

5. Carrier Transport

Oct. 31, 2018

Review

n-type

$$N_D > N_A \quad n = N_D - N_A$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D - N_A}$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = N_D - N_A$$

⇓

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D - N_A}\right)$$

p-type

$$N_A > N_D \quad p = N_A - N_D$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A - N_D}$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right) = N_A - N_D$$

⇓

$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A - N_D}\right)$$

Drift

drift velocity: $\vec{v}_{d,n} = -\mu_n \vec{E}$ $\vec{v}_{d,p} = \mu_p \vec{E}$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p)\vec{E} = \sigma\vec{E}$$

for Si: $\mu_n = 1500 \text{ cm}^2/\text{Vs}$
 $\mu_p = 450 \text{ cm}^2/\text{Vs}$

Drift

		E_g (eV)	μ_n (cm ² /V-s)	μ_p (cm ² /V-s)	m_n^*/m_0 (m_l, m_t)	m_p^*/m_0 (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm ³)	Melting point (°C)
Si	(i/D)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(i/D)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(i/W)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(i/Z)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(i/Z)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(i/Z)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(i/Z)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(d/Z)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(d/Z, W)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(d/Z)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(d/Z)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(d/Z)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(d/Z)	0.18	10 ⁵	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(d/Z, W)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(d/Z)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(d/Z)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(d/W, Z)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(d/W)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(d/Z)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(i/H)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(i/H)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(i/H)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

Solid state electronic devices, Streetman and Banerjee

$$\vec{v}_{d,n} = -\mu_n \vec{E} \quad \vec{v}_{d,p} = \mu_p \vec{E} \quad \vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = (ne\mu_n + pe\mu_p) \vec{E} = \sigma \vec{E}$$

Matthiessen's rule

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

↑
phonons, temperature dependent

↑ mostly temperature independent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

↑
doping increases the conductivity
by increasing the carrier density
but decreases the mobility

Mobility calculator

INPUTS

Semiconductor material	c-silicon	Excess electron conc. Δn	1	cm ⁻³
Dopant atom	boron	Excess hole conc. Δp	1	cm ⁻³
Ionised dopant conc. N_{dop}	1E+16	Electron eff. lifetime $\tau_{eff\ e}$	1E-4	s
Temperature T	300	Hole eff. lifetime $\tau_{eff\ h}$	1E-4	s

OUTPUTS

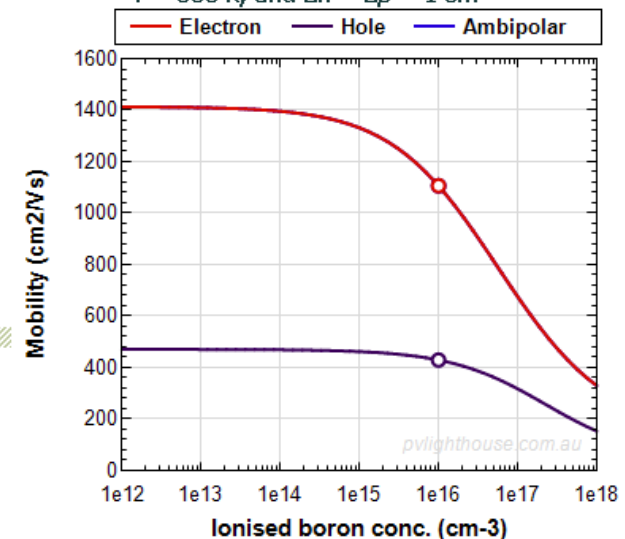
	Carrier concentrations			Carrier mobility etc.		
	Equilibrium n_0, p_0 (cm ⁻³)	Excess $\Delta n, \Delta p$ (cm ⁻³)	Net n, p (cm ⁻³)	Mobility μ_e, μ_h, μ_a (cm ² V ⁻¹ s ⁻¹)	Diffusivity D_e, D_h, D_a (cm ² s ⁻¹)	Diff Length L_e, L_h, L_a (cm)
Intrinsic	9300	1.0	9300	1107	28.61	5.349E-2
Majority	1.0E+16	1.0	1.0E+16	429.3	11.10	3.331E-2
Minority				1107	28.61	5.349E-2

	Resistivity (Ω -cm)	
Intrinsic	ρ_0	1.454
Majority	ρ	1.454

More inputs

Mobility	
Ionised dopant conc.	
	1E+12 cm ⁻³
	1E+18 cm ⁻³
Points	50

Mobility vs ionised dopant concentration for boron-doped c-silicon with $T = 300\text{ K}$, and $\Delta n = \Delta p = 1\text{ cm}^{-3}$



$$\mu = \mu_{min} + \frac{\mu_{max} - \mu_{min}}{1 + (N/N_{ref})^\gamma}$$

For Electrons:

$$\mu_{min} = 47 \left(\frac{T}{300}\right)^{-1,23} \frac{\text{cm}^2}{\text{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 1373 \left(\frac{T}{300}\right)^{-2,38} \frac{\text{cm}^2}{\text{Vs}}$$

$$N_{ref} = 1,05 \cdot 10^{17} \left(\frac{T}{300}\right)^{3,65} \text{cm}^{-3}; \gamma = 0,68 \left(\frac{T}{300}\right)^{-0,32}$$

For Holes:

$$\mu_{min} = 36 \left(\frac{T}{300}\right)^{-0,87} \frac{\text{cm}^2}{\text{Vs}}$$

$$\Delta\mu = \mu_{max} - \mu_{min} = 438 \left(\frac{T}{300}\right)^{-2,01} \frac{\text{cm}^2}{\text{Vs}}$$

$$N_{ref} = 2,85 \cdot 10^{17} \left(\frac{T}{300}\right)^{2,93} \text{cm}^{-3}; \gamma = 0,65 \left(\frac{T}{300}\right)^{0,26}$$

Resistivity calculator

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

Calculator

Advanced inputs

About

EN 中文

RELATED LINKS

Mobility calculator

PV Education: 4 Point Probe

PV Education: Base Resistance

Dopant concentration, N_D

1E+15 cm^{-3}

Resistivity, ρ

4.58 $\Omega \cdot \text{cm}$

MATERIAL INPUTS

Semiconductor: c-silicon

Dopant species: Phosphorus

Temperature T : 300 K

PLOT

Plot: Resistivity against Dopant concentration

Sweep limits and steps:

$N_D \text{ min}$: 1E+12 cm^{-3}

$N_D \text{ max}$: 1E+20 cm^{-3}

Steps: 100

Resistivity vs dopant concentration

Resistivity ($\Omega \cdot \text{cm}$)

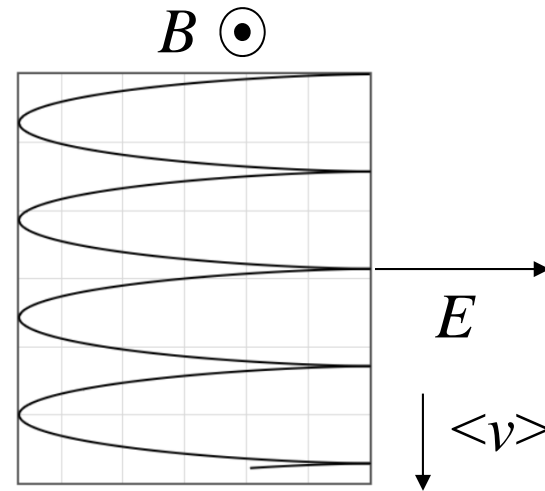
Dopant concentration (cm^{-3})

