7
pn junctions

November 11th, 2026

7
pn junctions

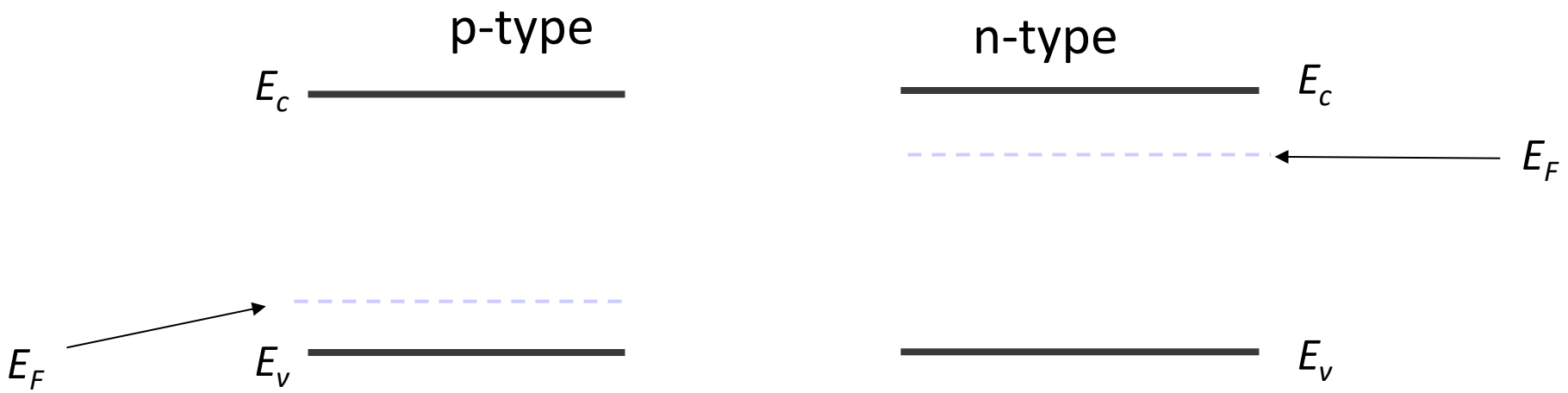
diodes
solar cells
LEDs

insulation

JFET
MOSFET
bipolar transitors

pn junction

isolated semiconductors



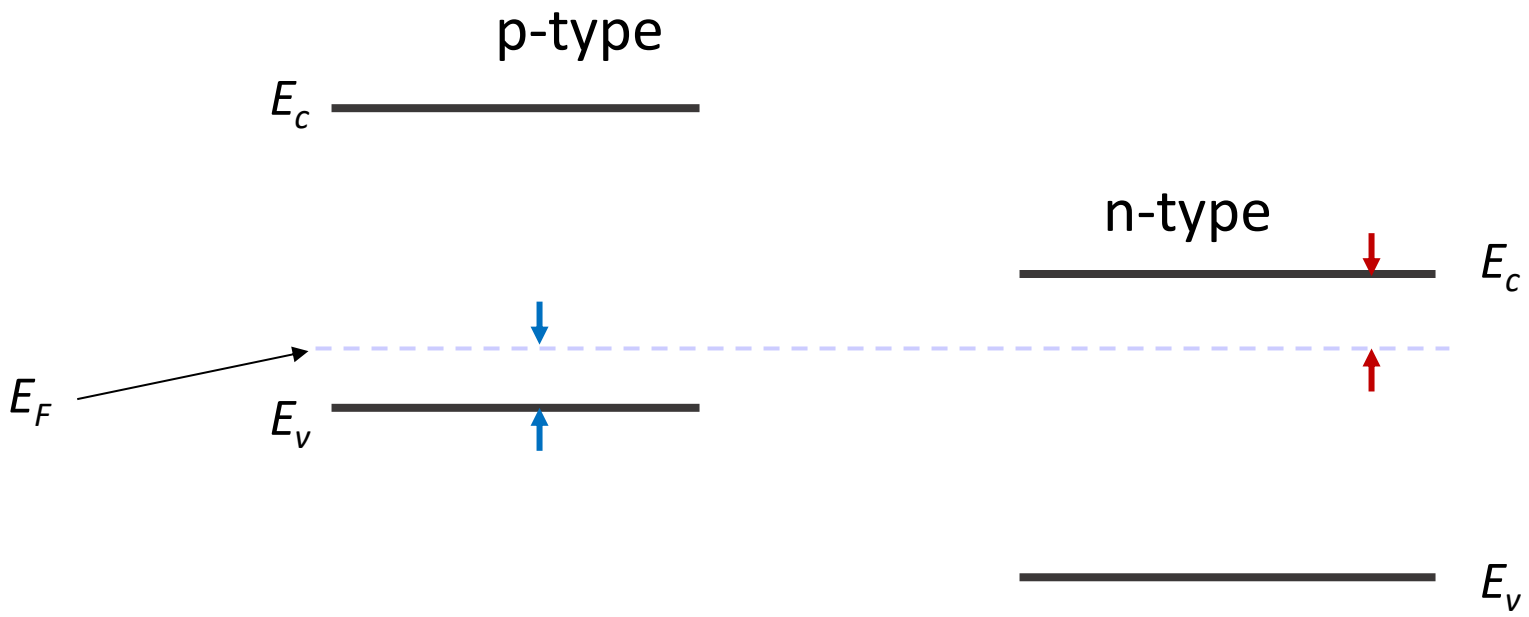
$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right) \qquad E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right) \qquad n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

pn junction

semiconductors in contact

electrons flow from n to p



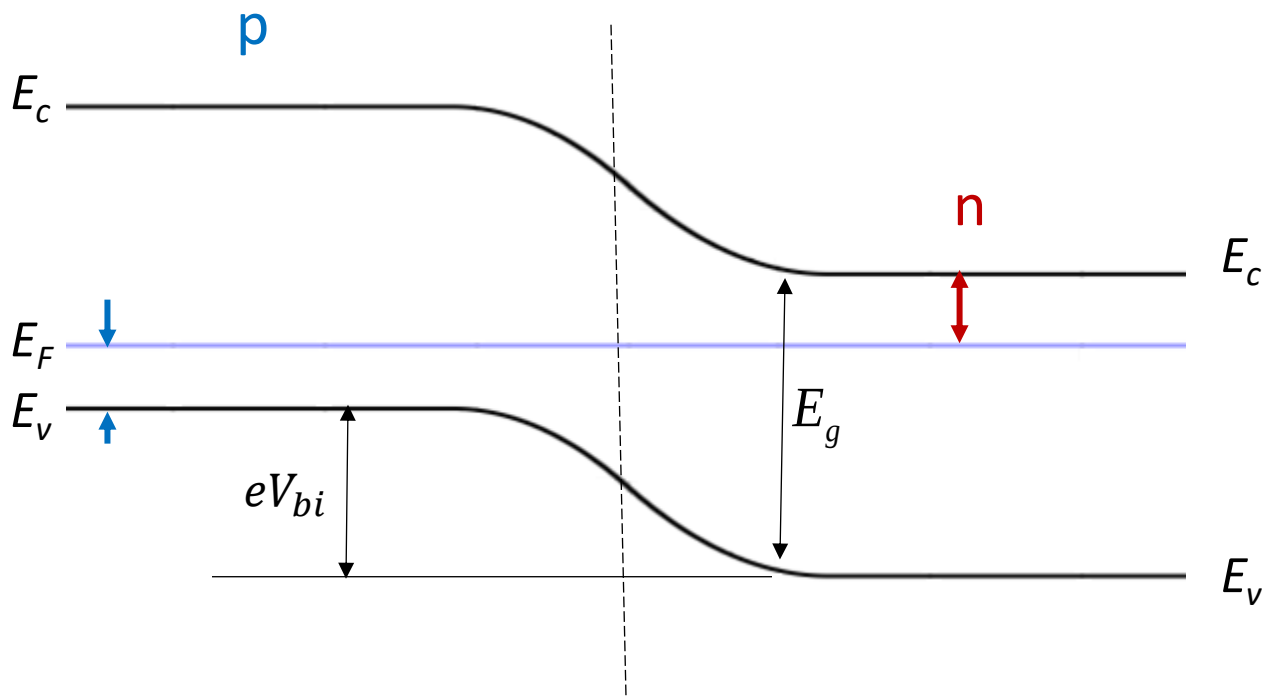
$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right) \approx N_A$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) \approx N_D$$

Built-in voltage V_{bi}

E_v in p-region

$$E_F - E_{vp} = k_B T \ln\left(\frac{N_v}{N_A}\right)$$



E_c in n-region

$$E_{cn} - E_F = k_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c}{N_{D,n} - N_{A,n}}\right) - k_B T \ln\left(\frac{N_v}{N_{A,p} - N_{D,p}}\right)$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{(N_{D,n} - N_{A,n})(N_{A,p} - N_{D,p})}\right)$$

V_{bi}

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{(N_{D,n} - N_{A,n})(N_{A,p} - N_{D,p})}\right)$$

$$n_i^2 = N_v N_c \exp\left(\frac{-E_g}{k_B T}\right) \qquad E_g = -k_B T \ln\left(\frac{n_i^2}{N_v N_c}\right)$$

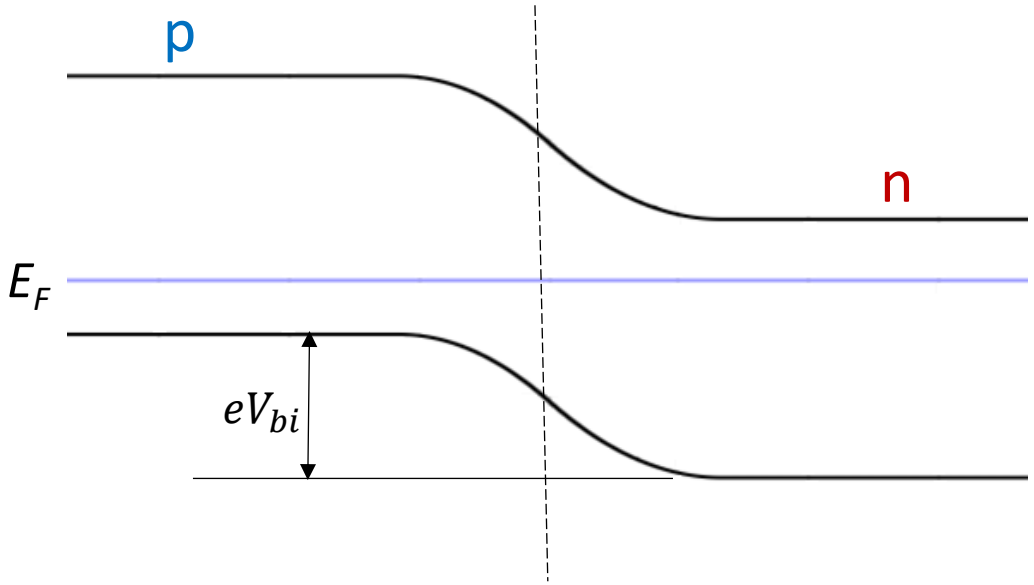
$$eV_{bi} = k_B T \ln\left(\frac{(N_{D,n} - N_{A,n})(N_{A,p} - N_{D,p})}{n_i^2}\right)$$

