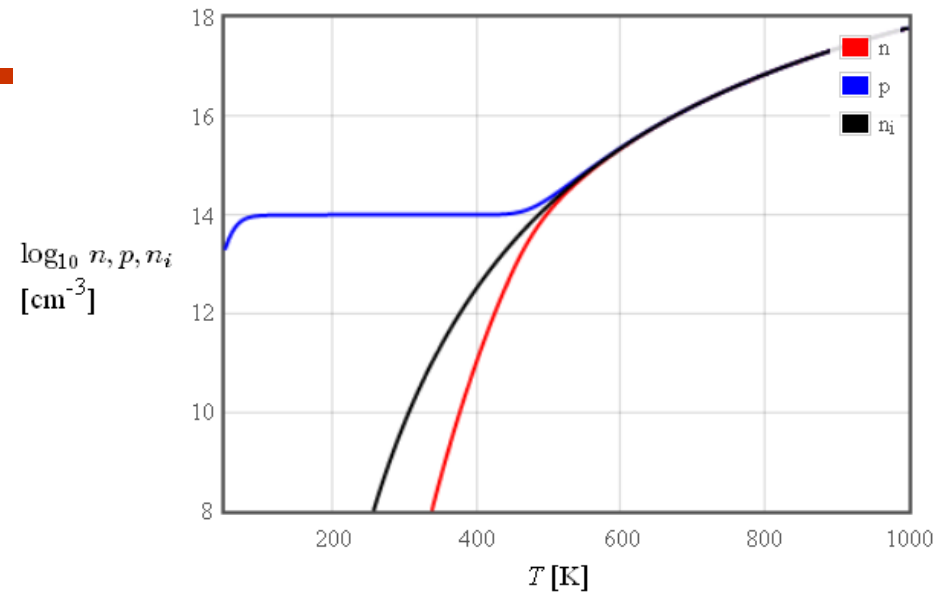
p-type

p-type $N_A > N_D$, $n \sim 0$

$$p = N_A = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

For p-type, $p \sim$ density of acceptors,
 $n = n_i^2/p$



Intrinsic / Extrinsic

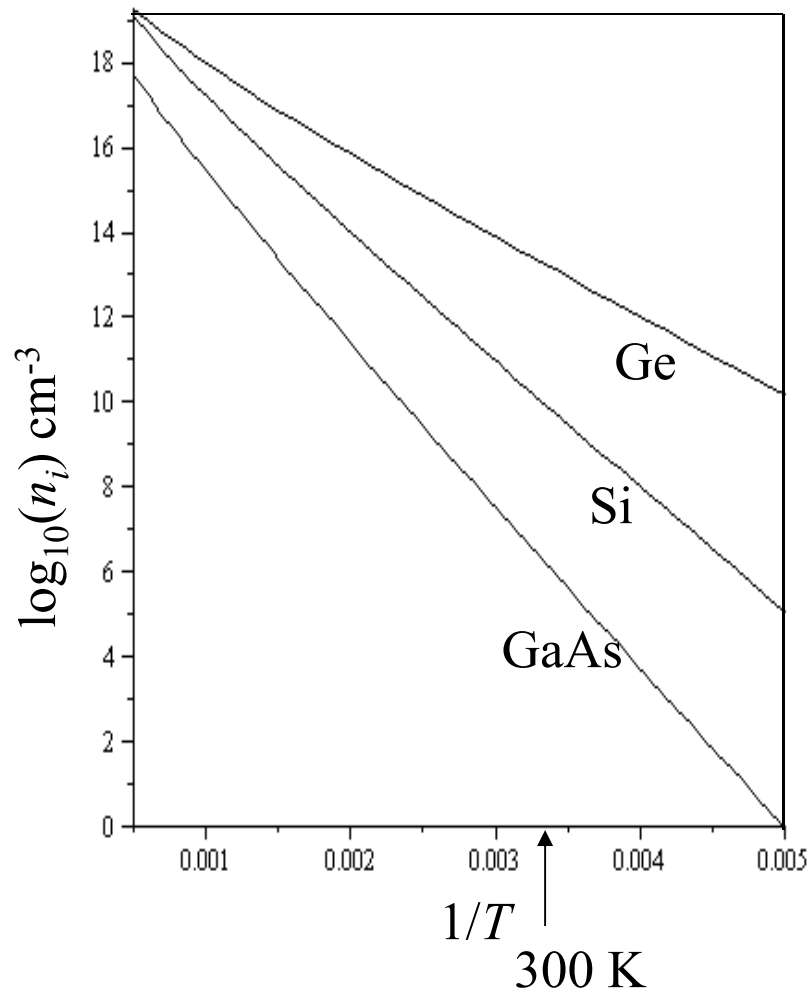
Intrinsic: $n = p$

Conductivity strongly temperature dependent near room temperature

Extrinsic: $n \neq p$