<http://www.pvlighthouse.com.au/calculators/Resistivity%20calculator/Resistivity%20calculator.aspx>

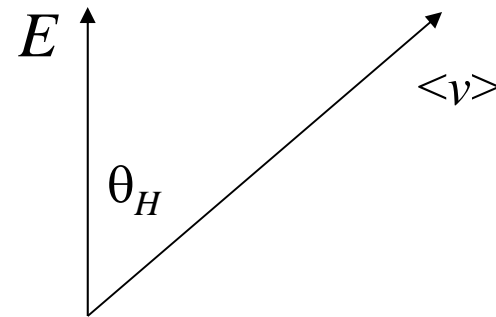
Crossed E and B fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e(\vec{E} + \vec{v} \times \vec{B})$$



Diffusive transport



Hall angle:

$$\theta_H = \tan^{-1} \left(-\frac{eB_z \tau_{sc}}{m^*} \right)$$

Magnetic field (diffusive transport)

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If B is in the z -direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

$$-\mu E_z = v_{dz}$$

Magnetic field

$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

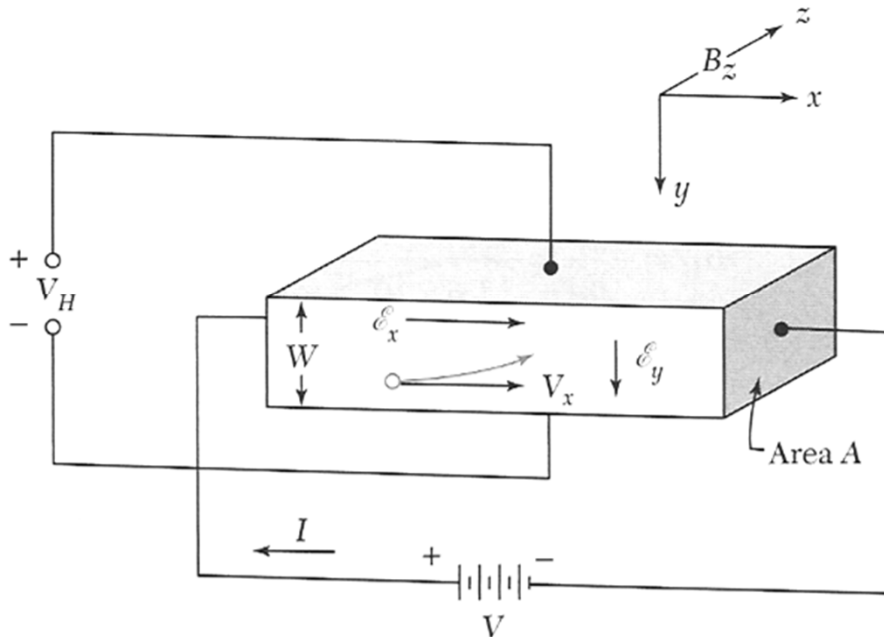
$$v_{d,z} = -\mu E_z$$

If $E_y = 0$,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan \theta_H = -\mu B_z$$

The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

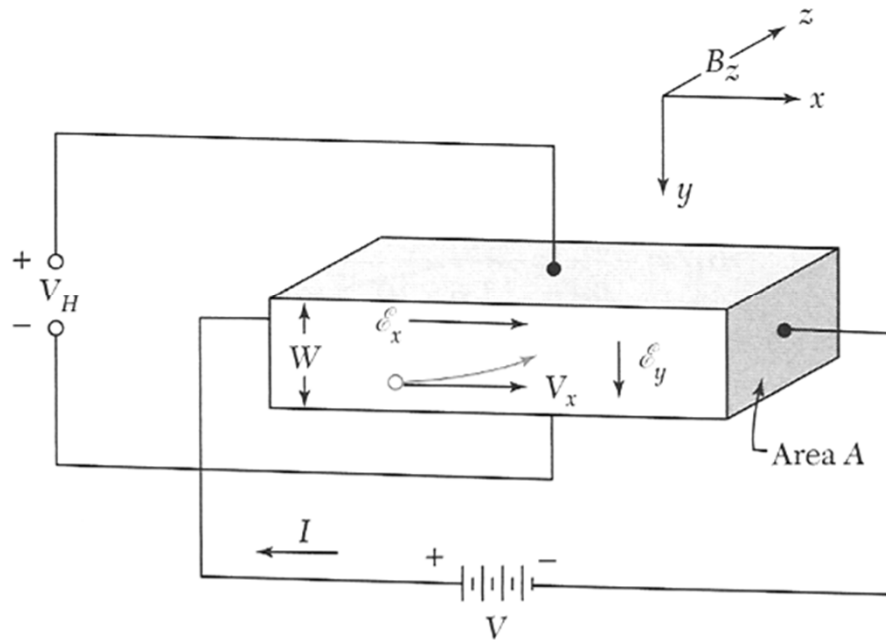
$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

If $v_{d,y} = 0$,

$$E_y = v_x B_z = V_H / W = R_H j_x B_z \quad V_H = \text{Hall voltage}, R_H = \text{Hall Constant}$$

The Hall Effect (diffusive regime)



$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$E_y = v_x B_z = V_H / W = R_H j_x B_z$$

V_H = Hall voltage, R_H = Hall Constant

$$j_x = I / A$$

$$v_x = -j_x / ne \quad \text{for n-type}$$

$$v_x = j_x / pe \quad \text{for p-type}$$

$$R_H = -1/ne \quad \text{for n-type}$$

$$R_H = 1/pe \quad \text{for p-type}$$

Ballistic transport in transistors

The mean free path ~ 100 nm $>$ gate length ~ 20 nm

v not proportional to E

$$\vec{v} = \mu \vec{E}$$

j not proportional to E

$$\vec{j} = \sigma \vec{E}$$

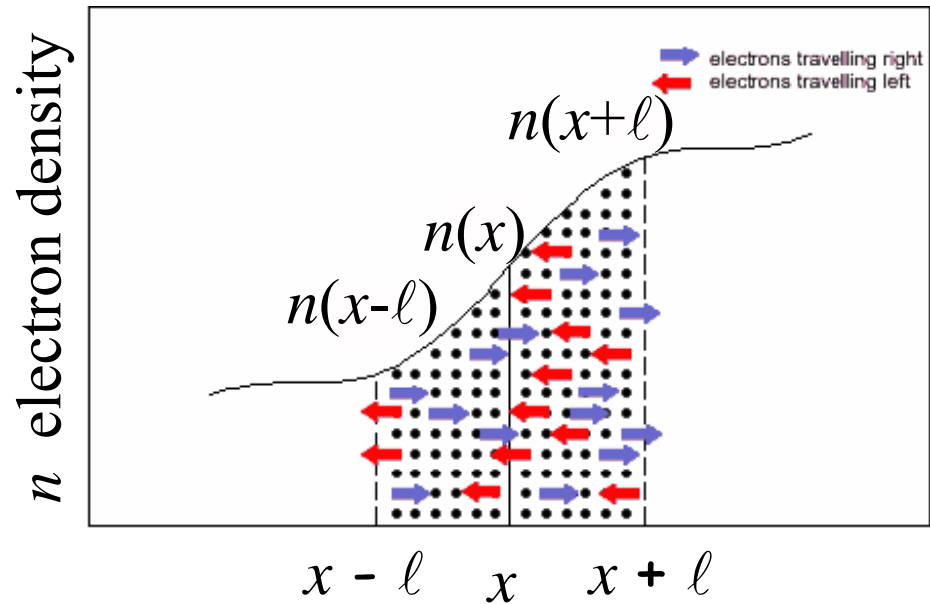
nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