for $N_{D,n} - N_{A,n} = N_D$ and $N_{A,p} - N_{D,p} = N_A$

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

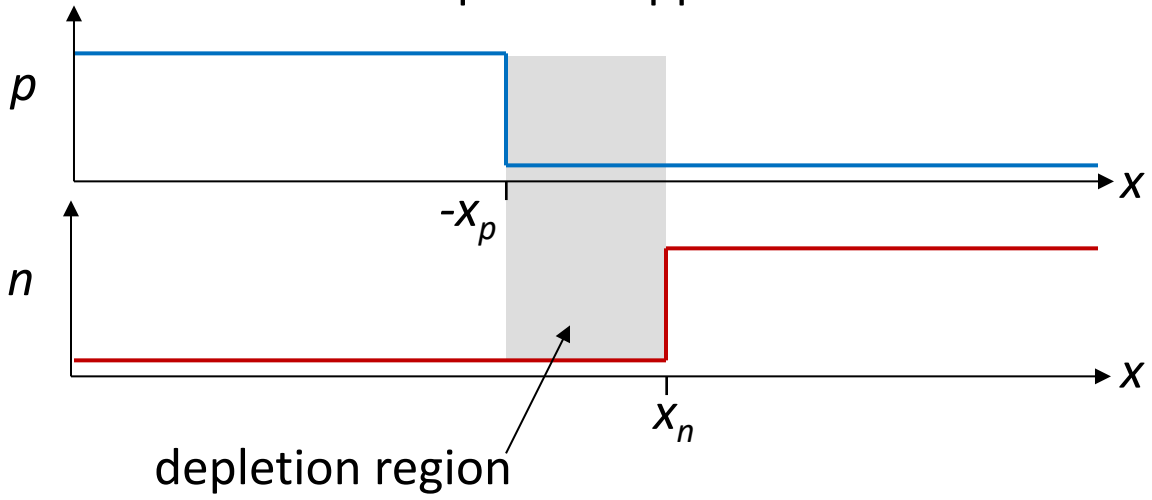
https://lampz.tugraz.at/~hadley/psd/L6/Vbi.html