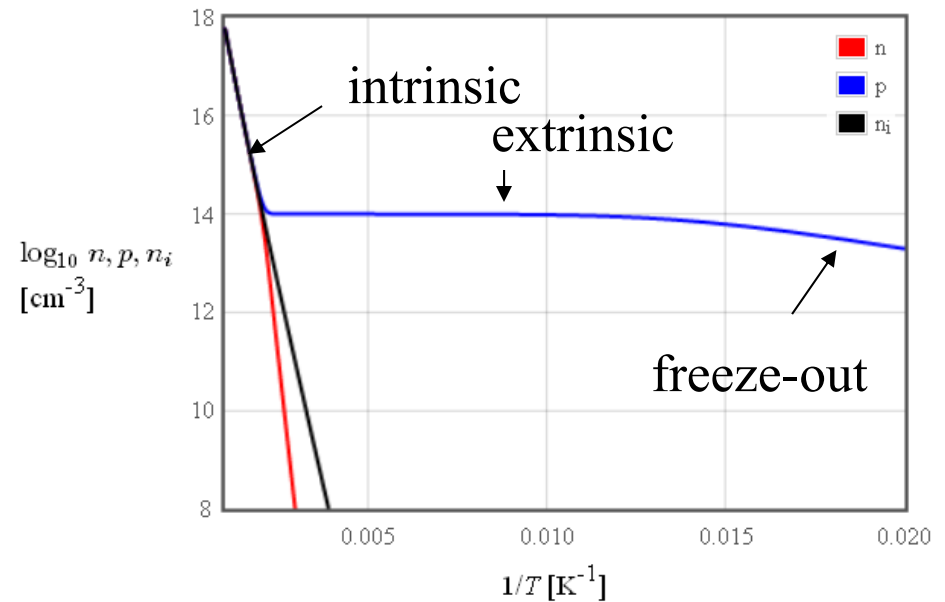
Conductivity almost temperature independent at room temperature

Intrinsic semiconductors



$$n_i = \sqrt{N_v N_c} \exp\left(-\frac{E_g}{2k_B T}\right)$$

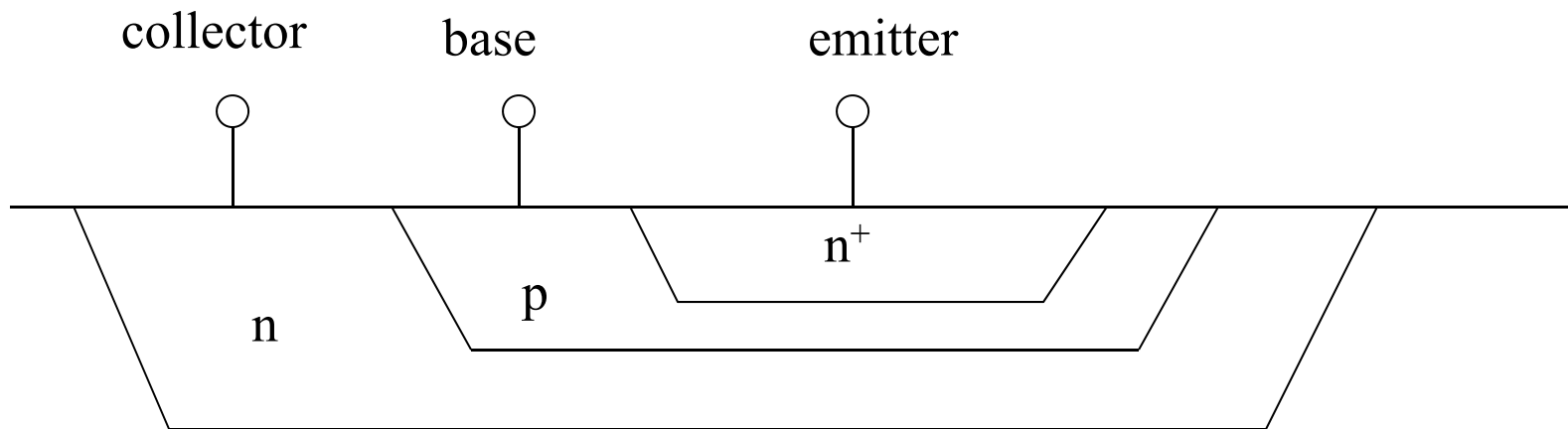
Extrinsic semiconductors



At high temperatures, extrinsic semiconductors have the same temperature dependence as intrinsic semiconductors.