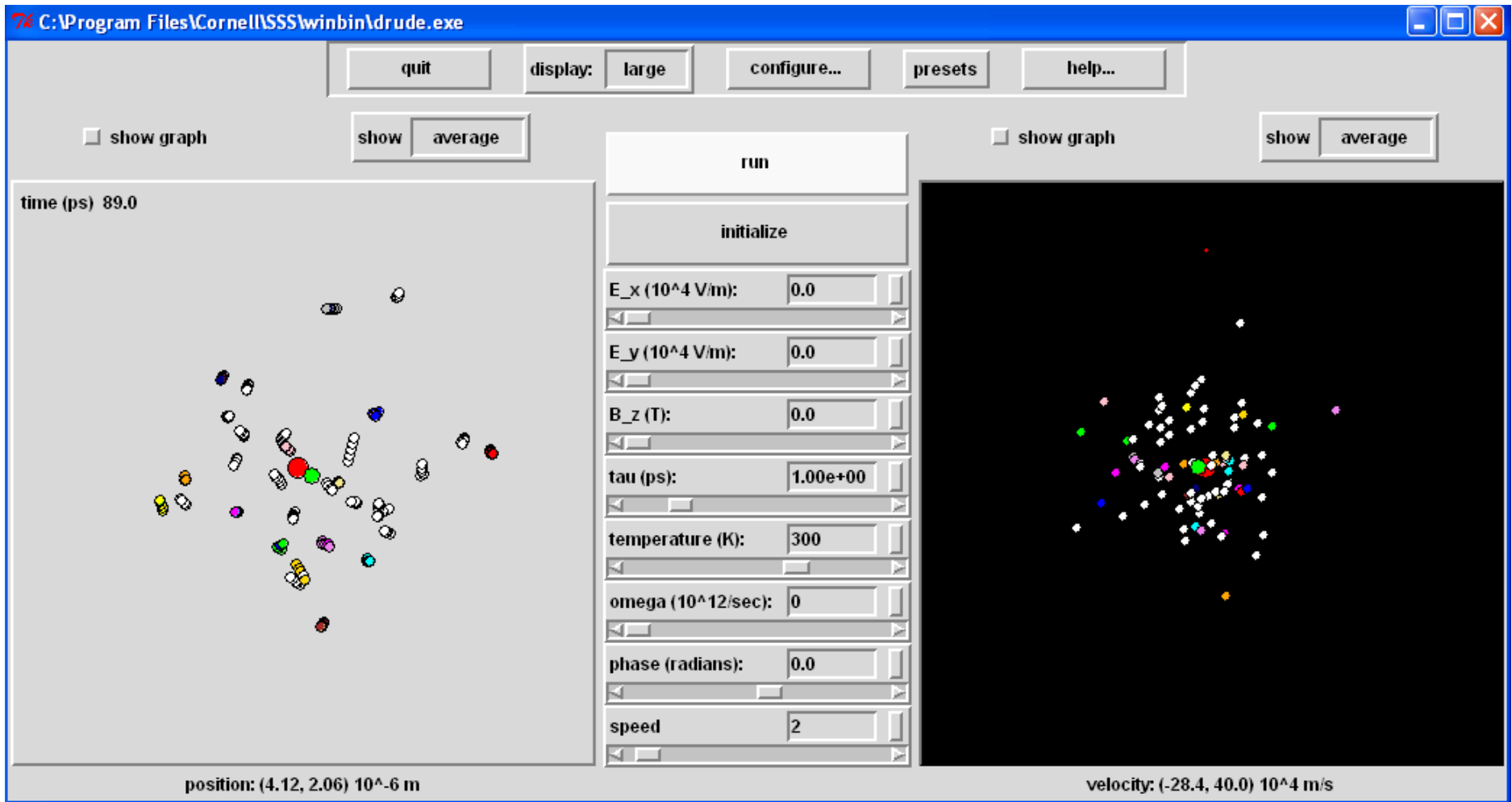
Diffusion

$$j_{n,diff} = |e| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -|e| D_p \frac{dp}{dx}$$




Diffusion is from high concentration to low concentration.



If no forces are applied, the electrons diffuse.
 The average velocity moves against an electric field.
 In just a magnetic field, the average velocity is zero.
 In an electric and magnetic field, the electrons move in a straight line at the Hall angle.

Einstein relation

$\vec{E} = -\nabla V$  $n(x) = A \exp\left(\frac{-eV(x)}{k_B T}\right)$ Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T} A \exp\left(\frac{-eV_{pot}}{k_B T}\right) \nabla V = -\frac{ne}{k_B T} \nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD \frac{ne\vec{E}}{k_B T} = 0$$

$$D = \frac{\mu k_B T}{e}$$

Über die von der molekular-kinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen

Current Density Equations

Drift



Diffusion



$$\vec{j}_n = -ne\mu_n\vec{E} + eD_n\nabla n$$

$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

Current Density Equations

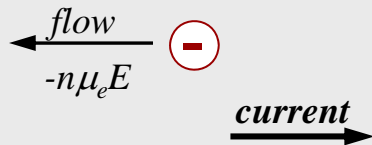
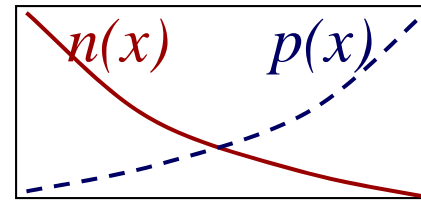
note: electron and hole currents have same direction

electric current = charge \times particle flow

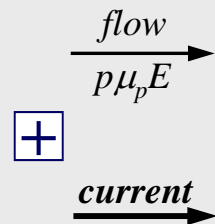
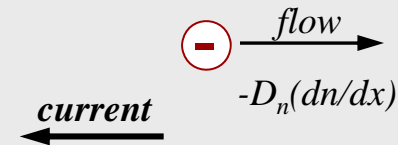
drift