p and n profiles



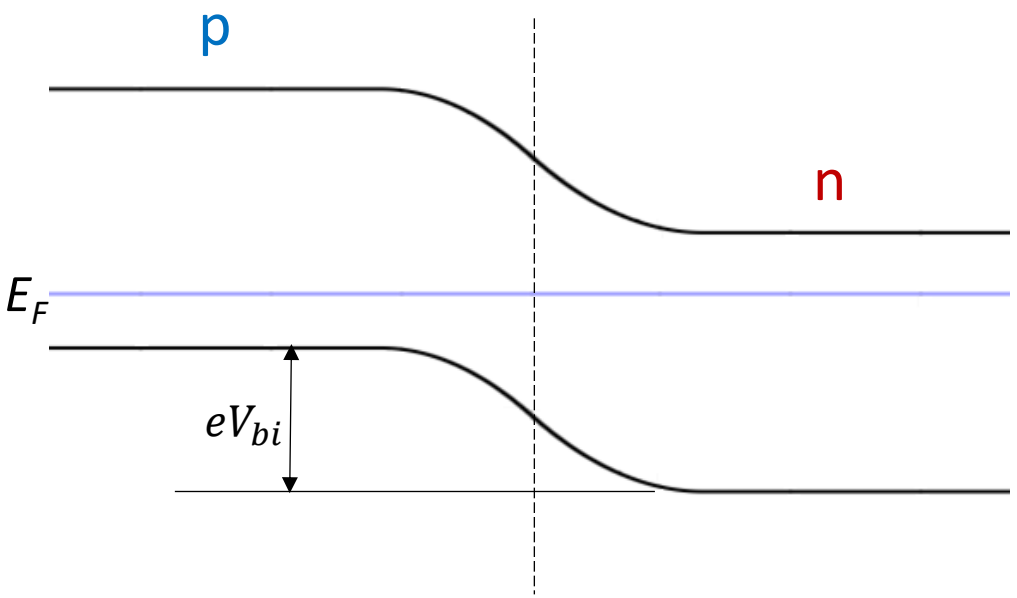
depletion approximation

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

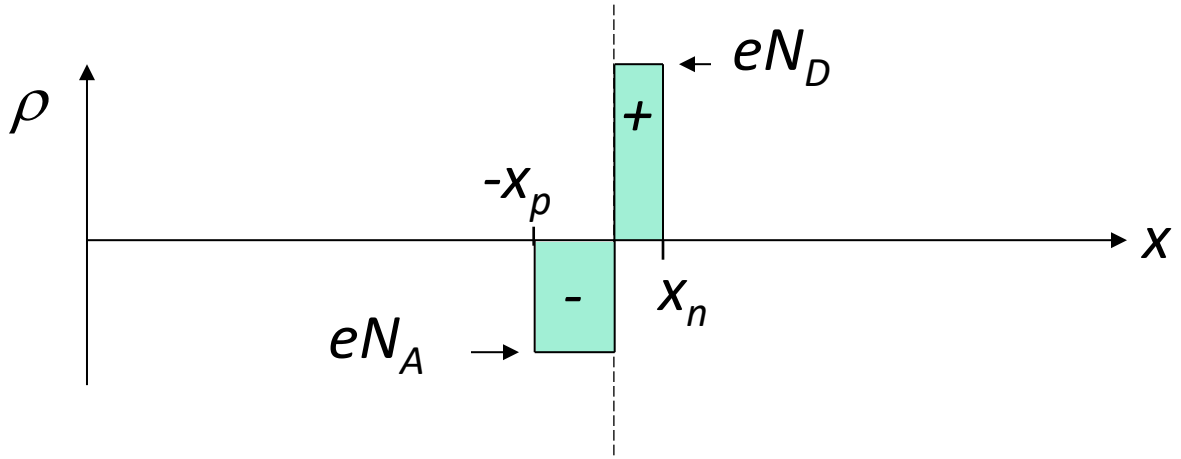


$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

space charge



exposed ionized dopants



$$N_A x_p = N_D x_n$$

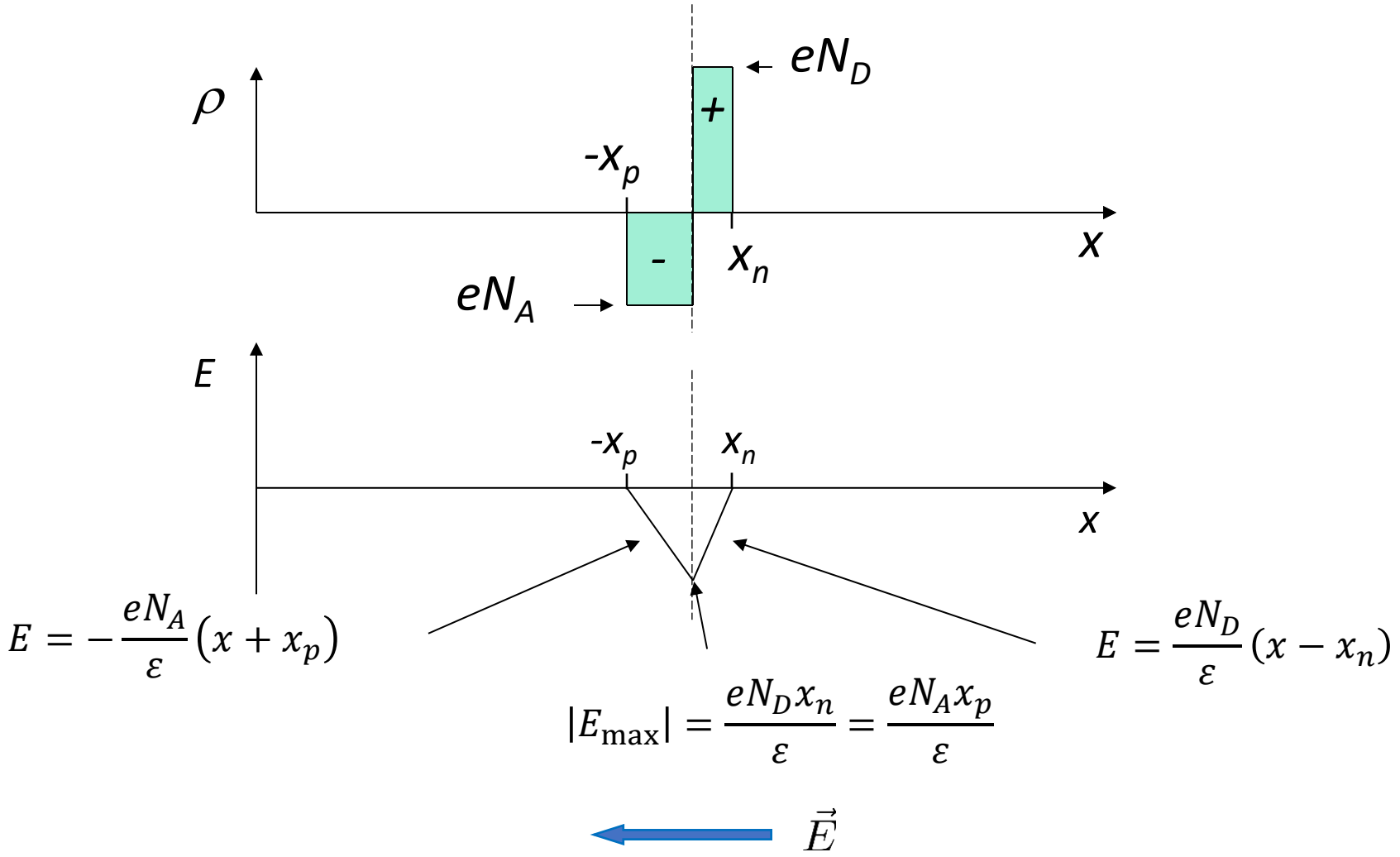
electric field

Gauss's law

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

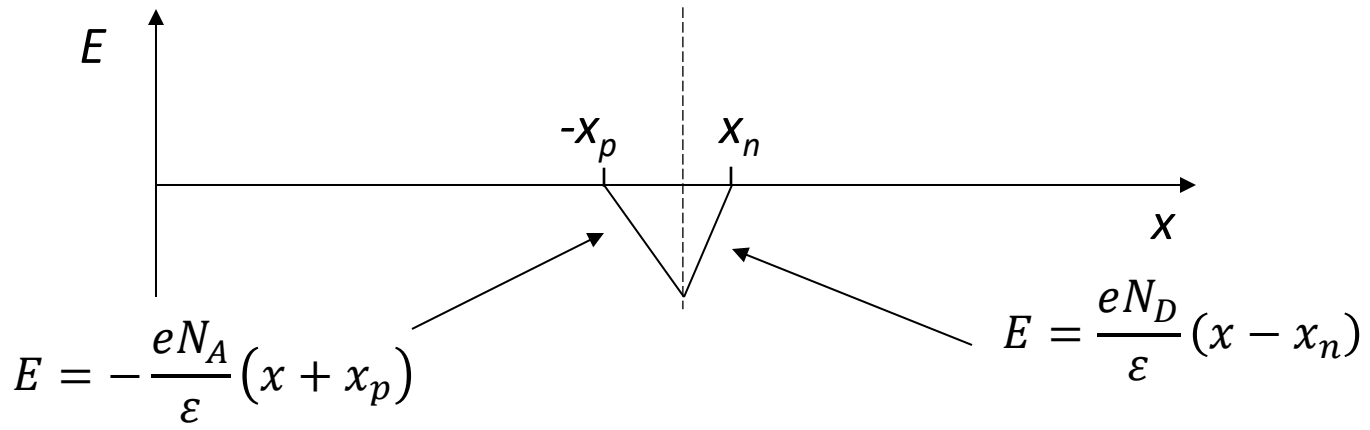
in 1-D is

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$



E pushes the holes towards *p* and the electrons towards *n*

electrostatic potential



$$\frac{dV}{dx} = -E$$

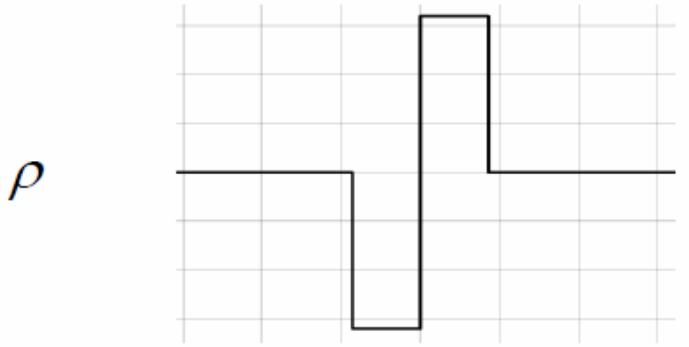
$-x_p < x < 0$	$0 > x > x_n$
$V = \frac{eN_A}{\varepsilon}\left(\frac{x^2}{2} + xx_p\right)$	$V = \frac{-eN_D}{\varepsilon}\left(\frac{x^2}{2} - xx_n\right)$

$$V(-x_p) = \frac{-eN_A}{2\varepsilon}x_p^2 \qquad V(0) = 0 \qquad V(x_n) = \frac{eN_D}{2\varepsilon}x_n^2$$

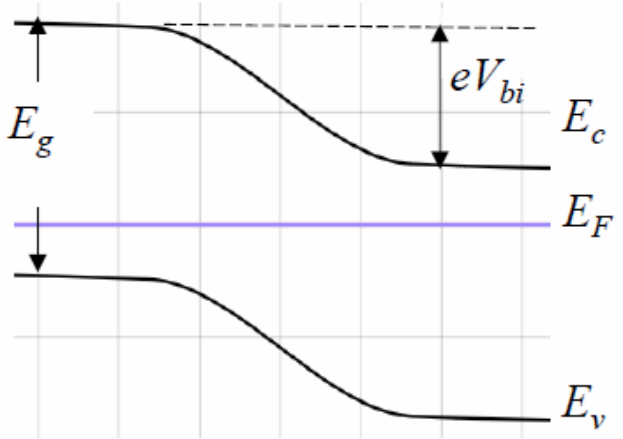
abrupt pn junction

$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= \frac{eN_A x_p^2}{2\varepsilon} + \frac{eN_D x_n^2}{2\varepsilon}$$







https://lampz.tugraz.at/~hadley/psd/L6/abrupt.html

Depletion width

$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{eN_A x_p^2}{2\varepsilon} + \frac{eN_D x_n^2}{2\varepsilon}$$

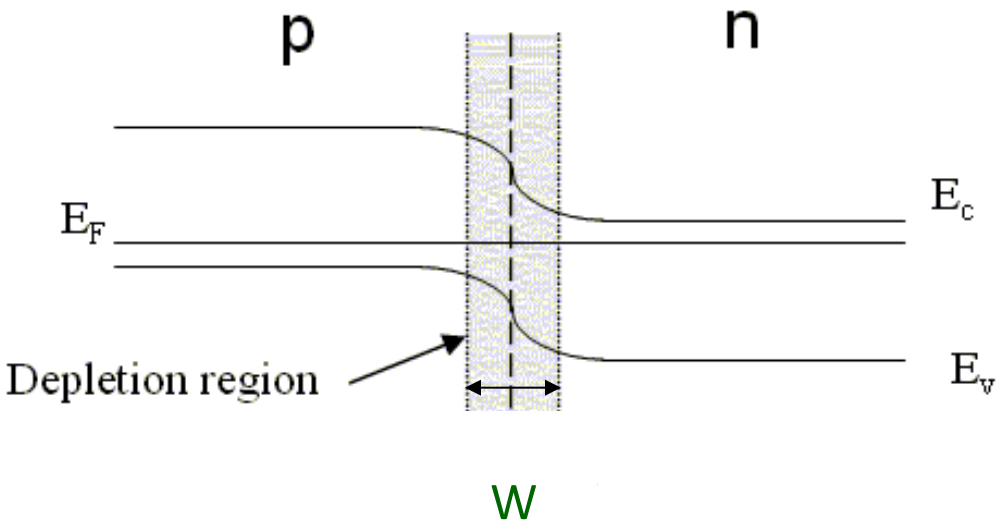
$$N_A x_p = N_D x_n = N_D(W - x_p) = N_A(W - x_n)$$

$$x_p = \frac{N_D W}{N_A + N_D} \qquad x_n = \frac{N_A W}{N_A + N_D}$$

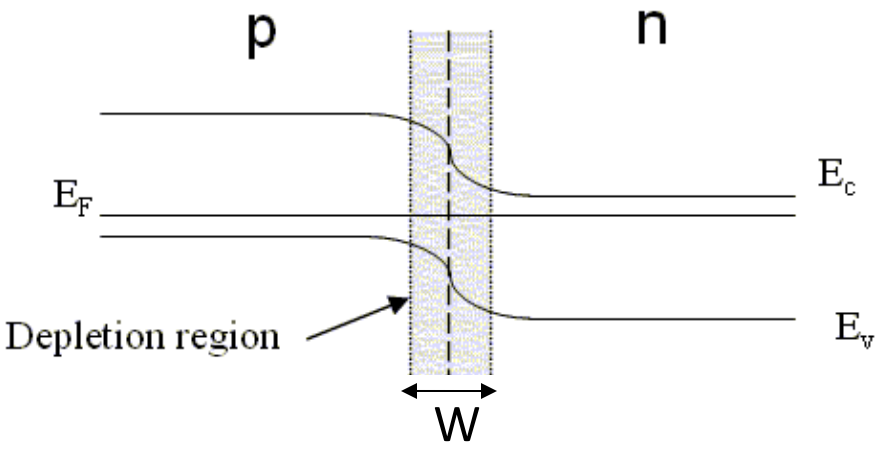
$$V_{bi} = \frac{e}{2\varepsilon} \frac{N_D N_A}{N_D + N_A} W^2$$

$$W = \sqrt{\frac{2\varepsilon(N_D + N_A)V_{bi}}{eN_D N_A}}$$

light doping: wide depletion width



Depletion width



V_{bi} ~ 1V

W ~ 10 nm - 10 μm

E_{max} ~ 10^4 V/cm

The electric field pushes the electrons towards the n-region and the holes towards the p-region.

Diffusion sends electrons towards the p-region and holes towards the n-region.

Abrupt pn junctions in the depletion approximation

In an abrupt pn junction, the doping changes abruptly from p to n. It is common to solve for the band bending, the local electric field, the carrier concentration profiles, and the local conductivity in the depletion approximation. In this approximation it is assumed that there is a depletion width W around the transition from p to n where the charge carrier densities are negligible. Outside the depletion width the charge carrier densities are equal to the doping densities so that the semiconductor is electrically neutral outside the depletion width. Using this approximation it is possible to calculate the important properties of the pn junction.

N_A = 1E15 1/cm³ N_D = 1E15 1/cm³ E_g = 1.166-4.73E-4*T*T/(T+636) eV

$N_v(300)$ = 9.84E18 1/cm³ $N_c(300)$ = 2.78E19 1/cm³ ϵ_r = 12 T = 300 K

μ_p = 480 cm²/V s μ_n = 1350 cm²/V s τ_p = 1E-10 s τ_n = 1E-10 s

V = -0.5 V

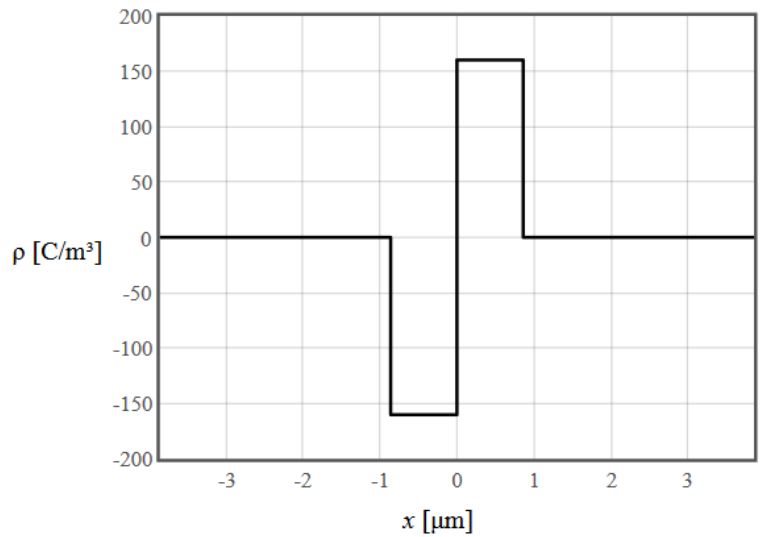
V

Submit

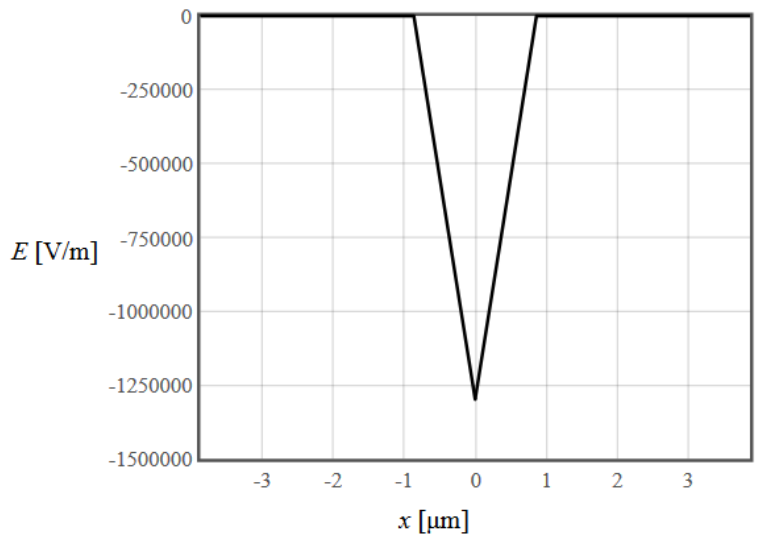
$E_g = 1.12$ eV $W = 1.72$ µm $x_p = -0.861$ µm $x_n = 0.861$ µm $V_{bi} = 0.618$ V $C_j = 6.17$ nF/cm²