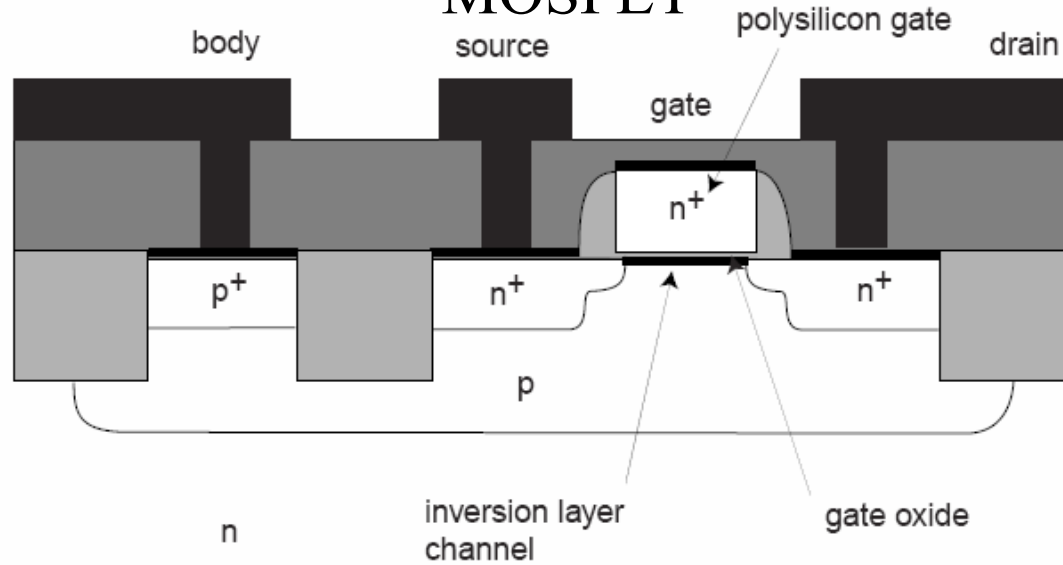
Why dope with donors AND acceptors?

Bipolar transistor

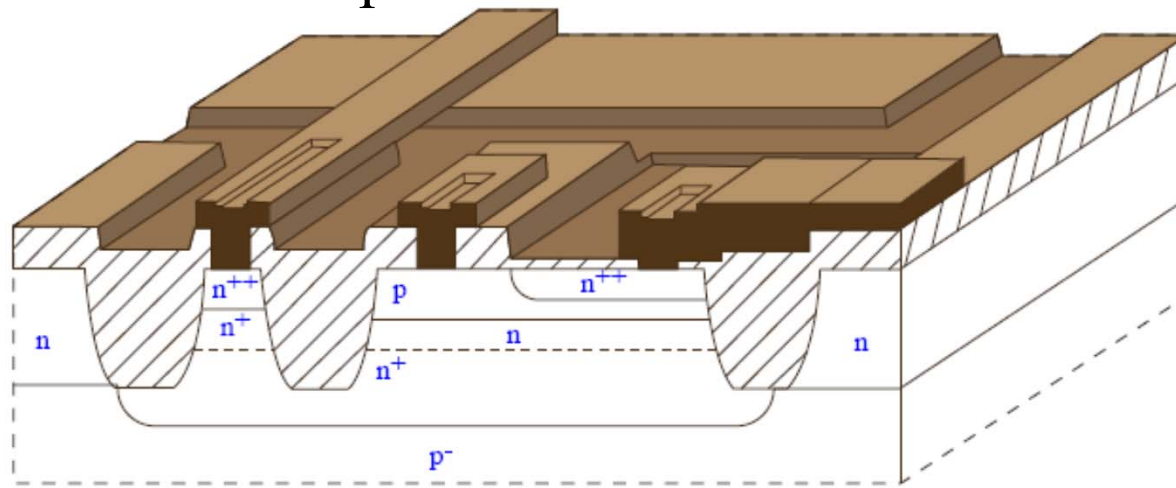


lightly doped p substrate

MOSFET



Bipolar Junction Transistor



Oxide isolated integrated BJT - a modern process

Ionized donors and acceptors

For $E_v + 3k_B T < E_F < E_c - 3k_B T$ Boltzmann approximation

$$N_D^+ = \frac{N_D}{1 + 2 \exp\left(\frac{E_F - E_D}{k_B T}\right)} \qquad N_A^- = \frac{N_A}{1 + 4 \exp\left(\frac{E_A - E_F}{k_B T}\right)}$$