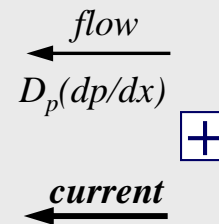
diffusion



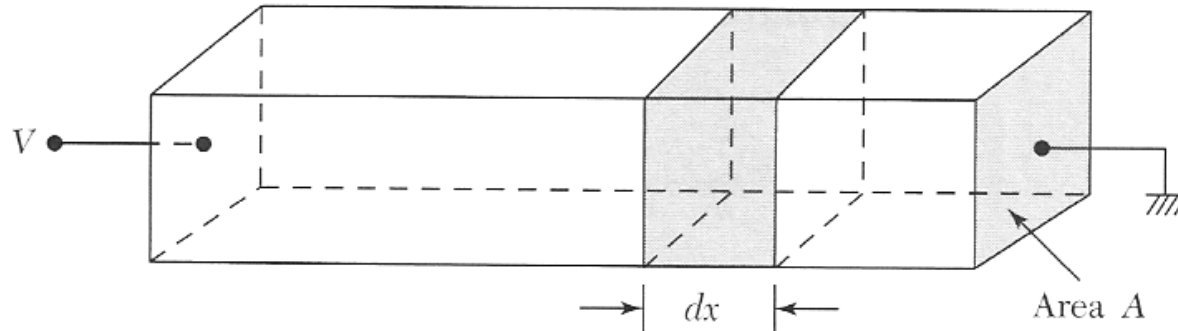
$$j_e = -e \times \text{flow}$$



$$j_p = e \times \text{flow}$$



Continuity equations

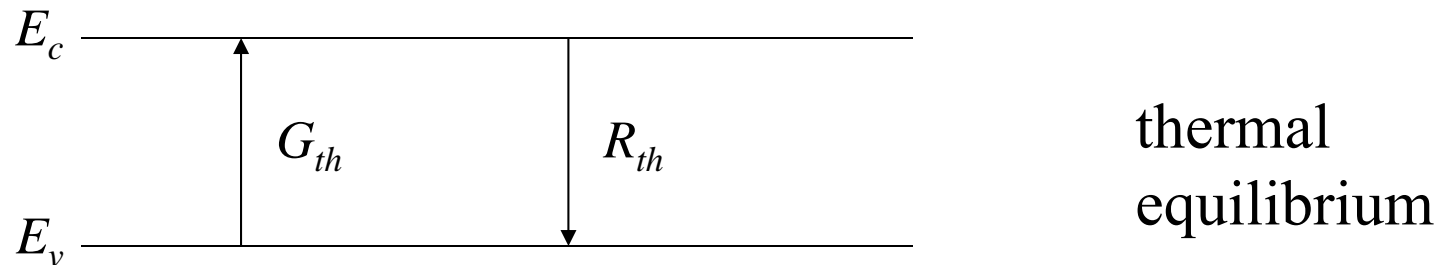


$$\frac{\partial n}{\partial t} = \frac{1}{e} \nabla \cdot \vec{j}_n + G_n - R_n$$

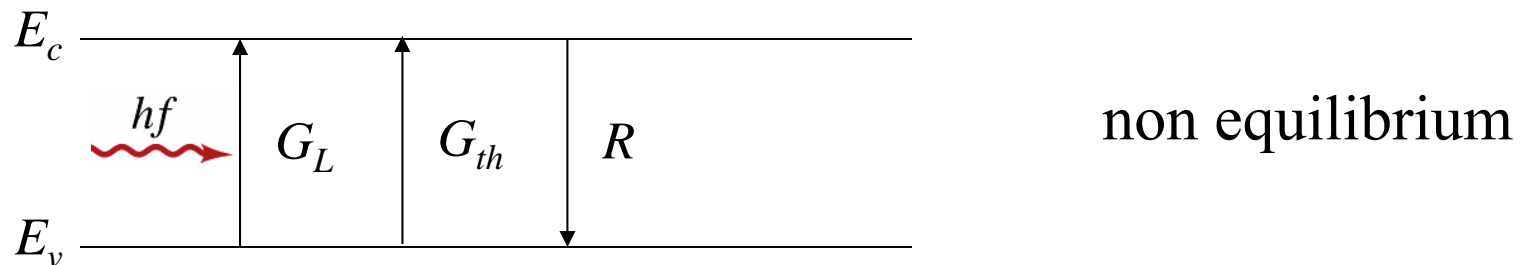
$$\frac{\partial p}{\partial t} = -\frac{1}{e} \nabla \cdot \vec{j}_p + G_p - R_p$$

j_n and j_p consist of drift and diffusion terms

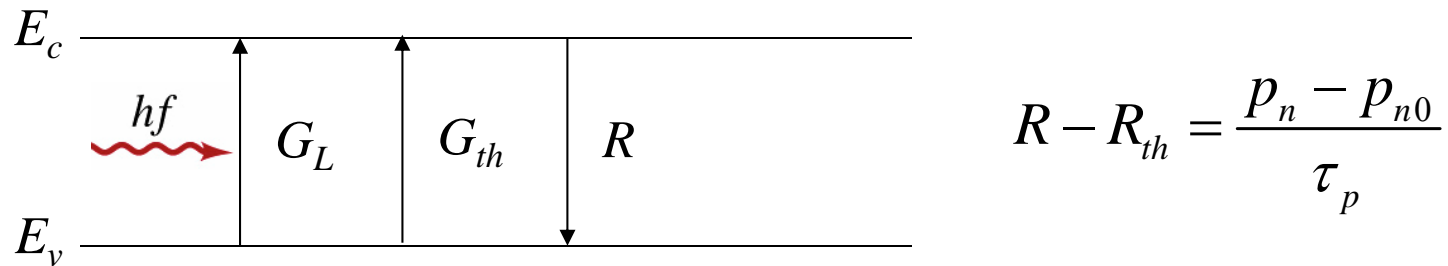
Generation and Recombination



Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



Recombination

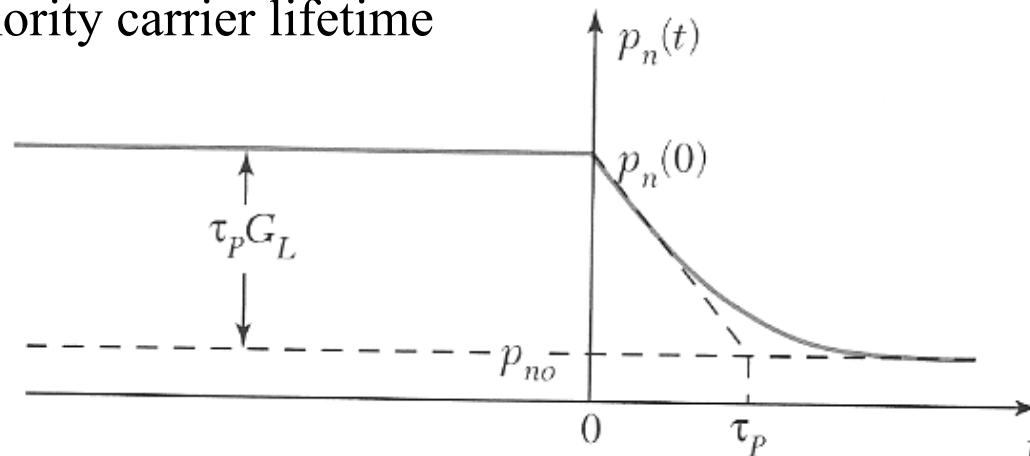


Recombination rate is limit by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration

p_{n0} (or n_{p0}) = equilibrium minority carrier concentration

τ_p = minority carrier lifetime



minority carrier lifetimes

p-type

$$n_p(t) = n_{excess} \exp(-t / \tau_n) + n_{p0}$$

n-type

$$p_n(t) = p_{excess} \exp(-t / \tau_p) + p_{n0}$$

minority carrier
lifetimes



$$np = n_i^2$$

Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{e} \nabla \cdot \vec{j}_n + G_n - R_n$$