$D_p = 12.4$ cm²/s $D_n = 34.9$ cm²/s $L_p = 0.352$ µm $L_n = 0.591$ µm

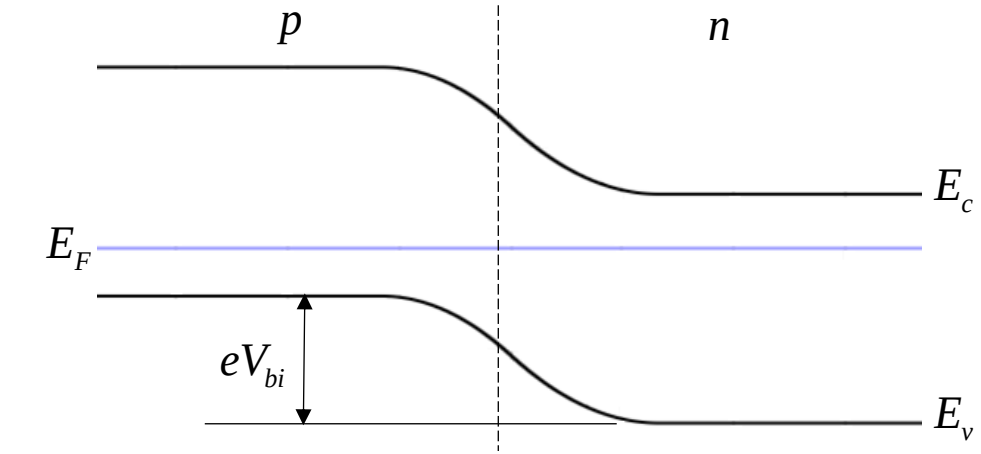
Charge density

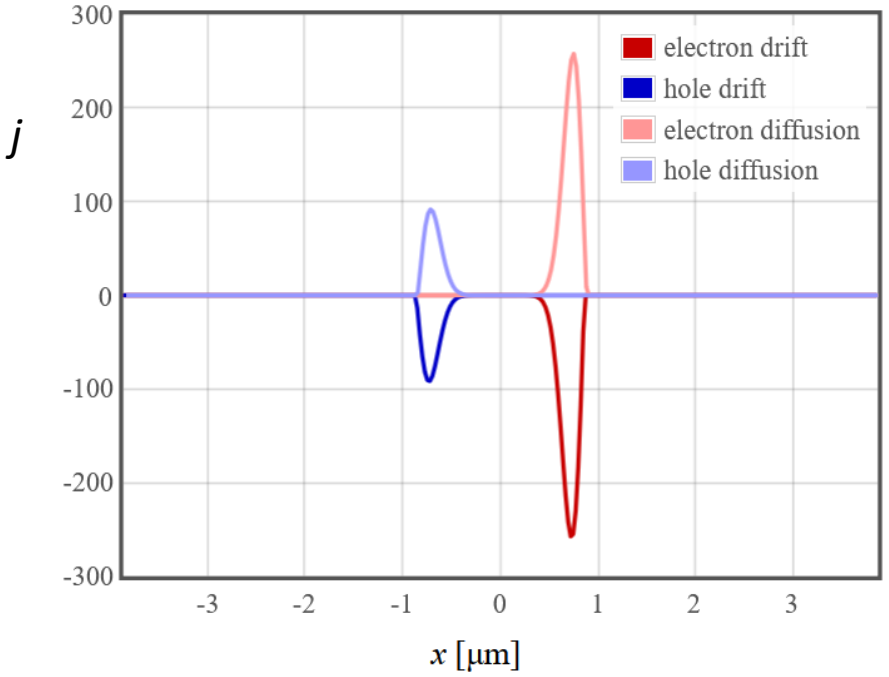


Electric field



Drift and diffusion





$$\vec{j}_n = en\mu_n\vec{E} + eD_n\nabla n$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$\nabla n = -\frac{\nabla E_c}{k_B T} N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = -\frac{\nabla E_c}{k_B T} n$$

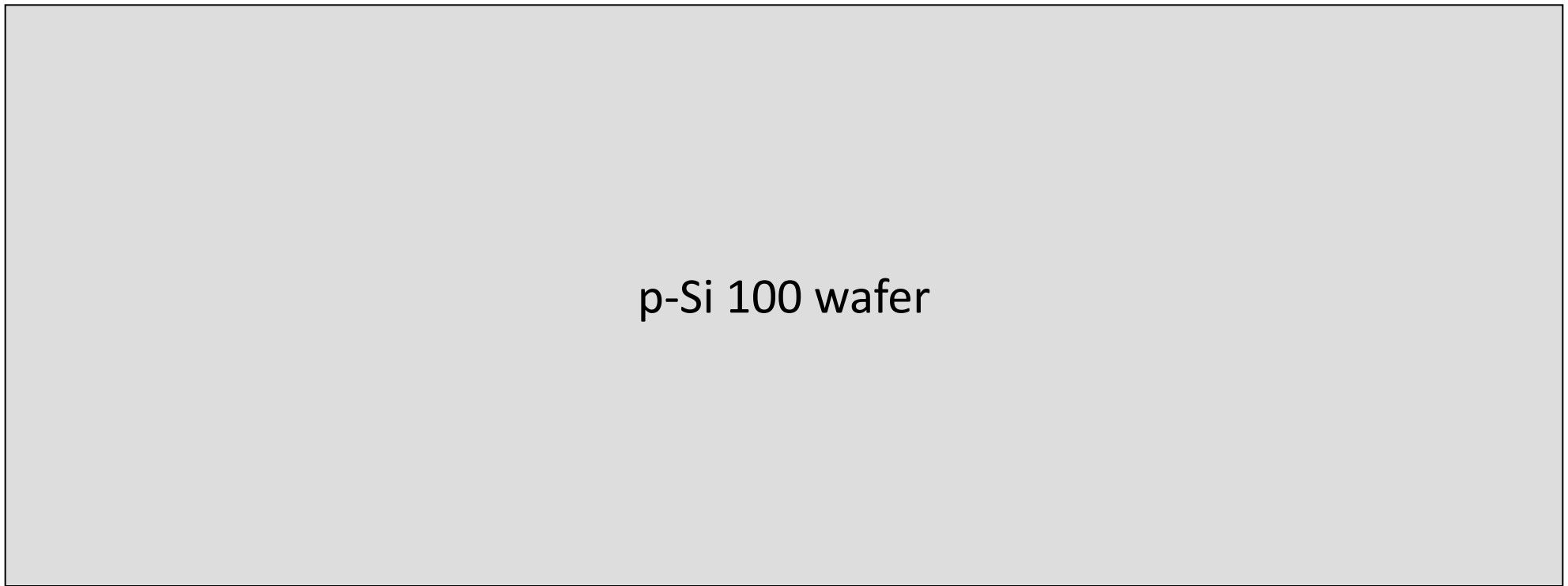
$$e\vec{E} = \nabla E_c$$

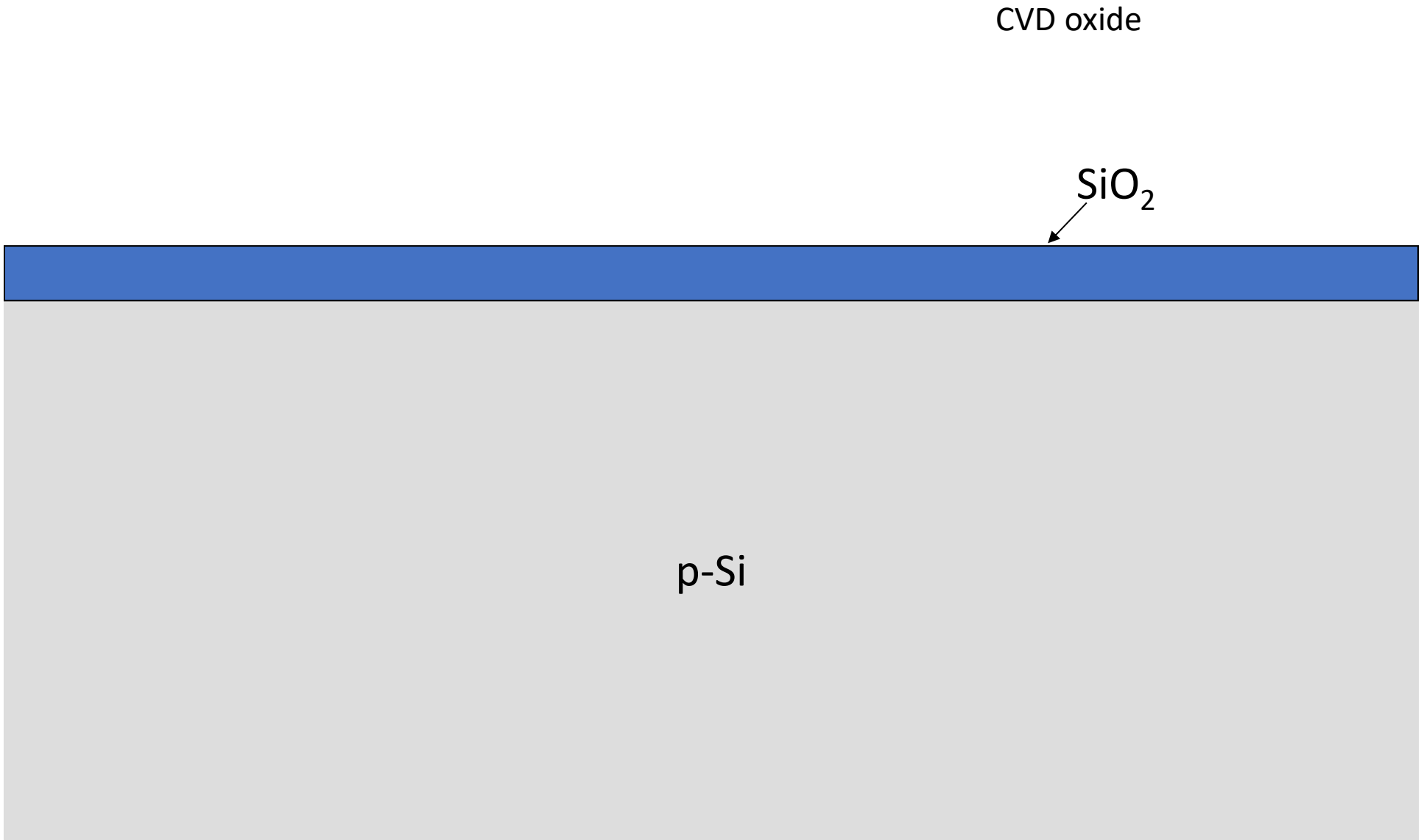
$$\vec{j}_n = n\nabla E_c \left(\mu_n - \frac{eD_n}{k_B T}\right)$$