4 for materials with light holes and heavy holes (Si)
2 otherwise

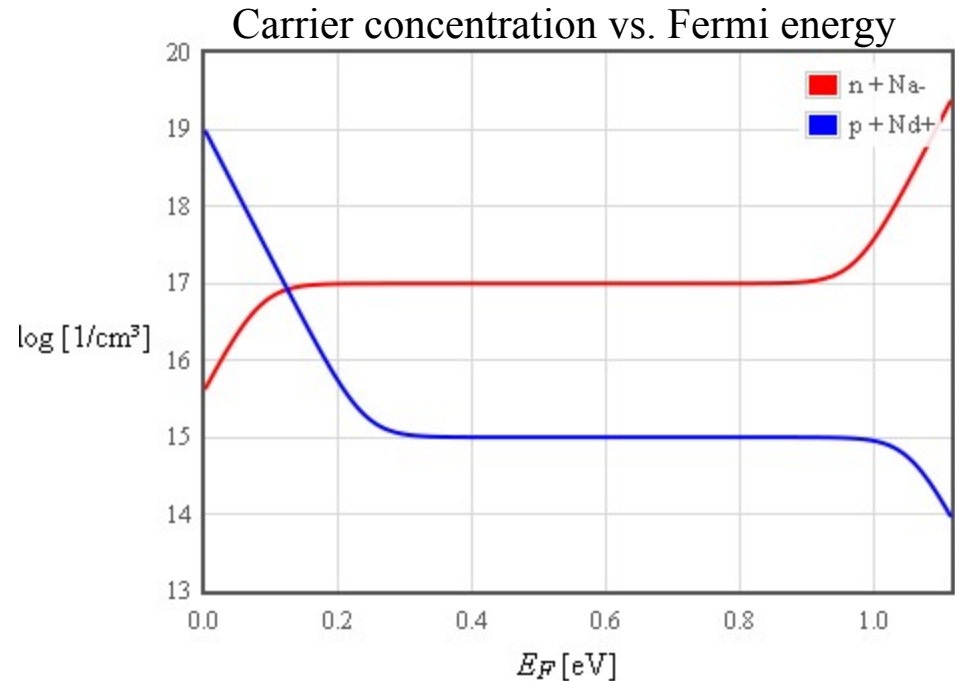
N_D = donor density cm^{-3} N_D^+ = ionized donor density cm^{-3}

N_A = acceptor density cm^{-3} N_A^- = ionized acceptor density cm^{-3}

Mostly, $N_D^+ = N_D$ and $N_A^- = N_A$

Charge neutrality

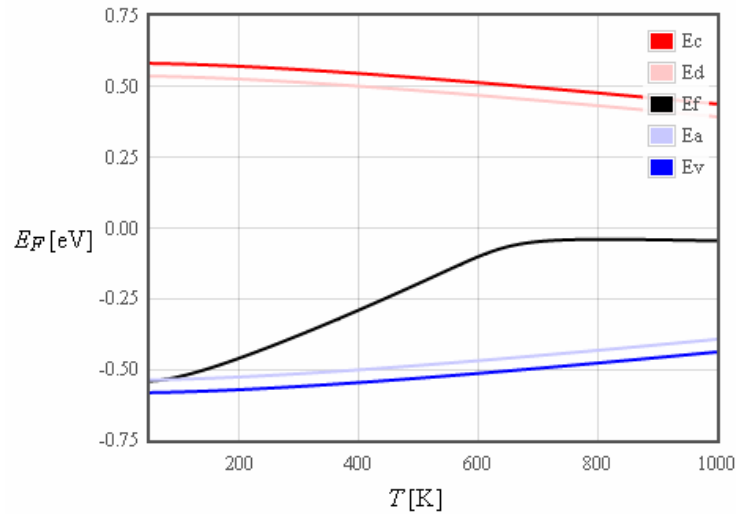
$$n + N_A^- = p + N_D^+$$



```
for ($i=0; $i<500; $i++) {
    $Ef = $i*$Eg/500;
    $n=$Nc*pow($T/300,1.5)*exp(1.6022E-19*($Ef-$Eg)/(1.38E-23*$T));
    $p=$Nv*pow($T/300,1.5)*exp(1.6022E-19*(-$Ef)/(1.38E-23*$T));
    $Namin = $Na/(1+4*exp(1.6022E-19*($Ea-$Ef)/(1.38E-23*$T)));
    $Ndplus = $Nd/(1+2*exp(1.6022E-19*($Ef-$Ed)/(1.38E-23*$T)));
}
```