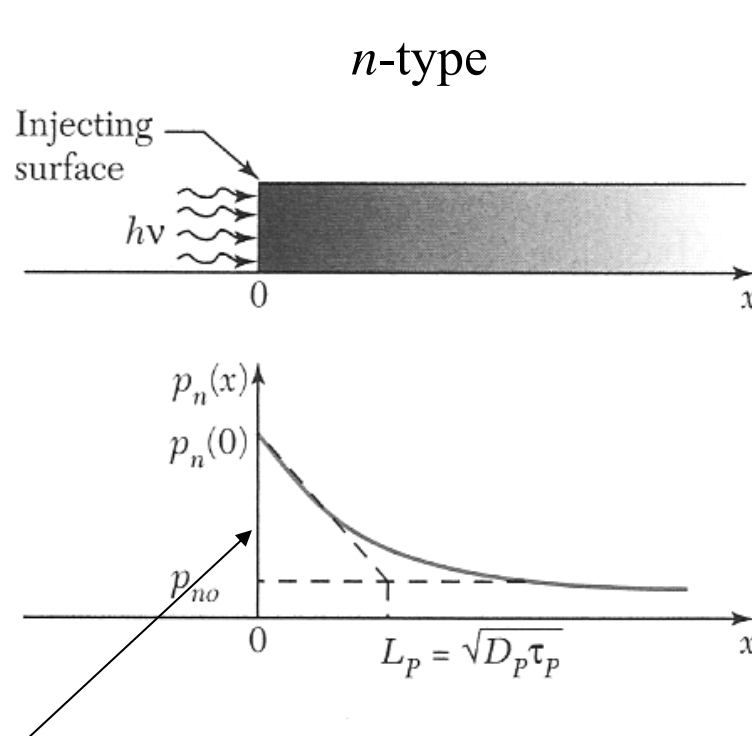
drift: $\vec{j}_n = -ne\mu_n\vec{E}$ $\nabla \cdot \vec{j}_n = -en\mu_n\nabla \cdot \vec{E} - e\nabla n\mu_n\vec{E}$

diffusion: $\vec{j}_{n,diff} = |e|D_n\nabla n$ $\nabla \cdot \vec{j}_{n,diff} = |e|D_n\nabla^2 n$

$$\frac{\partial n}{\partial t} = n\mu_n\nabla \cdot \vec{E} + \nabla n\mu_n\vec{E} + D_n\nabla^2 n + G_n - \frac{n - n_0}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p\mu_p\nabla \cdot \vec{E} - \nabla p\mu_p\vec{E} + D_p\nabla^2 p + G_p - \frac{p - p_0}{\tau_p}$$

Diffusion Length



Steady state

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion Length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \quad \Leftrightarrow \quad p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(\frac{-x}{L_p}\right)$$

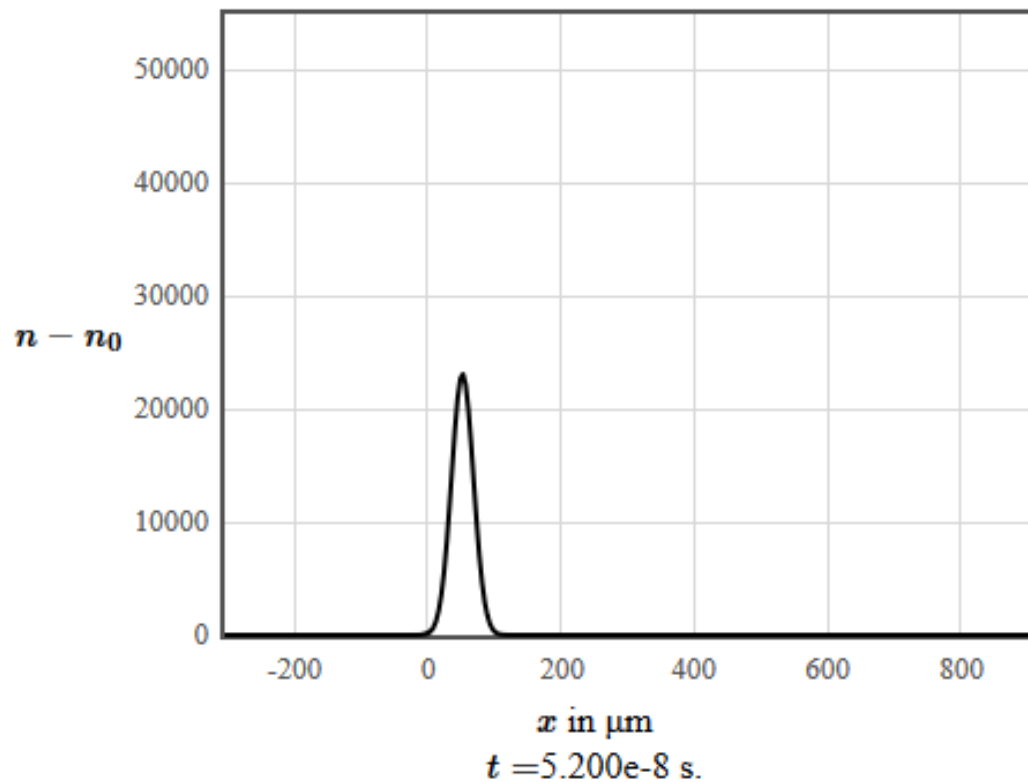
$$0 = \frac{D_p (p_n(0) - p_{n0})}{L_p^2} \exp\left(\frac{-x}{L_p}\right) - \frac{(p_n(0) - p_{n0})}{\tau_p} \exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