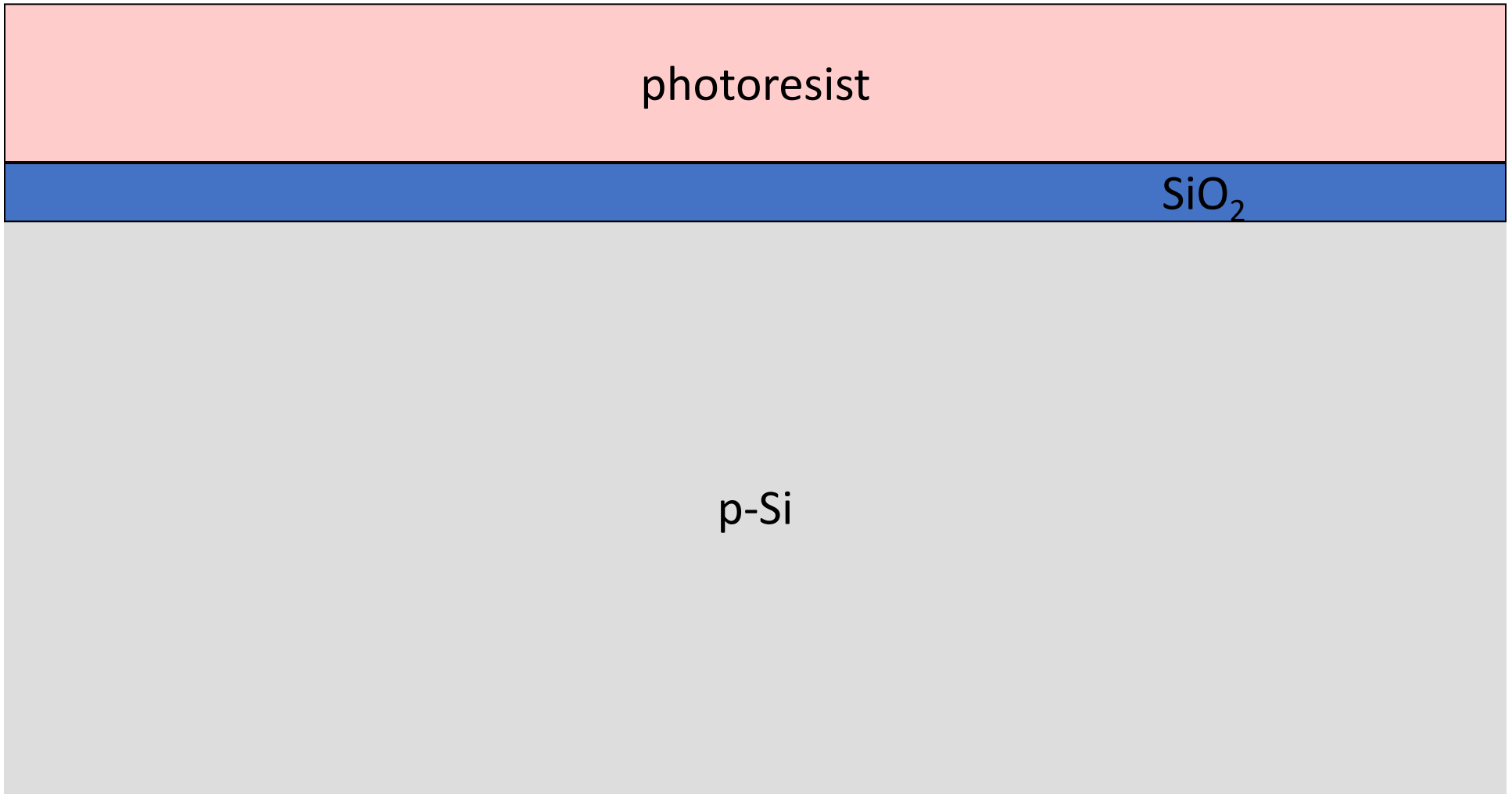
Einstein relation

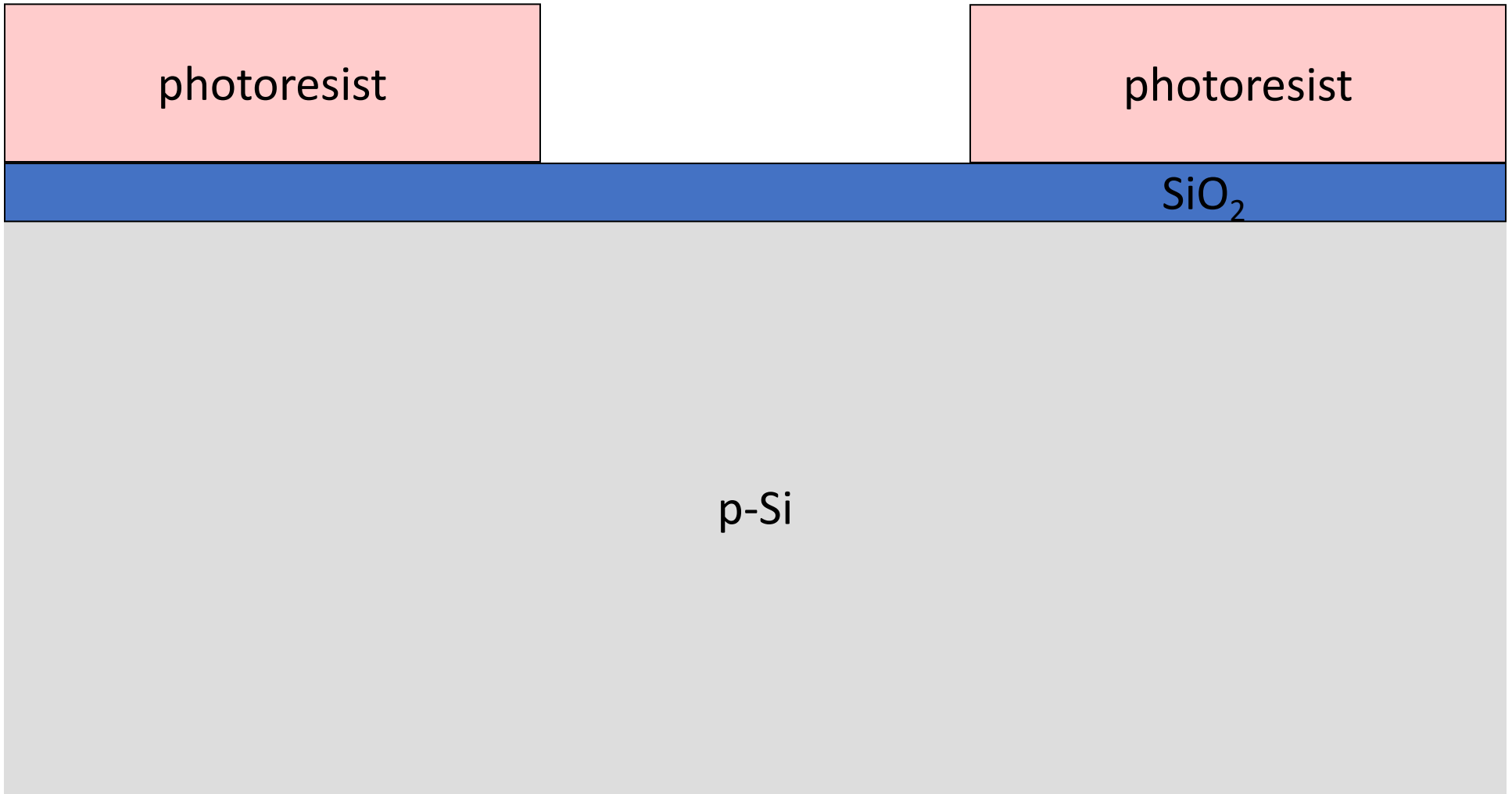
If the E_F is constant, net current density $j = 0$.