E_f	n	p	N_d^+	N_a^-	$\log(n+N_a^-)$	$\log(p+N_d^+)$
0	4.16629283405	9.84E+18	1E+15	4.19743393218E+15	15.622983869	18.9930392318
0.00224	4.54358211887	9.0229075682E+18	1E+15	4.56020949614E+15	15.6589847946	18.9553946382
0.00448	4.95503779816	8.27366473417E+18	1E+15	4.95271809535E+15	15.694843609	18.9177504064
0.00672	5.40375389699	7.58663741327E+18	1E+15	5.37710747619E+15	15.7305487171	18.8801065693
0.00896	5.89210460791	6.95665026215E+18	1E+15	5.8256000025E+15	15.7660076057	18.8404621605

Calculating $E_F(T)$ numerically



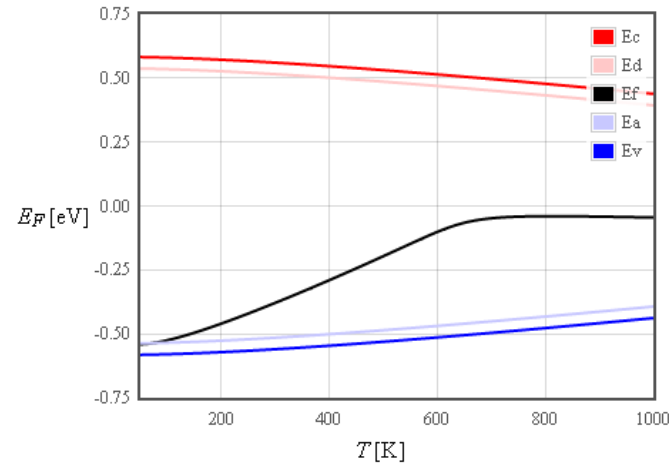
$N_c(300\text{ K}) = 2.78\text{E}19$	1/cm ³	Semiconductor <input type="button" value="Si"/> <input type="button" value="Ge"/> <input type="button" value="GaAs"/>
$N_v(300\text{ K}) = 9.84\text{E}18$	1/cm ³	
$E_g = 1.166 - 4.73\text{E-}4 * T * T / (T + 636)$	eV	
$N_d = 1\text{E}15$	1/cm ³	Donor <input type="button" value="P in Si"/> <input type="button" value="P in Ge"/> <input type="button" value="Si in GaAs"/>
$E_c - E_d = 0.045$	eV	
$N_a = 1\text{E}16$	1/cm ³	Acceptor <input type="button" value="B in Si"/> <input type="button" value="B in Ge"/> <input type="button" value="Si in GaAs"/>
$E_a - E_v = 0.045$	eV	
$T_1 = 50$	K	
$T_2 = 1000$	K	
<input type="button" value="Replot"/>		

[Source code](#)

E_f	n	p	N_d^+	N_a^-	T
0.0421442489848	2.40151024146E-36	1.42005993475E+16	1E+15	1.52148150693E+16	100
0.0432605499091	3.55188567934E-34	1.60496732529E+16	1E+15	1.70727565871E+16	104
0.0443992886238	3.6447725184E-32	1.79708501904E+16	1E+15	1.89867455992E+16	108
0.0455696761919	2.70091324524E-30	1.99342913907E+16	1E+15	2.0961617146E+16	112
0.046759290261	1.49291303432E-28	2.19530074854E+16	1E+15	2.29663635196E+16	116
0.0479826534344	6.34683409239E-27	2.39841986132E+16	1E+15	2.50167521691E+16	120
0.0492194372194	2.12451662366E-25	2.60634467518E+16	1E+15	2.70669670674E+16	124
0.0504951208621	5.73929239594E-24	2.81194143968E+16	1E+15	2.91550921724E+16	128