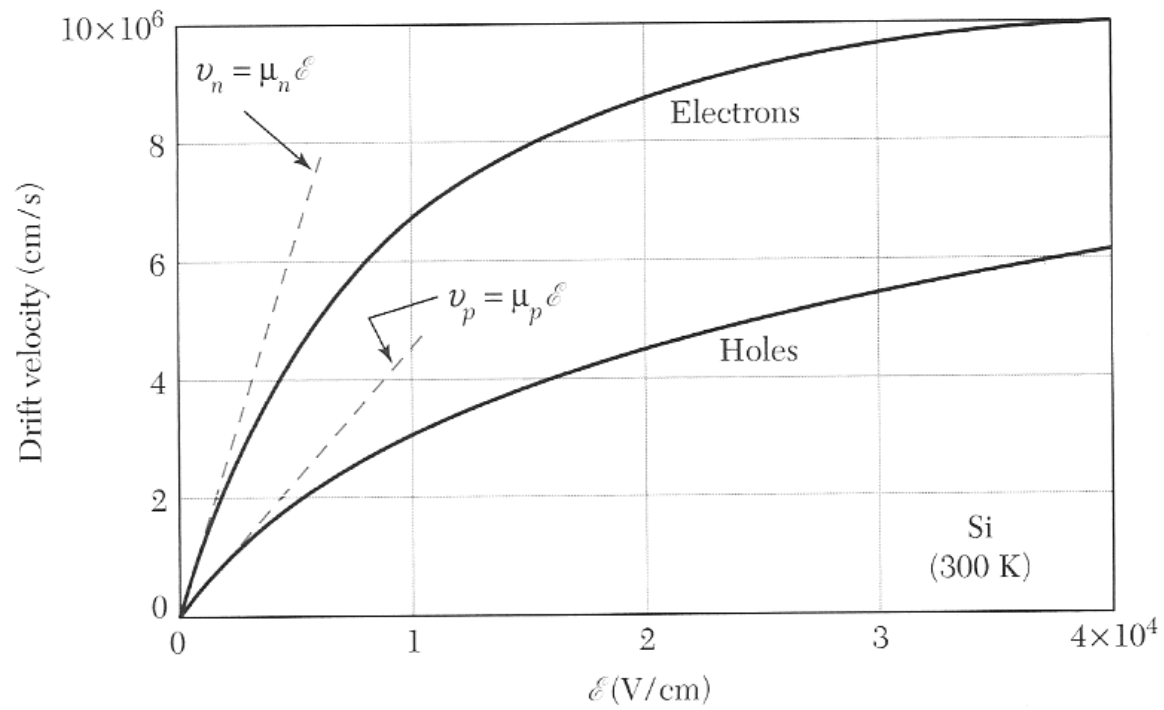
Haynes Shockley experiment

$$n_p(x, t) = \frac{n_{\text{generated}}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{(x - \mu_n E t)^2}{4D_n t}\right) \exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



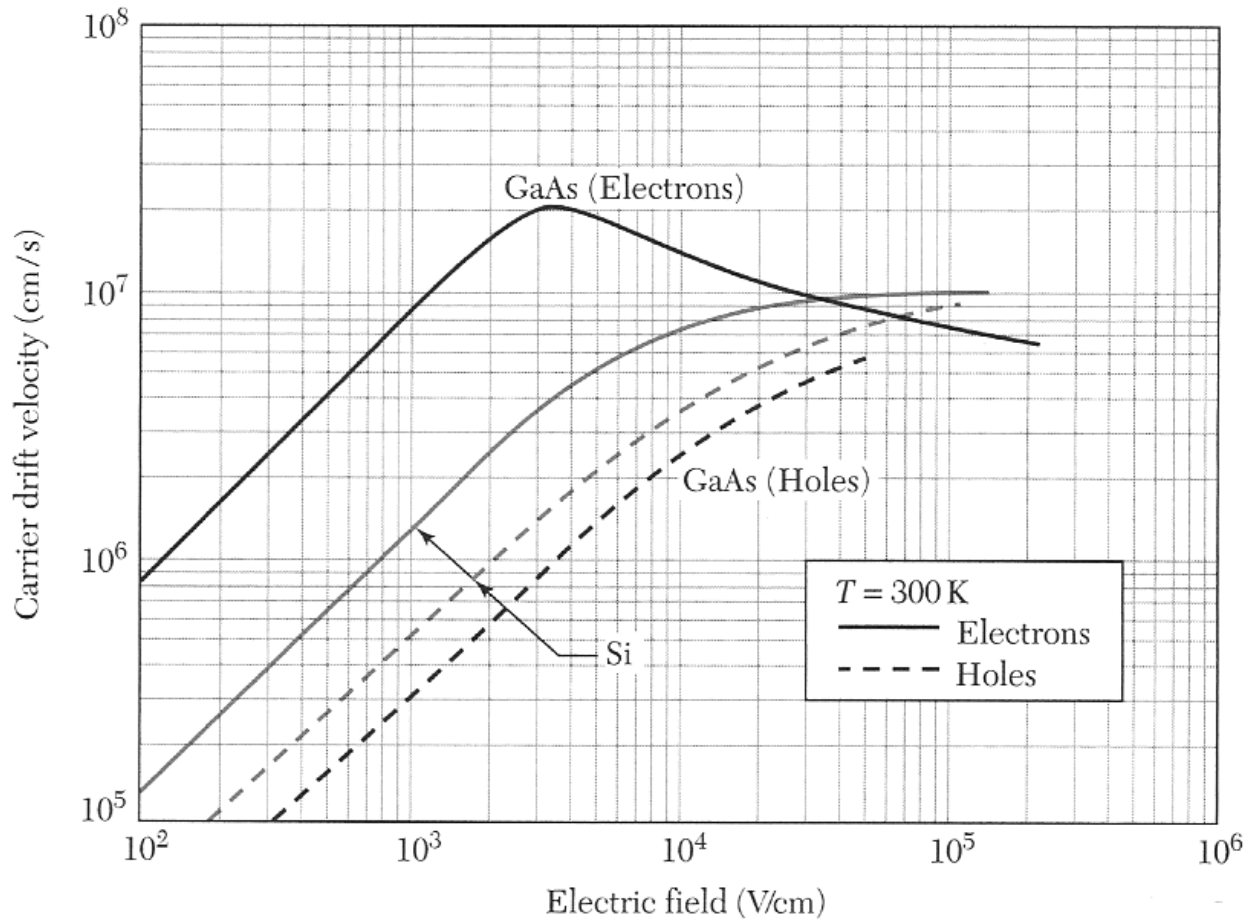
$\tau = 1\text{E-}6$ [s]
 $E = 100$ [V/cm]
 $\mu = 1000$ [$\text{cm}^2/\text{V s}$]
 $D = \mu k_B T / e = 0.00258$ [m^2/s]
 $L = \sqrt{D\tau} = 50.8$ [μm]