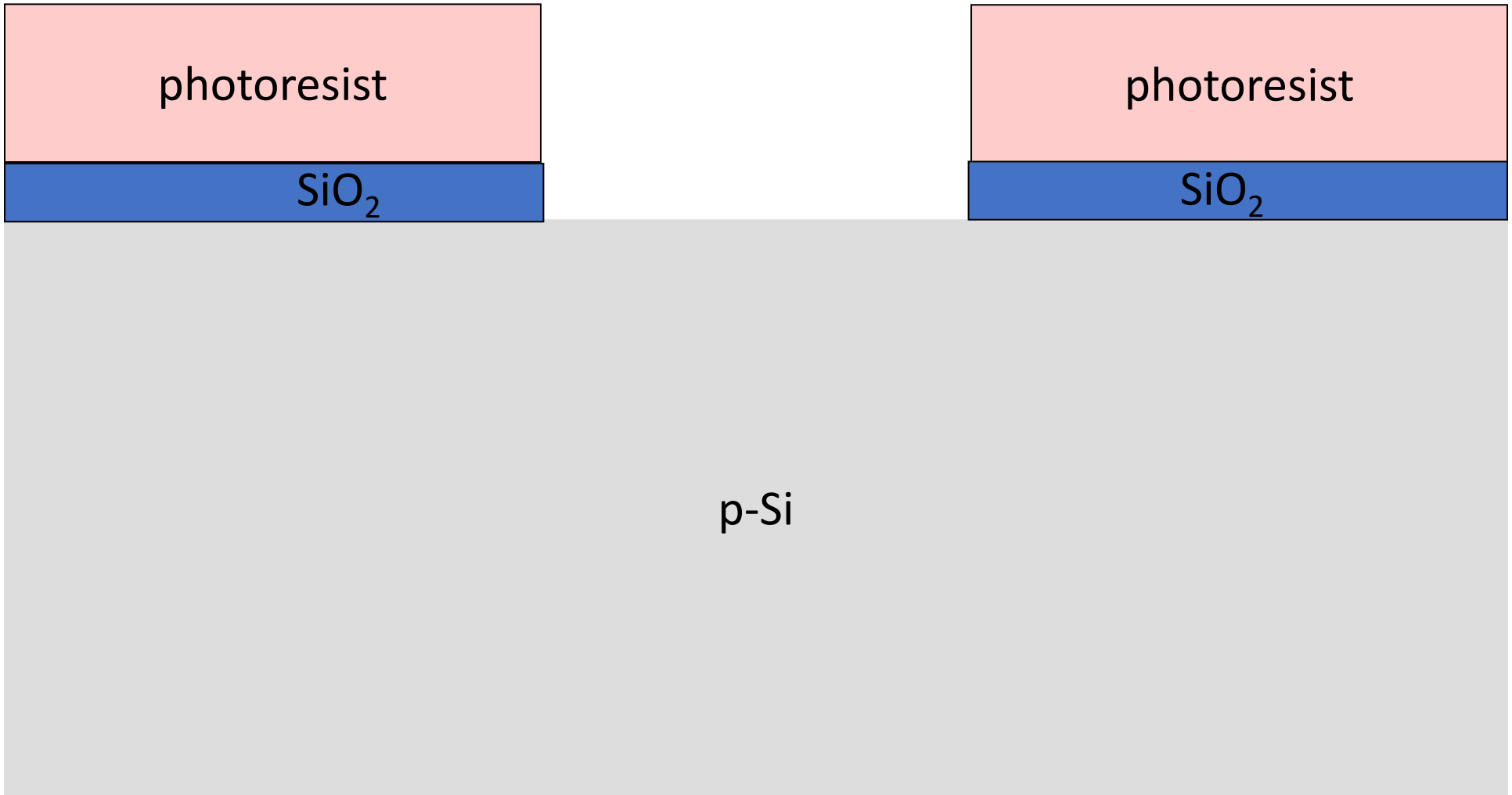
diode fabrication

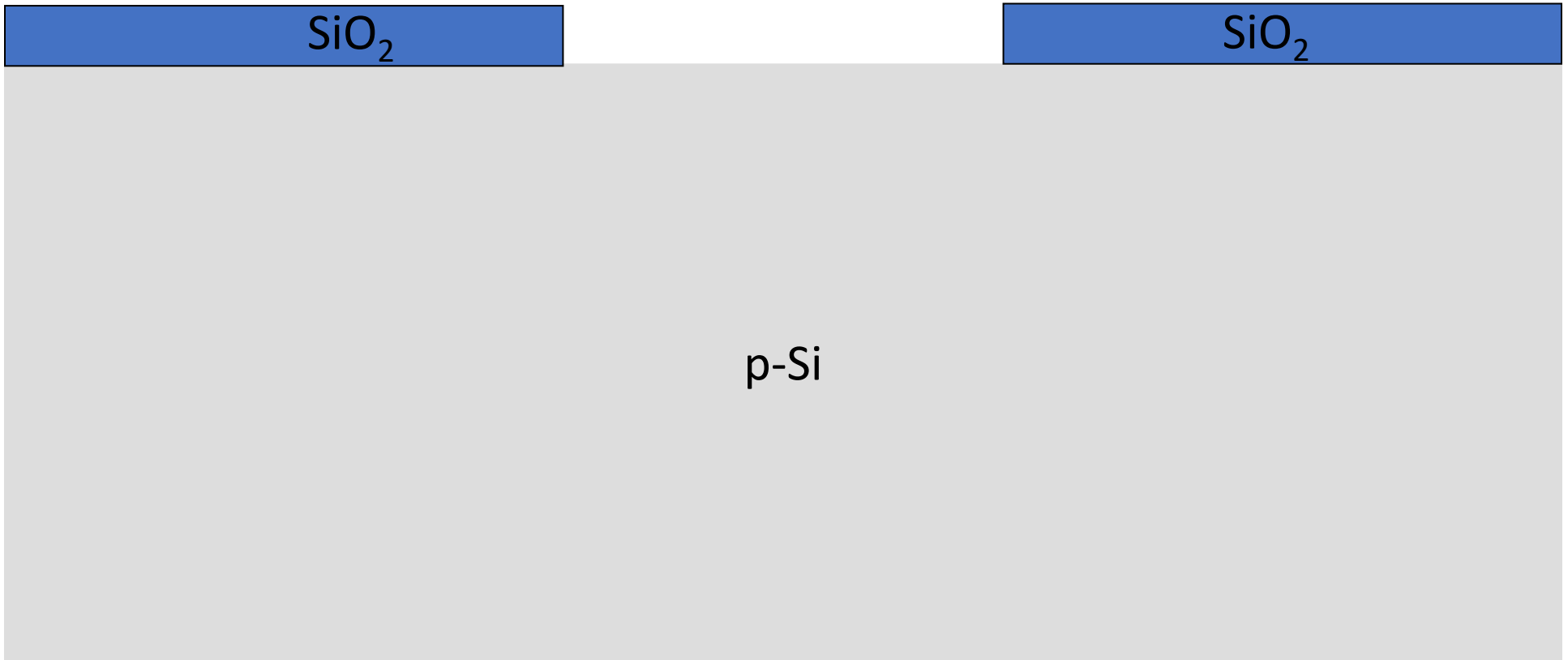



CVD oxide
SiO_2
p-Si

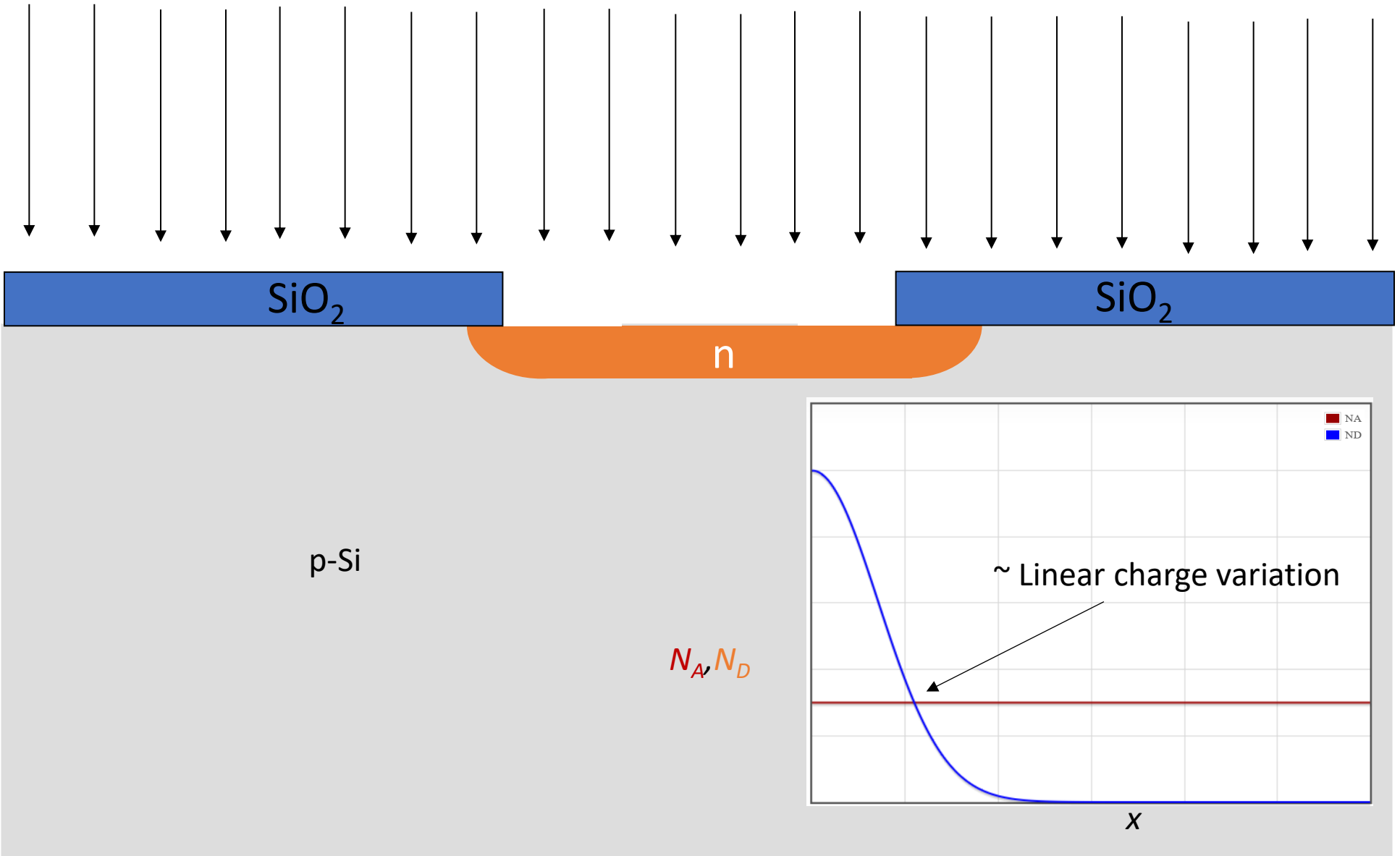

photoresist
SiO_2
p-Si


photoresist
photoresist
SiO_2
p-Si


photoresist
photoresist
SiO_2
SiO_2
p-Si

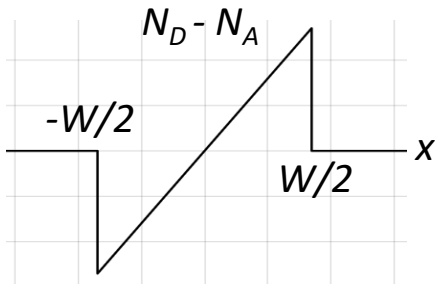

SiO_2
SiO_2
p-Si

Diffuse or Implant



linearly graded junction

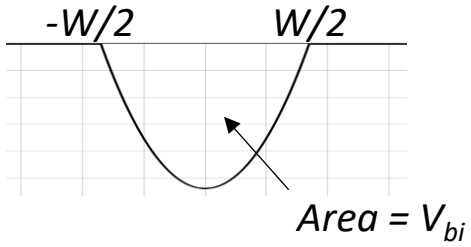
charge density



slope

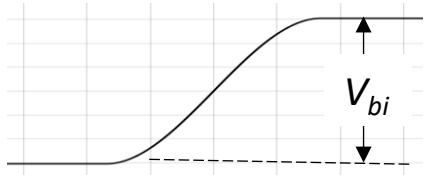
$$\rho = e(N_D(x) - N_A(x)) = eax$$

electric field



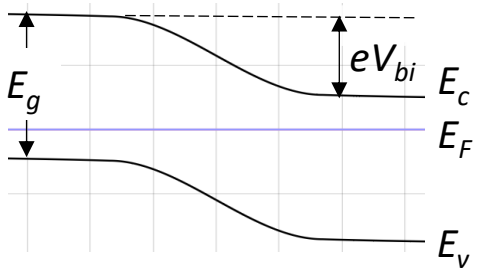
$$E = \int \frac{\rho}{\varepsilon} dx = \frac{-ea}{2\varepsilon}\left(\left(\frac{W}{2}\right)^2 - x^2\right) \qquad E_{max} = \frac{-eaW^2}{8\varepsilon}$$

potential



$$V = \int E\, dx = \frac{ea}{2\varepsilon}\left(\left(\frac{W}{2}\right)^2 x - \frac{x^3}{3}\right)$$