Fermi energy vs. temperature

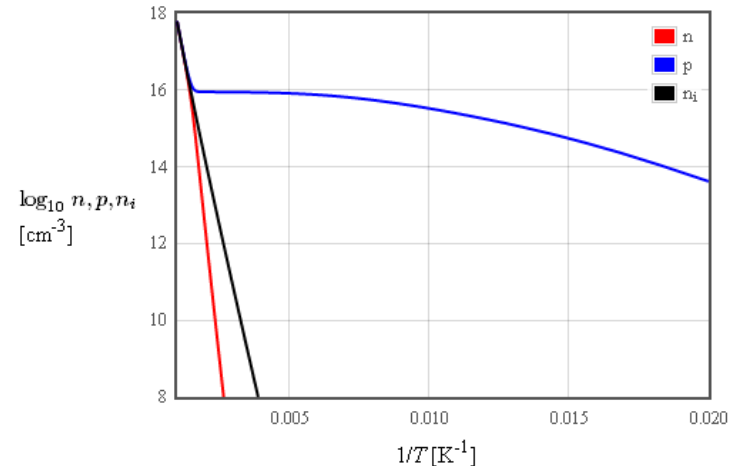
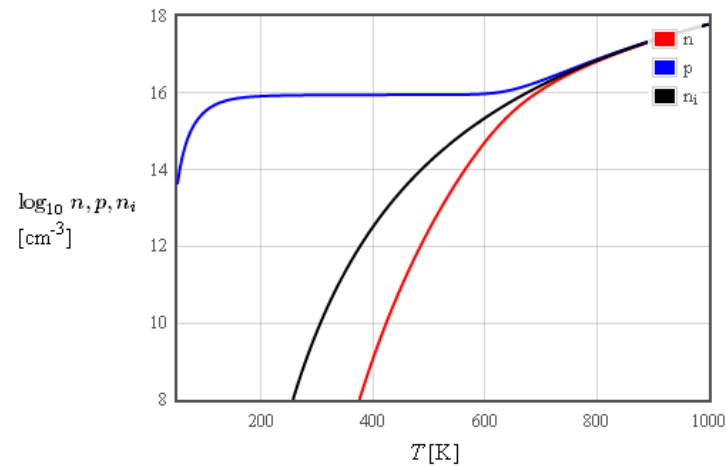
Fermi energy of an extrinsic semiconductor is plotted as a function of temperature. At each temperature the Fermi energy was calculated by requiring that [charge neutrality](#) be satisfied.



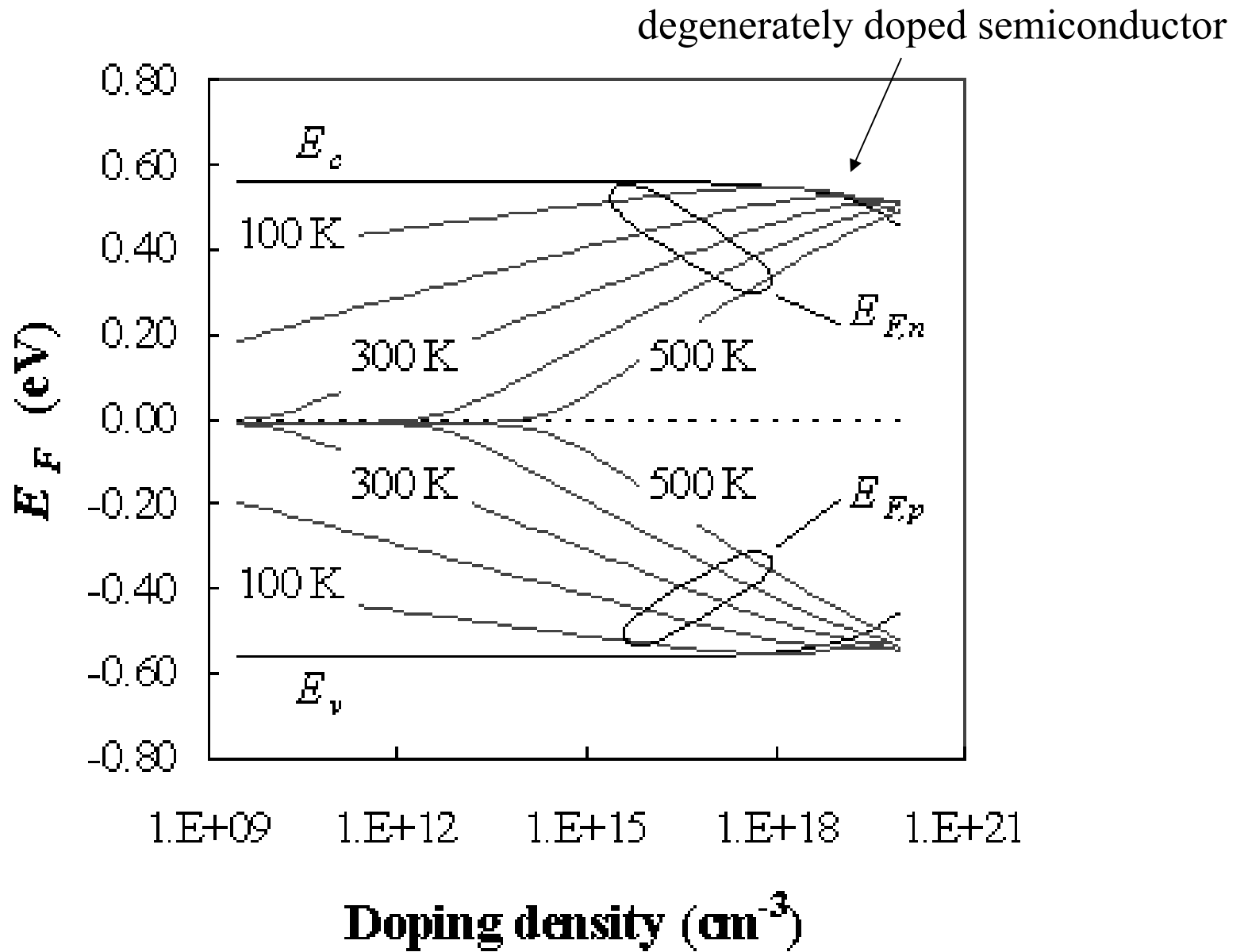
$N_c(300\text{ K}) = 2.78\text{E}19$	1/cm ³	Semiconductor <input type="button" value="Si"/> <input type="button" value="Ge"/> <input type="button" value="GaAs"/>
$N_v(300\text{ K}) = 9.84\text{E}18$	1/cm ³	
$E_g = 1.166 - 4.73\text{E} - 4 * T * T / (T + 636)$		eV
$N_d = 1\text{E}15$	1/cm ³	Donor <input type="button" value="P in Si"/> <input type="button" value="P in Ge"/> <input type="button" value="Si in GaAs"/>
$E_c - E_d = 0.045$	eV	
$N_a = 1\text{E}16$	1/cm ³	Acceptor <input type="button" value="B in Si"/> <input type="button" value="B in Ge"/> <input type="button" value="Si in GaAs"/>
$E_a - E_v = 0.045$	eV	
$T_1 = 50$	K	
$T_2 = 1000$	K	
<input type="button" value="Replot"/>		