High Fields

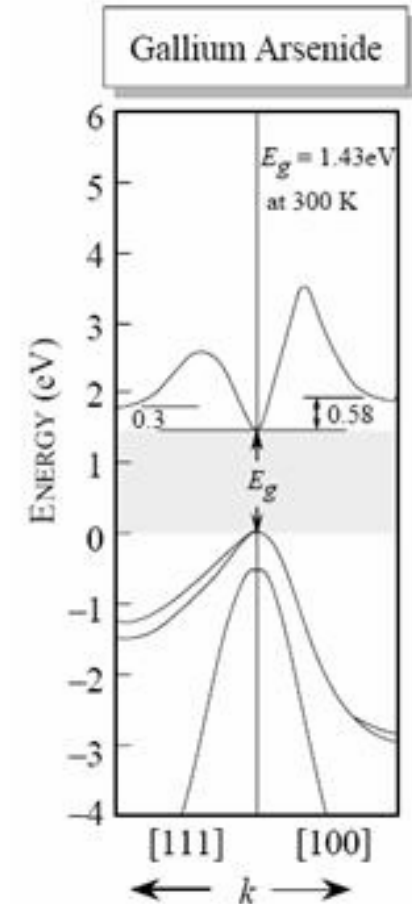


Silicon

High Fields



GaAs

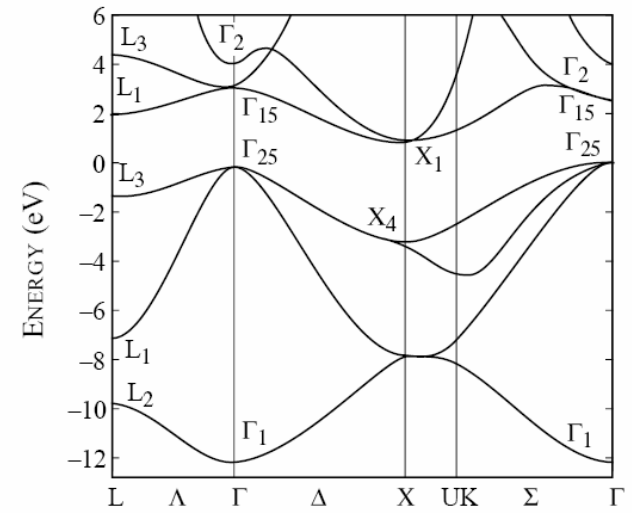


Impact ionization

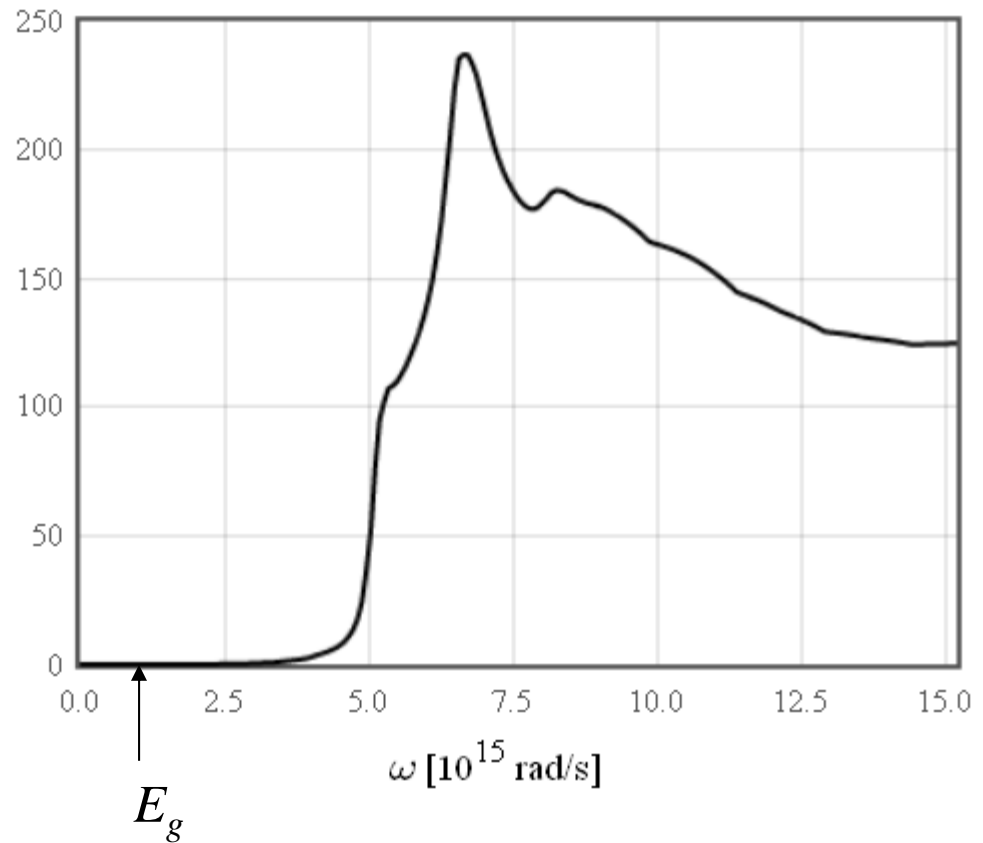
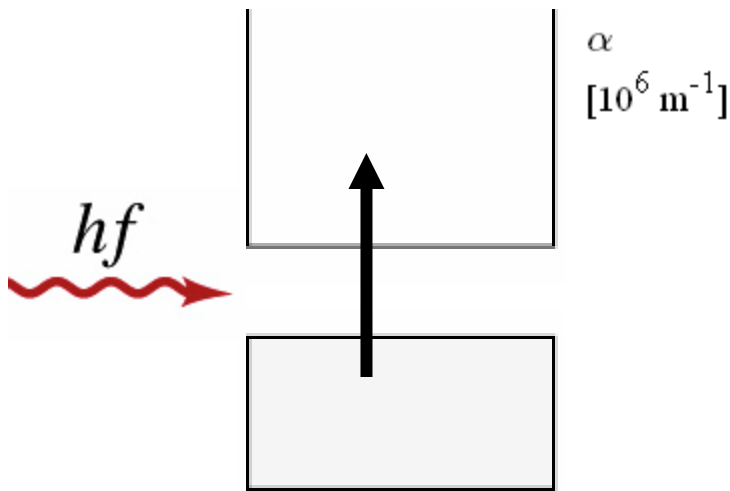
Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.