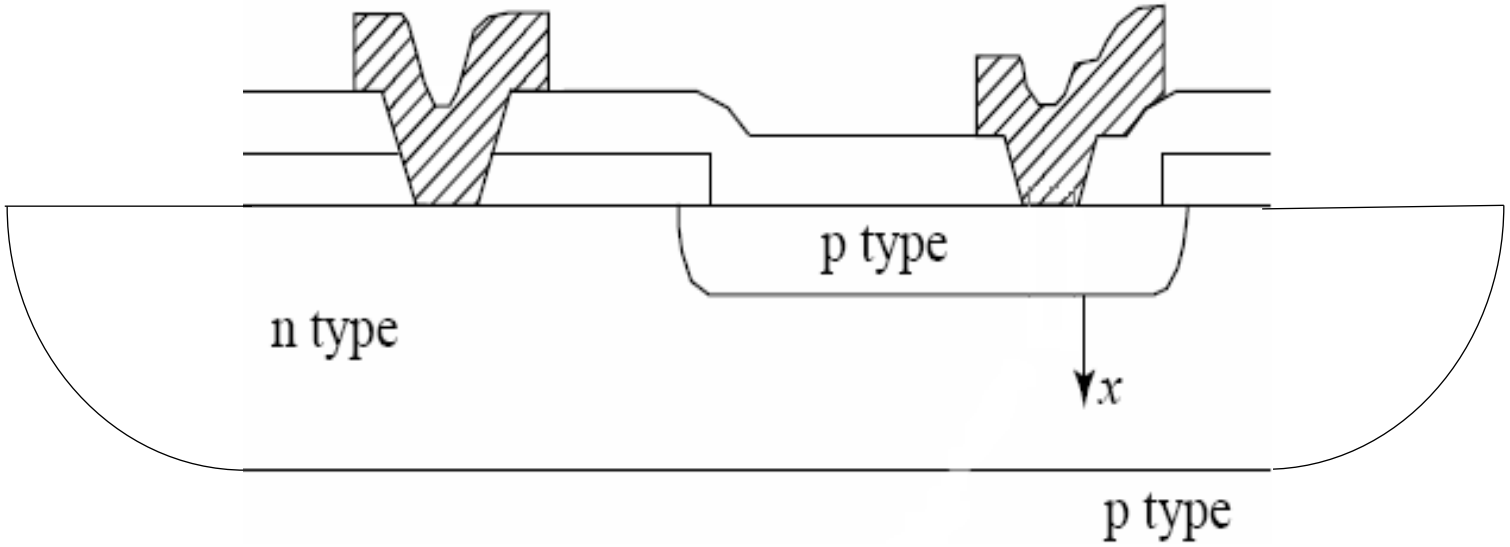
energy



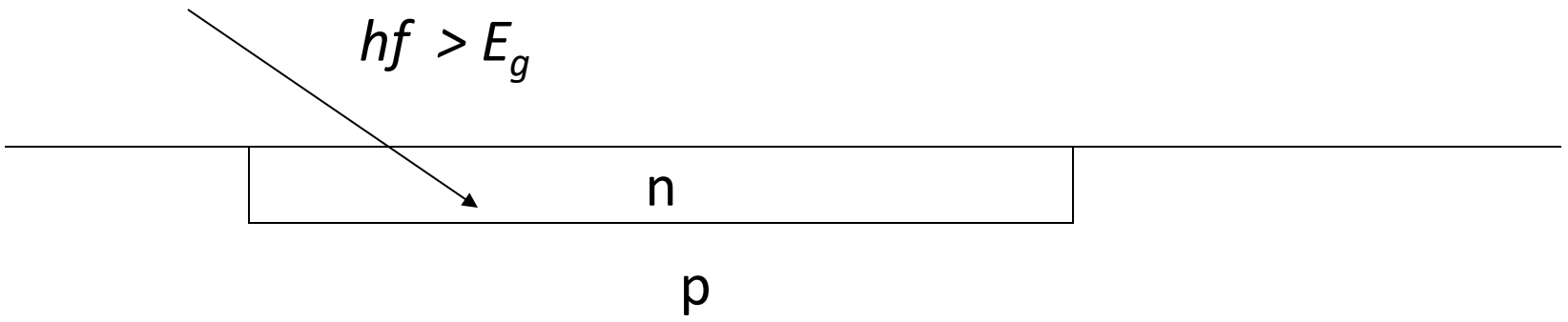
$$V_{bi} = \frac{eaW^3}{12\varepsilon}$$

http://lampz.tugraz.at/~hadley/psd/L6/linearly_graded.html

Isolation

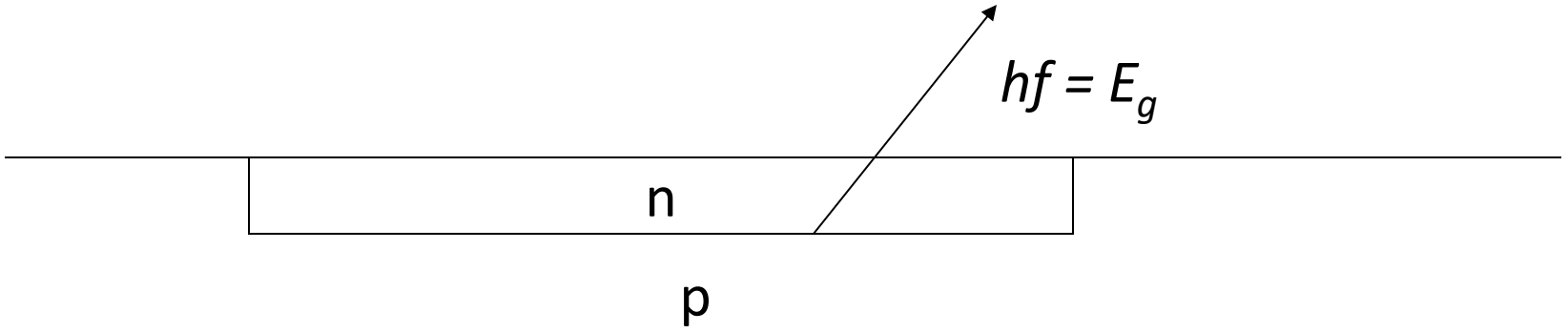


solar cell



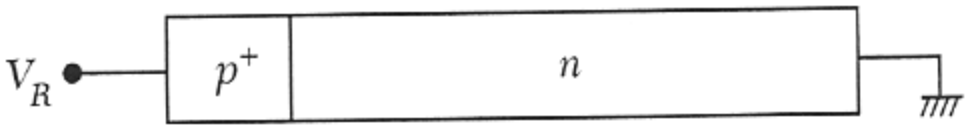
Light creates an electron-hole pair in the depletion region. The electric field sweeps the electrons towards the n-region and the holes towards the p-region.

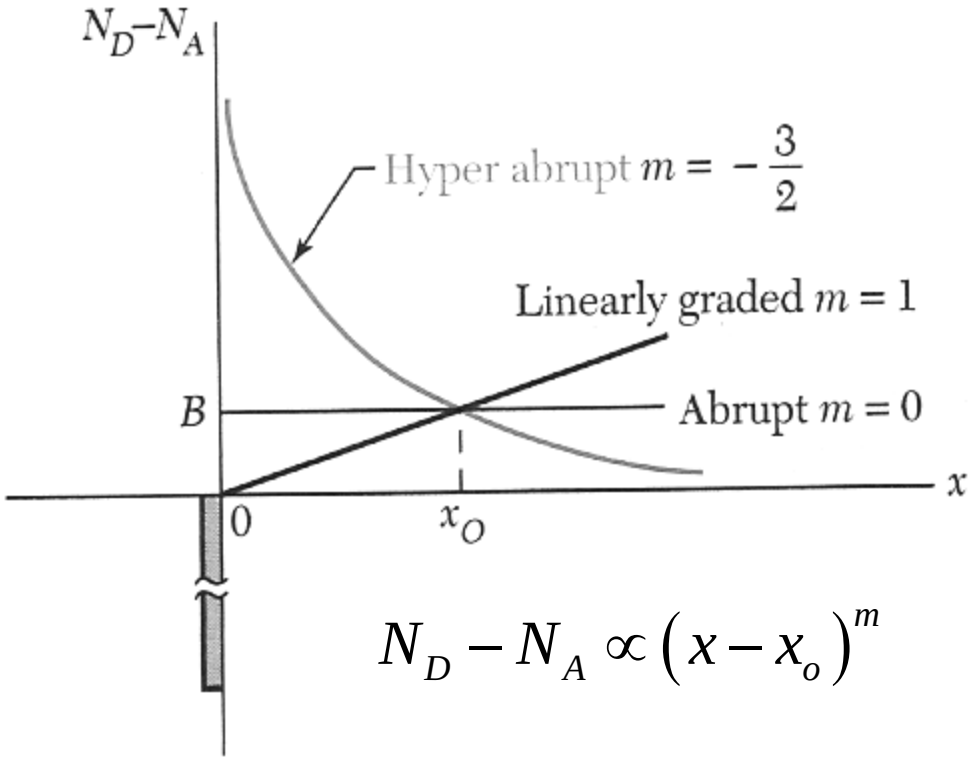
Light emitting diode



Electrons and holes are injected into the depletion region by forward biasing the junction. The electrons fall in the holes. For direct bandgap semiconductors, photons are emitted. For indirect bandgap semiconductors, phonons are emitted.

Varactor





$$N_D - N_A \propto (x - x_o)^m$$

$$C_j \propto (V_{bi} + V_R)^{-n}$$

abrupt: $n = 1/2$
linearly graded: $n = 1/3$

$n = 1/(m+2)$

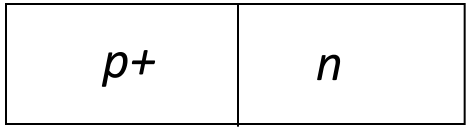


Capacitance-voltage characteristics

specific capacitance $C_j = \frac{\varepsilon}{W}$ $\mathrm{F\,m^{-2}}$

abrupt junction: $W = \frac{\varepsilon}{C_j} = \sqrt{\frac{2\varepsilon(N_D + N_A)(V_{bi} - V)}{eN_D N_A}}$

a one-sided abrupt junction in reverse bias:



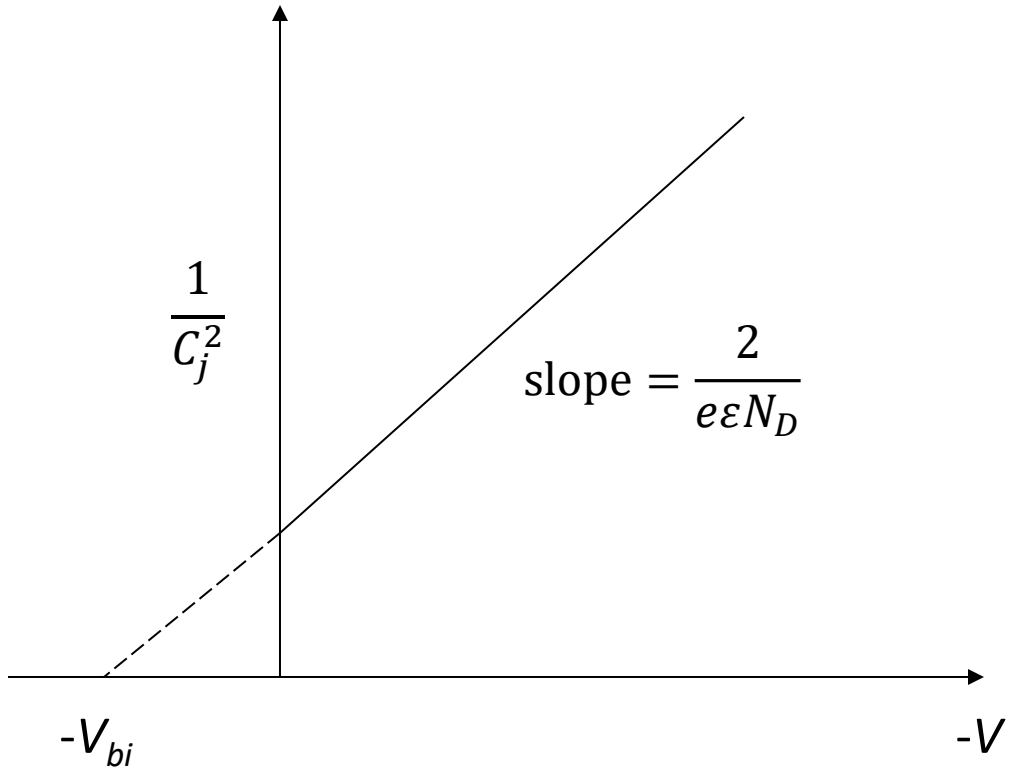
$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{e\varepsilon N_D}$$