Once the Fermi energy is known, the carrier densities n and p can be calculated from the formulas, $n = N_c \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_F - E_c}{k_B T}\right)$ and $p = N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{E_v - E_F}{k_B T}\right)$.

The intrinsic carrier density is $n_i = \sqrt{N_c \left(\frac{T}{300}\right)^{3/2} N_v \left(\frac{T}{300}\right)^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right)}$.



<http://lamp.tu-graz.ac.at/~hadley/psd/L4/eftplot.html>



Degenerate semiconductor

Heavily doped semiconductors are called degenerately doped

$N_D > 0.1 N_c \rightarrow E_F$ in the conduction band

$N_A > 0.1 N_v \rightarrow E_F$ in the valence band

Heavy doping narrows the band gap

The Boltzmann approximation is not valid

Degenerate semiconductors = metal

Carrier Transport

Ballistic transport

Drift

Diffusion

Generation and recombination

The continuity equation

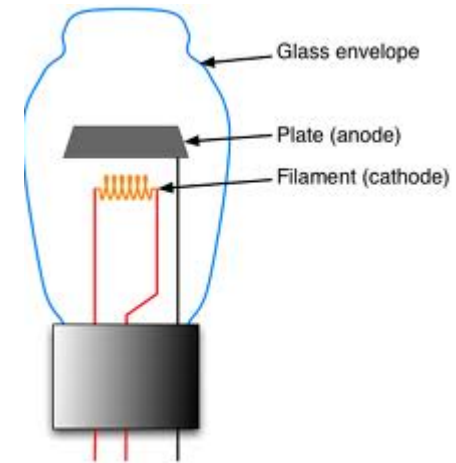
High field effects

Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m \frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0t + \vec{x}_0$$



Electrons moving in an electric field follow parabolic trajectories like a ball in a gravitational field.

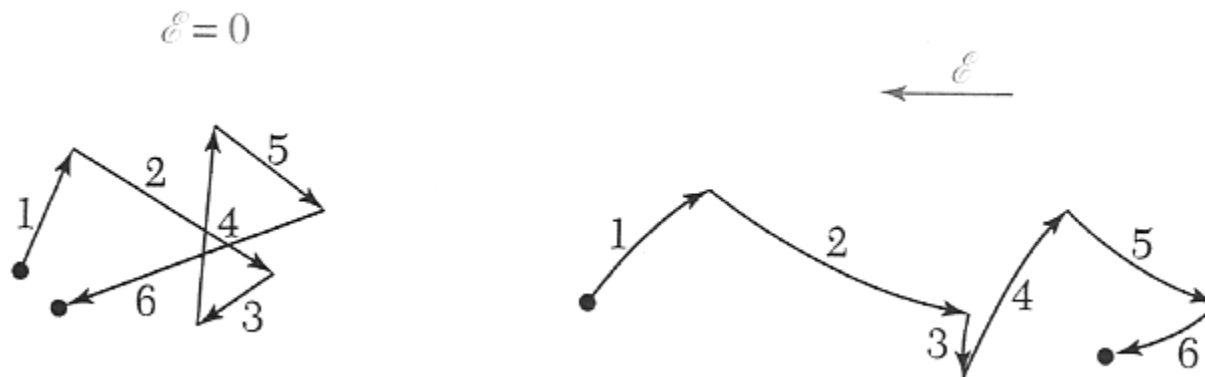
Drift

The electrons scatter and change direction after a time τ_{sc} .