Absorption

$$I = I_0 \exp(-\alpha x)$$

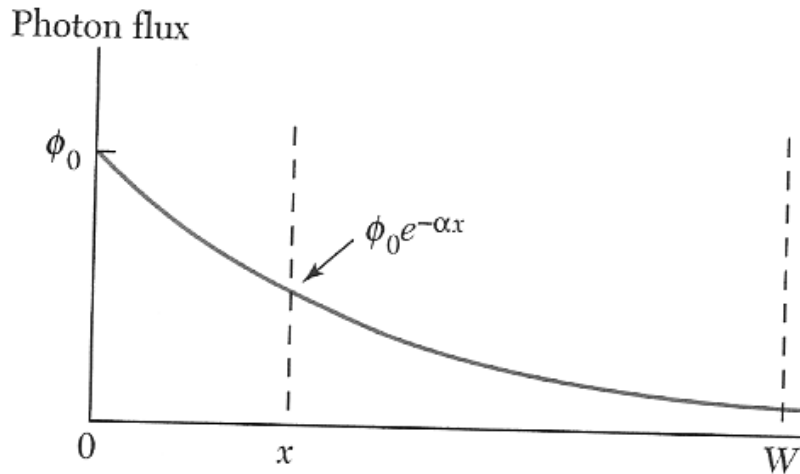


absorption coefficient α



Absorption

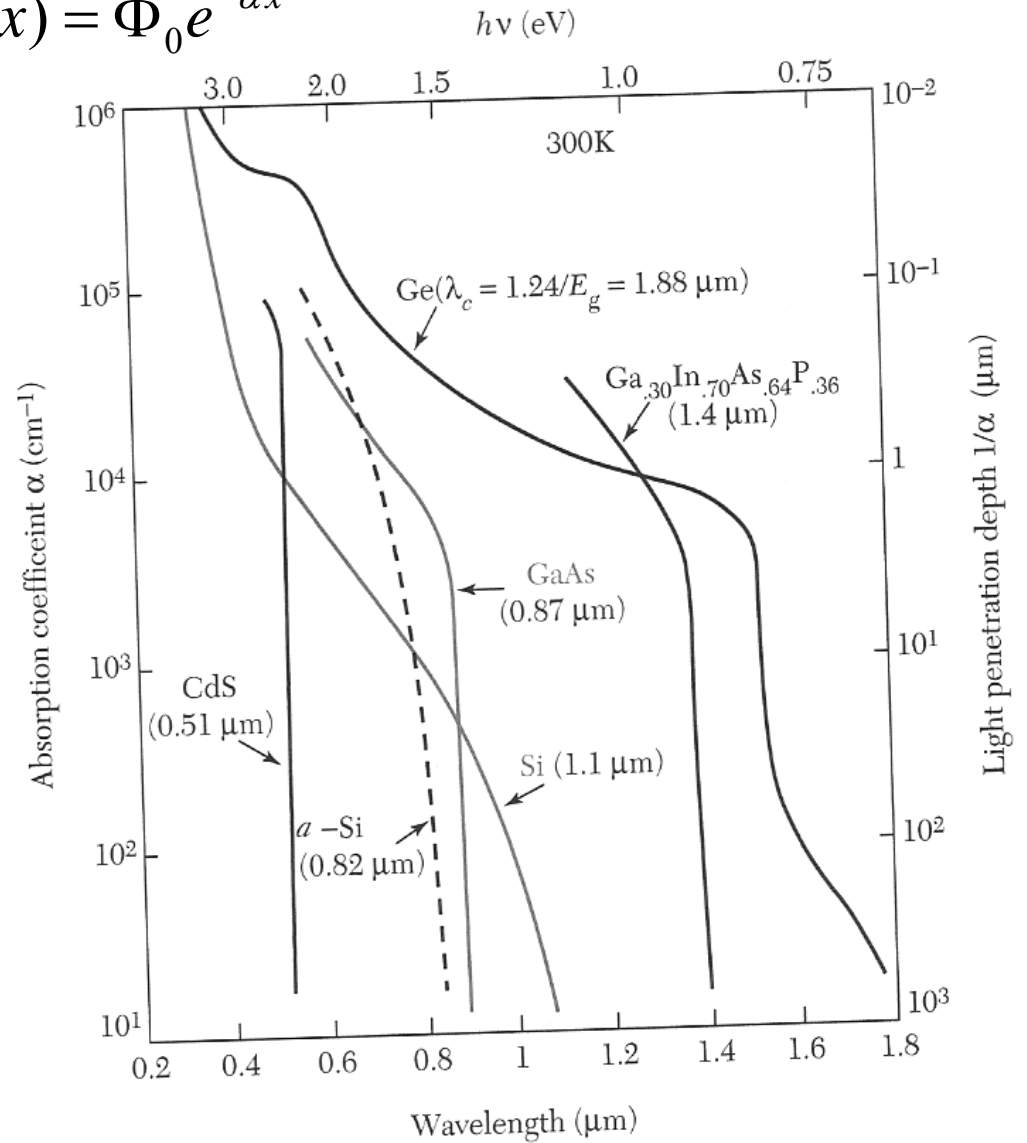
Photon flux: $\Phi(x) = \Phi_0 e^{-\alpha x}$



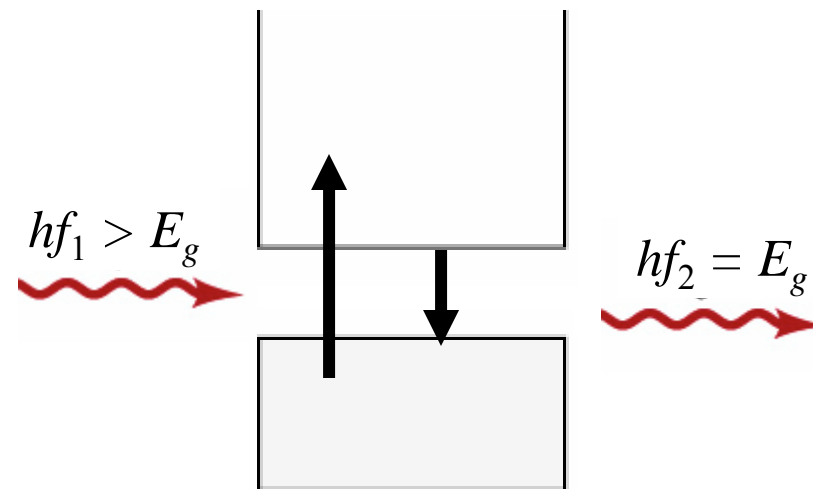
Sharp absorption edge for direct bandgap materials

$$\alpha \approx 3.5 \times 10^6 \left(\frac{m_r^*}{m_0} \right)^{3/2} \frac{\sqrt{\hbar\omega - E_g}}{\hbar\omega} \text{ cm}^{-1}$$

direct bandgap

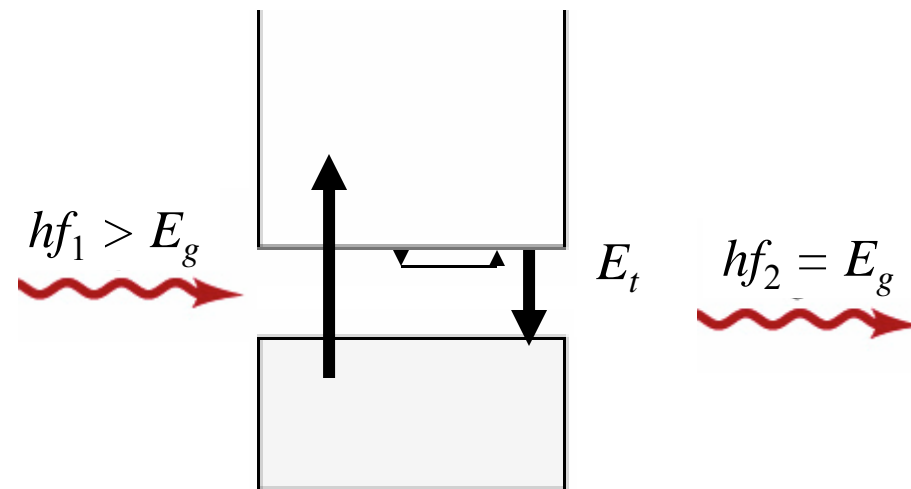


Photoluminescence



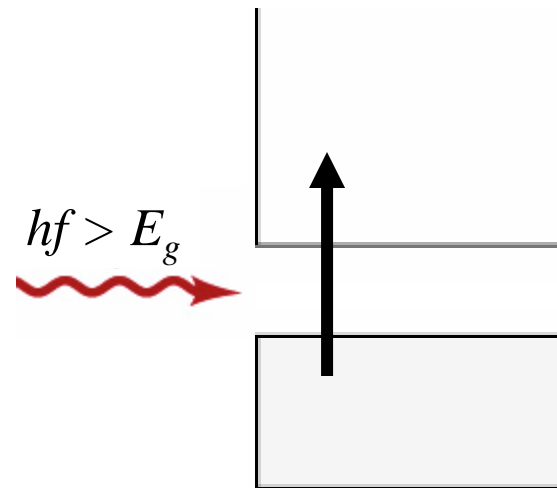
In a direct bandgap semiconductor, emission stops $\sim 10^{-8}$ s after excitation stops.

Phosphorescence



An electron falls into a trap and must be thermally excited out before it recombines and emits light.

Photoconductivity



$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

Laser printer