Capacitance-voltage characteristics

a one-sided abrupt junction in reverse bias:

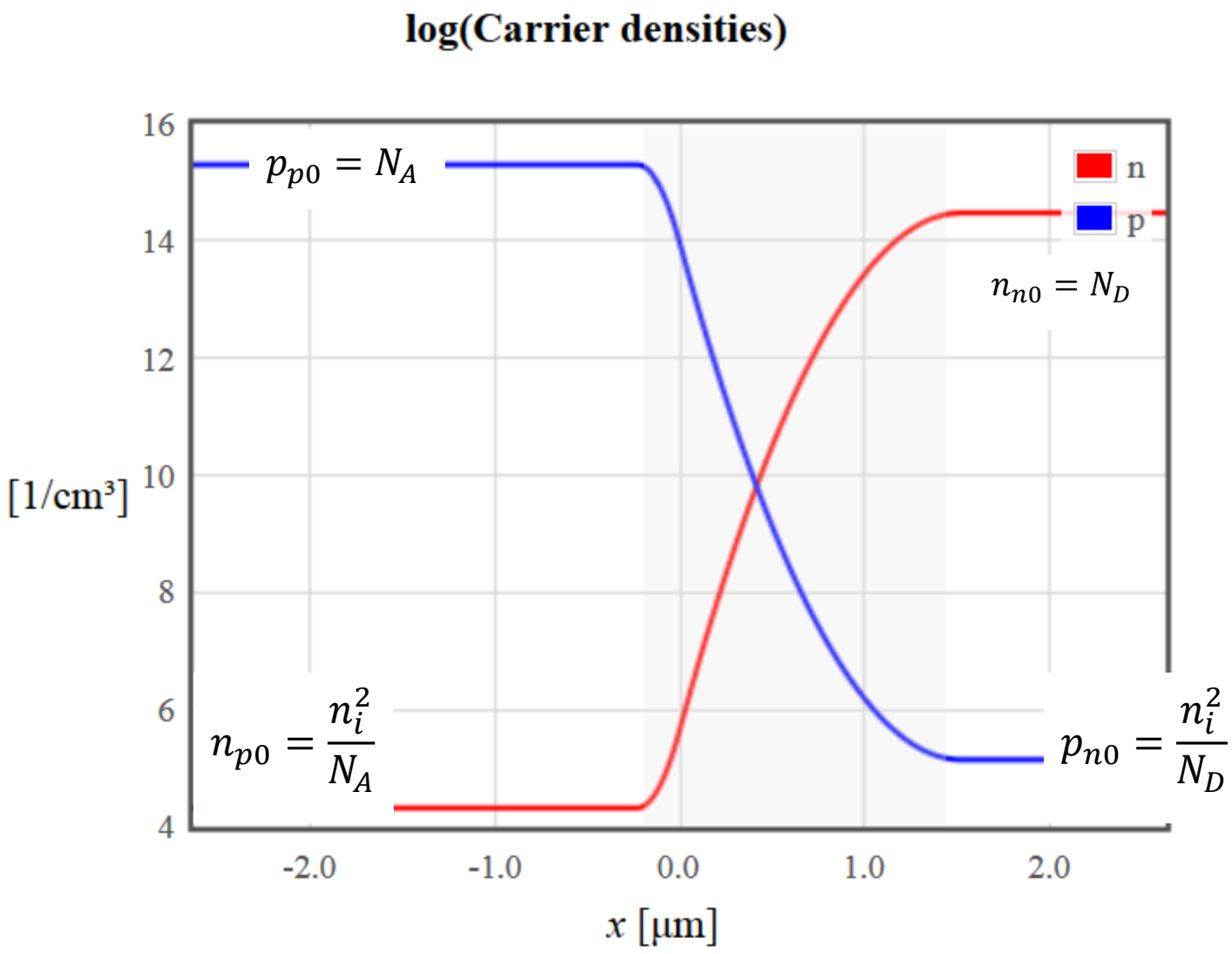
p+	*n*

$$\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{e\varepsilon N_D}$$



slope gives impurity concentration and the intercept gives V_{bi}

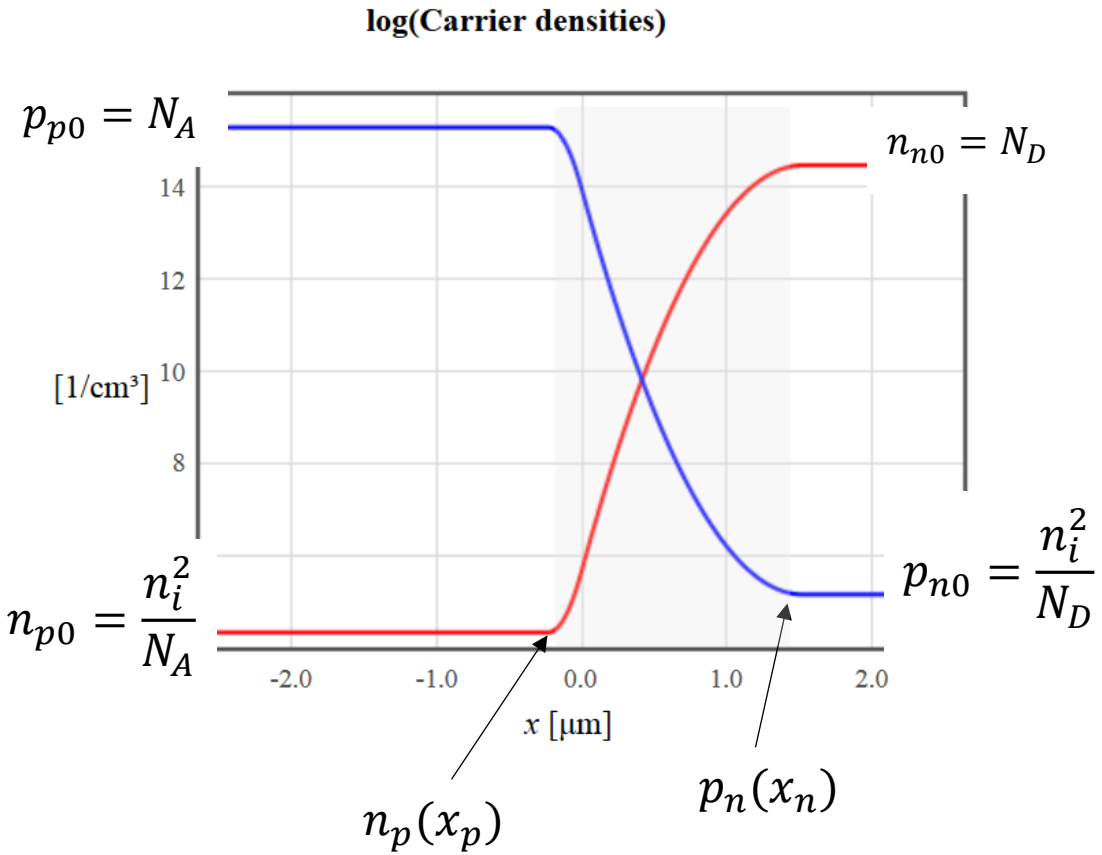
equilibrium concentrations, $V = 0$



$$n_{p0}p_{p0} = n_{n0}p_{n0} = n_i^2$$

bias voltage, $V = 0$

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) = k_B T \ln\left(\frac{N_D}{n_{p0}}\right) = k_B T \ln\left(\frac{N_A}{p_{n0}}\right)$$



$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$

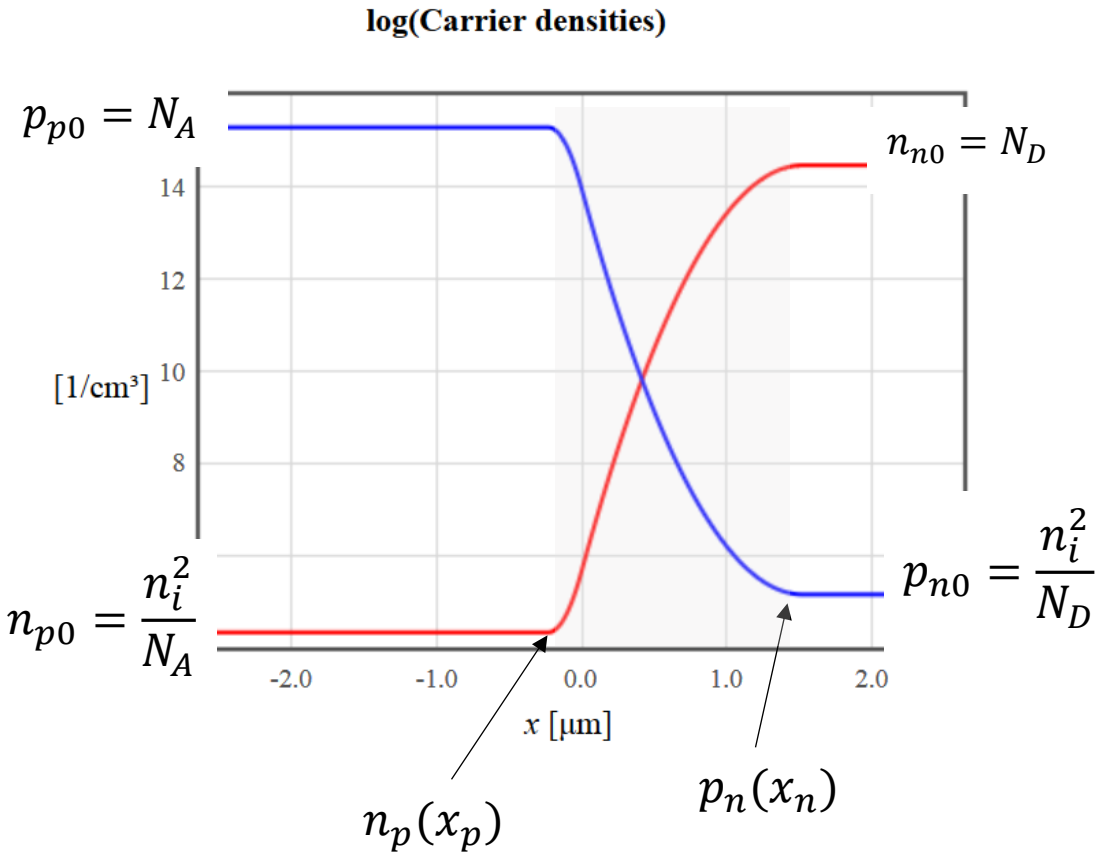
$V = 0$

$$n_{p0} = N_D \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$$p_{n0} = N_A \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

bias voltage, $V \neq 0$

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) = k_B T \ln\left(\frac{N_D}{n_{p0}}\right) = k_B T \ln\left(\frac{N_A}{p_{n0}}\right)$$



$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$

$V = 0$

$$n_{p0} = N_D \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$
$$p_{n0} = N_A \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$V \neq 0$

$$n_p(x_p) = N_D \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$
$$p_n(x_n) = N_A \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$