Classical equipartition: $\frac{1}{2} m v_{th}^2 = \frac{3}{2} k_B T$

At 300 K, $v_{th} \sim 10^7$ cm/s.

mean free path: $\ell = v_{th} \tau_{sc} \sim 10$ nm ~ 200 atoms



Drift (diffusive transport)

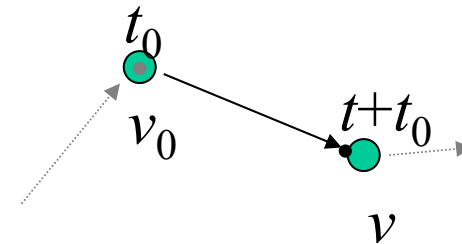
$$\vec{F} = -e\vec{E} = m^* \vec{a} = m^* \frac{d\vec{v}}{dt}$$

$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*} (t - t_0)$$

$$\langle \vec{v}_0 \rangle = 0$$

$$\langle t - t_0 \rangle = \tau_{sc}$$

time between two collisions



$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v}$$

drift velocity: $\vec{v}_{d,n} = -\mu_n \vec{E}$

$$\vec{v}_{d,p} = \mu_p \vec{E}$$

Drift

drift velocity: $\vec{v}_{d,n} = -\mu_n \vec{E}$ $\vec{v}_{d,p} = \mu_p \vec{E}$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p) \vec{E} = \sigma \vec{E}$$

$$\mu = \frac{-e\tau_{sc}}{m^*} = \frac{-e\ell}{m^* v}$$

for Si: $\mu_n = 1500 \text{ cm}^2/\text{Vs}$
 $\mu_p = 450 \text{ cm}^2/\text{Vs}$

For $E = 1000 \text{ V/cm}$ $v_d = 10^6 \text{ cm/s}$

C:\Program Files\Cornell\SSS\winbin\drude.exe



quit

display:

large

configure...

presets

help...

show graph

show

average

run

show graph

show

average

time (ps) 32.3



position: (4.14, -0.66) 10^{-6} m

initialize

E_x (10^4 V/m): 10

E_y (10^4 V/m): 10

B_z (T): 2

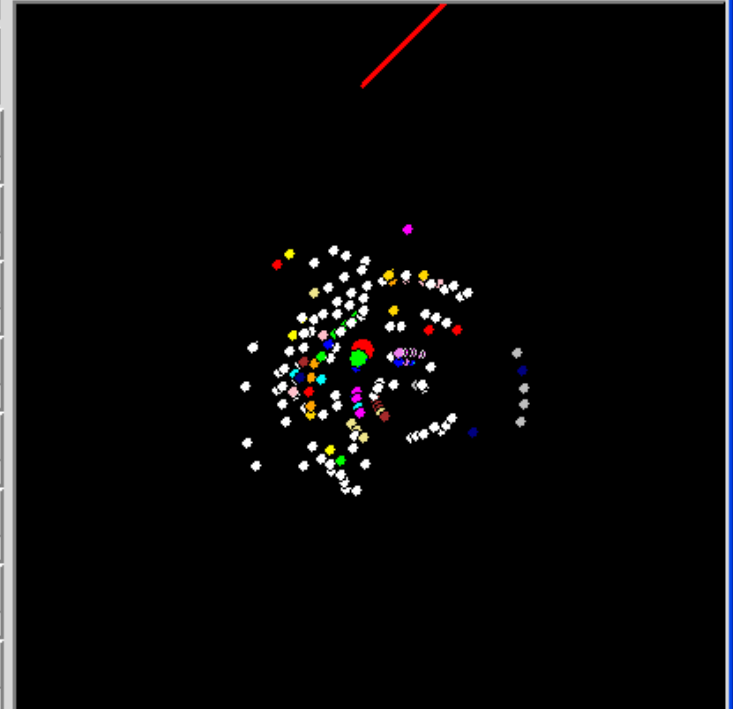
tau (ps): 1.00e+00

temperature (K): 300

omega (10^{12} /sec): 0

phase (radians): 0.0

speed 2



velocity: (0,0) 10^4 m/s